STABILITY AND BIFURCATIONS OF THE TWO-MASS MODEL OSCILLATION: ANALYSIS OF FLUID MECHANICS EFFECTS AND ACOUSTICAL LOADING

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ABSTRACT

We present some extensions to the results found by Lucero (1993) concerning the analysis of the large-amplitude oscillation of the vocal folds using the two-mass model. We focus on two points which were not considered in that work: the introduction of a more realistic model of the fluid mechanics aspects of the glottal flow, and the effects of the acoustical loading of the vocal tract. A numerical technique is presented for finding the equilibrium points and analysing their stability for generic aerodynamic and mechanical models of the vocal folds, including as well a representation of the acoustical impedance of the vocal tract. Our results confirm the interest of an analysis of stability of equilibrium points to obtain the oscillation regions of the vocal-folds, but also indicates to the need of better aerodynamic and acoustical models.

INTRODUCTION

Over the years, several researchers have been trying to quantify vocal fold vibration. One of the main question one is interested in answering is: Given a mechanical, aerodynamical, and acoustical model of the vocal folds and the vocal tract, under which conditions of the control parameters (e.g. lung pressure and stiffness of the laryngeal muscles) will the vocal folds oscillate? As even the simplest models of the vocal tract (e.g. Ishizaka and Flanagan 1972) are described by non-linear differential equations for both the mechanical and the aerodynamical parts, direct analytical analysis are difficult to be carried out. The difficulties are expected to increase as more realistic models of the larynx will be developed. This are the main reasons according to which previous works have been focused on small-amplitude analysis of vocal fold vibration (Titze 1988). The drawback of this kind of technique is the linearization of the equations of motion, making the conclusions hardly extensible to the large-amplitude oscillation behaviour.

More recently, some non-linear techniques have been applied to the study of vocal fold vibration. They range over a wide variety of mathematical tools. Awrejcewicz (1990) uses characteristic multipliers to change 'bifurcation' parameters in order to discover new periodic solutions via Hopf bifurcation. Weakly nonlinear analyses are done by Jensen (1990) to investigate the instability of the flow in a collapsed tube. Empirical orthogonal eigenfunctions are extracted from biomechanical simulations of the vocal folds by Berry et al. (1993); those authors show that chaotic oscillation can arise as a result of desynchronization of the low-order modes. Although those works represent a real progress with respect to the the former small-amplitude analysis, they lack the cleverness of fully analytical techniques.

In this respect, Lucero (1993) presented an analytically-based analysis of the large-amplitude oscillation of the vocal folds using the two-mass model. This technique consists, at first, in finding the equilibrium points of the dynamical equations of motion. As a second step, an analysis of stability is carried out, essentially by linearizing the system about those equilibrium points and by examining the sign of the real part of its characteristic equation. Although the results obtained were quite promising, the referred work was based on an oversimplified model of both the fluid mechanics aspects of the glottal flow and the geometry of the vocal folds. Furthermore, the coupling between the vocal fold oscillation and the acoustical loading of the vocal tract, as well as the effects of viscous resistances, were neglected.

The goal of the present study is twofold: first, we will redo the analysis of Lucero (1993) showing that some of his conclusions are due to the introduction of a 'spurious' element of the fluid mechanics. Second, we will apply a numerical version of the analysis of stability of the equilibrium point using a more realistic model of the glottal flow and including the effect of vocal tract loading.

ANALYSIS OF EQUILIBRIUM POSITIONS FOR THE TWO-MASS MODEL

We will proceed to a verification of the results of Lucero (1993) by eliminating the loss due to sharp edges (flow separation in vena-contracta effect; for more details see Pelorson et al. 1994). We will use the same notations as in Lucero (1993) and we ask the reader to refer to that paper for the meaning of the mathematical symbols. In the case of an open glottis ($x_1 > -x_{10}$ and $x_2 > -x_{20}$, where x_i and x_{i0} are the position and the rest positions for the masses 1 and 2), the driving force on the mass 1 is given by $F_1 = l_g d_1 P_S f_p$, where l_g is the width of the glottis, d_1 the length of mass 1 and P_S the sub-glottal pressure. The term f_p depends on the position of the masses and on a factor κ (see Ishizaka and Flanagan 1972) for sharp edges ($\kappa = 0.37$):

$$f_p = \frac{(x_1 + x_{10})^2 - (x_2 + x_{20})^2}{(x_1 + x_{10})^2 + \kappa(x_2 + x_{20})^2}.$$

