L.F.M. ten Bosch I.J. Bonder L.C.W. Pols

Institute of Phonetic Sciences, University of Amsterdam

## ABSTRACT

A phonetic/phonological model has been developed for describing the structure of natural vowel systems in terms of configurations consisting of $N$ points in the formant space. These configurations (abstract vowel systems) are defined as solutions of an optimalisation algorithm. This search algorithm uses an optimality strategy that is based upon two extralinguistic principles, one dealing with the articulatory effort, the other with perceptual ease. The model is evaluated by comparing the model results with available phonological data.

## INTRODUCTION

The model that we present is developed in order to find basic structure principles underlying the architecture of vowel systems. It uses as a starting-point the dispersion model of Liljencrants and Lindblom (1972). They tried to describe natural vowel systems by maximizing an acoustic distance measure between $N$ points, all of them positioned within a predefined fixed region in the formant space. The novelty of the present model is the extension of the acoustic principle (with respect to vowel dispersion only) with an articulatory minimal effort principle.

In the following three sections, we will gradually unfold the model. Section 1 poses the two basic structure principles we are using. Section 2 describes the model itself: 2.1 deals with the technical translation of the basic principles into an appropriate mathematical formulation and a search algorithm for the abstract vowel systems; 2.2 describes the comparison of these abstract systems with the vowel systems from natural languages; and 2.3 will briefly deal with the implementation of dynamic aspects of vowel systems: the long/short-opposition and the diphthongs. In section 3 we will give a summary of the present results. In section 4 we conclude with a discussion.

## 1. THE PRINCIPLES

We use two principles dealing with the structure of vowel systems which are supposed to be of primary importance:
(a) : minimality of effort of (static) vowel pronunciation;
(b) : minimality of inter-vowel confusion.

Vowel systems are said to be 'optimal' if they optimally satisfy both principles simultaneously.
Evidently, the consequences of these principles separately are conflicting: (a) yields minimal overall articulatory vowel distances, whereas (b) leads to maximal inter-vowel distances. In order to be able to handle both principles in an appropriate way, they have been translated into specific mathematical formulae. Some of these formulae directly deal with both the formant position of vowels and the vocal tract area function, other ones are based upon arguments concerning probability and optimalisation techniques (see section 2.1, the search algorithm).

## 2. THE MODEL

### 2.1. The Search Algorithm

Each vowel system is represented as a point in a so-called 'state space', in which principles (a) and (b) define an optimality strategy. The search for optimal vowel systems can be considered as looking for stable solutions in this state space. In order to specify the search algorithm, we introduce the following formuiae (classified into basic, derived and evaluational ones) :
2.1.1. basic formulae

These formulae play the most elementary role in the model.

The inter-vowel acoustic distance dr between $v 1$ and $v 2$ is defined as follows:
$(d F)^{2}=\left(\log \left(F_{1}(v 1)\right)-\log \left(F_{1}(v 2)\right)\right)^{2}+$
$\left(\log \left(F_{2}(v 1)\right)-\log \left(F_{2}(v 2)\right)\right)^{2}$

Only the relative positions of vovels in a vowel system are relevant. The logarithms the perceptual behaviour of the basilar membrane. This closely relates dF to empirically determined acoustic distan
measures involving mel or bark scales.

The expression for the inter-vowel confu-
sion probability p(v1, v2) reads: sion probability $\mathrm{p}(\mathrm{v} 1, \mathrm{v} 2)$ reads:
$p(v 1, v 2)=\exp (-\alpha * d F(v 1, v 2))$
$\alpha$ being a positive scaling parameter. Before actually evaluating vowel systems
we first introduce the following probwe first introduce the following prob-
abilistic concept. We hypothesize an exponential relation between the inter-vowe
confusion probability $p$ and the confusion probability $p$ and the inter-
vowel acoustic distance dF. This relation
can be can be globally verified by inspecting the perceptual vowel
several languages.
We define the articulatory effort dA :
$\mathrm{CA}=\Sigma(\mathrm{Si}-1)^{2} \quad(\mathrm{i}=1, \ldots, 4) \quad$ (3)
This expression relates the shape of the vocal tract (which is approximated by the
straight 4 -tube, consisting of 4 segment
 figure 1).
2.1.2. derived system formulae

In order to be able to define the structure principle for vowel systems as a
whole, we introduce the system counterarts of dA and dF .

