Fundamental frequency recording represents an area in which the interests of phonetics join with those of musical acoustics. However, a quantitative difference between these two aspects still remains being affected, in general, by the well-known principle of uncertainty (1, 2): while considerably higher time resolution, of the order of few milliseconds, is mostly required in speech research, in musicology, on the other hand, an extreme frequency resolution is primarily claimed.

It is hoped that some comment concerning high-accuracy frequency recording, given by a musicologist, may perhaps be of interest to phoneticians.

In speech intonation research, as well as, for example, in ethnomusicology, a durable analog record of fundamental frequency as a function of time proved to be a most valuable aid. Disregarding for the present purpose the spectral, autocorrelation, and digital methods we have at disposal two classes of simple analog means: first, integration methods which, however, are inferior in recording rapid changes of instantaneous frequency; second, timing methods, first described by Grützmacher and Lottermoser (3), which make use of particular timing waveforms to determine the duration of each fundamental period.

It is the logarithmic frequency scale that has been found most advantageous in several respects; one additional will be given later. Apparently, in the field of our interest, only the methods yielding the logarithm of the instantaneous value of the input quantity are to be taken into account.

I shall now briefly discuss three basic methods for obtaining the logarithmic display at the output of a frequency recorder.

(a) application of a logarithmic four-terminal network with inherent logarithmic characteristic (4, 5)
(b) approximation by a piecewise linear (polygonal) function (6, 7)
(c) approximation by a linear combination of exponential functions (8, 9)

(a) Logarithmic networks are based on more or less accurate logarithmic relation between two physical quantities in vacuum tubes or semiconductors. The former can
hardly be exploited economically in the problem under consideration, due to large
drift. The latter are easier to use, despite of the undesired temperature dependence.
There were many semiconductor logarithmic networks described in the literature,
covering often as much as seven decades of the input quantity with an accuracy
of several per cent. For the present purpose one decade would be far enough but
a higher accuracy is required. I would like to present a very simple circuit giving
good logarithmic response over two octaves with unselected general-purpose ger-
nium diodes. The average error of approximation was estimated to be about
one half per cent.

\[
\begin{align*}
V_i & \rightarrow \text{input voltage from an instantaneous frequency meter based on (3)} \\
V_o & \rightarrow \text{logarithmic output fed to a differential amplifier with high input impedance} \\
V_b & \rightarrow \text{fixed bias voltage. Instead of one, several diodes are connected in series to increase } V_b.
\end{align*}
\]

Fig. 1. Simple quasilogarithmic network transforms the initial, approximately linear, portion
of an exponential function into logarithmic function within the range from \(f\) to \(4f\). \(V_i\) — input
voltage from an instantaneous frequency meter based on (3), \(V_o\) — logarithmic output fed to
a differential amplifier with high input impedance, \(V_b\) — fixed bias voltage. Instead of one, several
diodes are connected in series to increase \(V_b\).

(b) Piècewise linear approximation based on nonlinear voltage dividers known
from diode function generators is to be preferred when large frequency range (up to
two decades) is needed along with relatively suppressed claims to accuracy. In
high-precision instruments this technique tends to become somewhat complicated.
It is often employed in biological research, for example, in cardiotachometers and
in pulse rate meters for electromyography, usually approximating the hyperbolic
function (10, 11).

(c) Third fruitful approach to the problem is in approximating the logarithmic
function by a linear combination of exponential functions with different decay
constants which are available with high accuracy and reproducibility by charging
capacitors in simple RC circuits connected to a common output. Very good results
were obtained in wide frequency range instruments.

On the other hand, it may be shown that in two-octave range the improvement
resulting from application of two different time constants, as compared to a single
RC circuit, is not substantial. Approximation by a single exponential curve (3, 12, 13)
is, of course, only very rough for ranges greater than, say, sixteen semitones. A phone-
tician and a musicologist would naturally welcome very large frequency ranges
operated without switching, even more than two octaves. However, the errors in
measurement due to the limited accuracy of analog display will generally be expressed
as a constant fraction of the full scale deflection. The experimenter is thus forced
to reduce reasonably the display range. A compromise must be made in order to
improve the accuracy without sacrificing a sufficiently wide range. (The instru-
ments are almost always designed as multirange ones.)

Fortunately, as anticipated above, the logarithmic frequency scale has an extra
advantage with regard to the problem just discussed. I would like to suggest that
one-octave range could be entirely satisfying since melodies exceeding that interval
may simply be recorded by repetition employing successively all contiguous one-
 octave bands occupied by the melody as a whole. Finished records are then properly
combined (fastened, glued) in parallel with respect to the time axis so that they
form a composite record having just the range needed by the melody but, of course,
with the accuracy valid for one-octave range.

Then, one single exponential function will be sufficient for the high-accuracy
approximation. The calculated nonlinearity of the semitone scale does not exceed
5 cents in terms of musical intervals or, in other words, maximum error amounts
approximately 0.4% of the full scale deflection, assuming that the time constant
equalled the reciprocal of the center frequency of the range.

\[
\begin{align*}
\text{Fig. 2. Departure from linearity of the semitone scale obtained by an exponential approximation} \\
\text{with the time constant equal to the period of the center frequency. Horizontal axis: log frequency} \\
\text{in semitones re center frequency; vertical axis: deviation of the actual scale from a linear one} \\
\text{in semitones. This may be taken into account when calibrating the instrument or, in one-octave} \\
\text{range, it may be neglected being less than } \pm0.5\text{C.}
\end{align*}
\]

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REFERENCES