As the contraction at the entrance of the vocal folds is smooth, we believe that there is no reason for the vena-contracta effect, κ having to be set to zero. In this case, equation (8) of Lucero (1993), obtained by setting the derivatives of the equations of motion to zero, becomes $(y_{1e} - 1) = H(1 - y_{2e}^2/(\beta^2 y_{1e}^2))$, where $\beta = x_{10}/x_{20}$, and H is a constant that depends on several parameters of the model, including the mass stiffnesses (k_1 and k_2). $y_{i0} = 1 + x_i/x_{i0}$ are the normalized mass displacements. The final solutions for $\beta = 1$ (rectangular prephonatory glottis) are (i) $y_{1e} = y_{2e} = 1$ (rest positions), and (ii) the solutions of the following equation

$$y_{1e}^2 + (1-\alpha)(1+\alpha)Hy_{1e} - (1-\alpha)^2 H = 0.$$

As H and $\alpha = k_c/(k_2 + k_c)$ are always positive, it is straightforward to prove that the roots of the above equations are always real and one of them is always negative. This invalidates the result of Lucero (1993), where there was possibility for the existence of three simultaneous equilibria. The main conclusion is that there will be always two equilibrium positions for any value of the command parameters of the model (the stiffnesses of the masses, related to α and the subglottal pressure).

NUMERICAL ANALYSIS OF STABILITY OF EQUILIBRIA

Lucero (1993) did an analysis of stability of the equilibrium points and found an analytical formulation for obtaining the bifurcation points and the regions of stability for the command parameters space. We extend the technique to a more realistic two-mass model (Pelorson et al. 1994) including the effects of moving flow separation point and viscosity losses, thanks to a more elaborated model of the changing geometry of the vocal folds. The acoustical loading at the outlet of the two-mass model is taken into account by modelling the input acoustical impedance of the vocal tract as a linear filter (plane wave propagation in the vocal tract is assumed). Hence the dynamical equations of motion for the two-mass model together with the dynamical

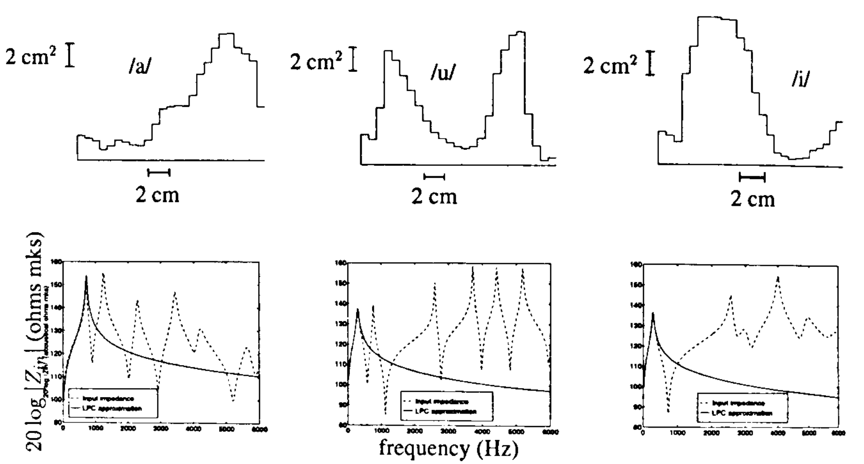

Figure 1: Area functions (top panel) and input impedances (bottom panel) for three French vowels: /a/ (left), /u/ (middle), and /i/ (right).

equation describing the input impedance filter compose the global equations of motion of the system.