The expression for the total articulatory
system effort DA reads:
$\mathrm{DA}=\max (\mathrm{dA})$
(4)

The articulatory effort value of a vowel system is defined as the maximal value of
the articulatory
offort values of $i t s$ members.


Fig 1. An example of a general $n$-tube with
segment areas Si .

The ictal perceptual system discriminality
DF will be $\left.\left.D_{F}=\prod_{\left(1-p\left(v_{i},\right.\right.} v_{j}\right)\right) \quad(1 \leqslant i<j \leqslant N)$

1-p(v1, v2) denotes the probability of
vowel v1 and vowel v2 not being mutually confused. Therefore DF is a measure for system. Consequently we have $\mathrm{DF}=1$-vowel case of perfect discriminality and $D F=0$ 2.1.3. evaluation formulae
we have to minimize We have to minimize the articulatory lity measure DF simultaneously. Therefor we introduce the penalty parameter $Q$ rela-
ting both aspects:

- (Da) $2+{ }^{2}$
$Q=(D A)^{2}+S *(D F-1)^{2}$
(6)

This type of expressions is well-known
from optimality theory and is in fact a natural choice here. Indeed, minimization of $Q$ logically implies minimization of $D$ towards zero and optimization of DF to convergence of this process is controlled by the slack variable s (s being a large positive number). Optimal vowel systems
are locally found by iteratively improving are locally found by iteratively improving
the position of all vowels in the systen while decreasing the value of $Q$.
2.2. Evaluation Part

The evaluation part of the algorithm des cribed above in fact consists of a meas urement of the goodness of fit of the more phonologically specified data fro language databases ([3], [4]). For the time being we confine the evaluation to
vowel systems without dynamic structure (without short/long opposition, without diphthongs). Presently, these latter effects contribute less to a general insight In the model a method is implemented fo actually effectuating the phonetic/phonological comparison. It is based upon essentially the same probabilistic motiva
tions as already used in formula (5). Th result of the comparison is expressed in terms of the similarity probability (de-
noted SP) of the respective abstract pho netic vowel system and a phonological sys
nem tem after having optimally paired each un
labelled $v_{i}$ in the model system with abelled $v_{i}$ in the model system with a system. $v_{j}$ in the phonological reference
$S P=\Pi \exp \left(-\alpha * d\left(v_{i}, v_{j}\right)\right)$
(7)

If $\mathrm{SP}=1$, the similarity is perfect. The
model evaluation now consists of the emodel evaluation now consists of the e-
valuation of all $S P$ values between a model solution containing N vowels and all known phonological $N$-vowel systems. The present result of
figure 2.

 Fig. 2 . Goodness of fit of the present
model in terms of the Sp value.
The heavy line a connects all the found maxima, b shows some possible ramifications. N denotes the number of
in the model system and the phono logical reference system.
one observes the decreasing SP value
for increasing values for increasing values of N. Probably
this phenomenon can be traced to the declining fit of the model itself
the increasing number of linguistic the increasing number of
possibilities for large

> 2.3. Dynamics The description of the dynamic part of voel systems appears to involve more linguistic details then are contained in the model described above. The model has
proved to proved to be inadequate for predicting
actual diphtongs and long vowels in a specific language, but it merely defines
and bounds the set of physical possibiand bound the set of physical possibiInties out of which a language may select.
$\mathrm{I}_{\text {order }}$ ordo study these possibilities in more detail we use a vowel structure ma-
trix of which the entries represent the trix of which the entries represent the ong vowels and diphthongs. The short
vowels constitute the elements along the wo axes. Evidently, long vowels emerge as geminates along the main diagonal and
diphthongs off the diagonal. In order to diphthongs off the diagonal. In order the acoustic gain relative to the articulatory
effort. We give the results of such a cal effort. We give the results of such a cal
culation in figure 3. One may observe preference for diphthongs to start in the a/-region (i.c. to show decreasing first la/-region (i.c. to
formant frequency).