More specifically, we approximated the effect of vocal-tract input impedance by a filter which takes into account only the first formant. Fig. 1 shows the area functions for three French vowels (/a/, /i/, and /u/) and the associated input impedances as a function of frequency $Z_{in}(f)$ computed from the area functions using the plane wave propagation hypothesis (dashed lines in Fig. 1). The poles and zeros of the vocal-tract impedance were computed from the impedance spectrum by a LPC approximation (solid lines in Fig. 1). We plan to include formants of higher order in a future work. By now, we are interested just in the effect of the first formant on vocal fold vibration and we believe that they will be more marked than the effect of the other formants.

The acoustical loading is modelled then by the pressure at the input of the vocal tract P_{CV}, which is the result of filtering the glottal flow U_g through the linear filter $Z_{in}(f)$. P_{CV} varies thus with time, perturbing the pressure difference across the glottis $P_S - P_{CV}$ (as a first approximation, P_S is considered independent of the glottal flow U_g in the present study). The whole model can be described by a set of augmented differential equations. The technique we used for finding equilibrium points and for analysing their stability consists essentially in linearizing the model about the equilibrium points using a perturbation analysis.

The system of non-linear differential equations can be compactly described using the notation:

$$\dot{u} = F(u),$$

where u are the state variables of both the mechanical and acoustical parts. About any equilibrium point $\overline{u}$, it is possible to linearize the system (see Guckenheimer and Holmes 1986 for details):

$$\dot{\xi} = D\xi$$

where $u = \overline{u} + \xi$ and D is the Jacobian of the function F about $\overline{u}$. Stability of the linearized system on state variables ξ depends on the eigenvalues of D. Specifically, in order to have a stable the system about the equilibrium point $\overline{u}$, the real part of all eigenvalues of D have to be negative.

We carried out this analysis varying two parameters of the model: P_S and k_1 (the stiffness of mass 1). The other free parameters were kept constant to typical values given in the literature (see Pelorson et al. 1994). Starting from the rest position for the masses, we found the equilibrium positions, which corresponded always to a convergent configuration

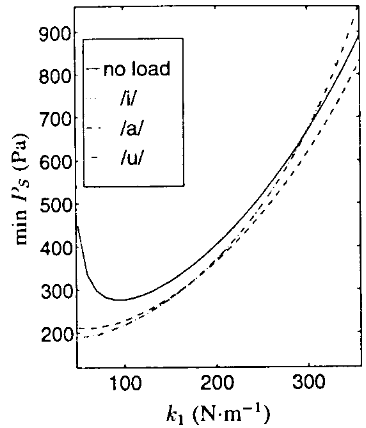

Figure 2: Minimum subglottal pressure for vocal fold oscillation as a function of the stiffness of mass 1.

for the vocal folds. For different values of k_1, we searched by a bisection algorithm the value of P_S in the boundary between the oscillating and non-oscillating regions. The results are shown in Fig. 2. It is possible to see that the region of stability is increased due to the acoustical loading of the vocal tract. The effect is more accentuated for the vowel /a/ and almost the same for vowels /i/ and /u/.

CONCLUSIONS

We extended the technique presented by Lucero (1993) for analysing the stability of equilibria of the vocal folds using a more realistic model of the flow through the glottis and including the effects of acoustical loading. We considered the analysis of the equilibrium points done by Lucero (1993) and we studied influence of some aerodynamical effects. We proposed a numerical technique for obtaining the stability of equilibria, being able to determine regions of larynx/lungs command space for which the vocal folds will oscillate. The main virtue of the proposed technique is the ability to determine the bifurcation boundaries in the control space (including vocal tract configuration) without having to run temporal simulations. Our results shows the importance of the aeroacoustics in such an analysis. We agree with Lucero (1993) that the next logical step would be the study of more realistic models. However we emphasize that in parallel to improvements on the mechanics, much has to be done concerning the aerodynamical and acoustical descriptions.

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