Fig 3. Gain of acoustic contrast in ffort. The transitions are now desribed as concatenations of two short vowels out of the indicated set of four short vowels. Horizontally, we an vertically the short vowels in final position are shown. All entries quotients of acoustic contrast and rticulatory effort) have been They give an indication of the pref rence of the corresponding combi-
ration of short vowels. nation of short vowels. erence for transitions to start in the a-like region of the formant space is demonstrated by the values figuring
in the first column relative to those in the first column . The overallpreference for gemination can be de duced from the values along the
diagonal. In general it does not have diagonal. In general it does not have correspond to actual long vowels such
like /a/, /e/ etc. This identificalike is in fact a phonological item. The quotients have been specified up to only one decimal place in order They only have relative significance.
3. RESULTS OF THE MODEL

In the figures 4, 5 and 6 we give the preand 7 respectively. The closed contours represent contour lines of the articuatory effort function dA. One observes:

- the preference for the vowel /a/,
the preference for
followed by $/ i /$ and $/ \mathrm{u} /$;
- the preference for vowels along the lines $/ a /-/ i /$ and $/ a /-/ u /$;
the limitation of the available vowel
space without predefining
boundary in the formant space.

| F 2 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 kHz |
| :--- | :--- | :--- | :--- | :--- | :--- |





Figures 4, 5 and 6 show the model solution in the formant space. For reference the grey area indicate the region which is used by most languages. The staight line denotes the line $\mathrm{F} 1=$ F2. The other two lines are contour lines of the articulatory effort function dA, which gives an idea of the theoretically shaped vowel space by using an effort principle (see the text). In case of the 7 -vowel system, some of the vowels are positioned outside the grey area, as a consequence of the subtile imperfection of the balance between the two principles (a) and (b) (see the text).

## 4. DISCUSSION

In our project, we explicitly deal with the model in relation to other recent vowel dispersion theories as well as with recent improvements. The present results have led to the following two suppositions:
a) natural vowel systems may adequately be considered as derivations of specific 'abstract' vowel systems, while
b) the structure of these abstract vowel systems is defined by two extra-linguistic principles:

- reduction of perceptual vowel confusion probability and
- reduction of articulatory effort.

The present model certainly does not pretend to be the final answer to the question of the structure of vowel systems in general but it may stimulate a further fundamental approach to the subject. In our presentation we will briefly mention some of the parallels with recent phonological theories, e.g. [5]. Our model does not predict all linguistic details of vowel systems as it is not based upon such linguistic or other language-sensitive principles. However, some important tendencies are clearly demonstrable: tendencies in the appearance and behaviour of vowel systems are described by combining a few, indeed simple arguments concerning articulation and perception. The main question will be the search for a convincing theory relating vowel systems as they are actually observed on the one hand to the results of a stipulative or normative model at the other hand.

## ACKNOWLEDGEMENTS

This project is sponsored by the Netherlands Crganization for the Advancement of Pure Research zwo (project no. 300-161030).

## REFERENCES

[1] Dunn, H.K. (1950). The calculation of vowel resonances, and an electrical vocal tract. J. Acoust. Soc. Amer., vol. 22., pp. 740-753.
[2] Bonder, L.J. (1983). The n-tube formula and some of its consequences. Acustica vol. 52, pp. 216-226.
[3] Crothers, J. (1978). Typology and universals of vowel systems. In: Universals of human language. Ed. J.H. Greenberg. Vol. 2. Stanford University Press, Calif., pp. 93-152.
[4] Maddieson, I. (1984). Patterns of sounds. Cambridge studies in speech science and communication. Cambridge University Press.
[5] Kaye, J., Lowenstamm, J. and Vergnaud, J.-R. (1985). Vowel systems. Paper presented at the 1985 GLOW colloquium, Brussels.

