Linguistic Inference and Textual Entailment

Modelling Inference
through Logical Deduction
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Levels of Logical Method

- Model-theoretic interpretation
- Deduction calculi
- Deduction procedures
- Implemented deduction systems

Model-theoretic Interpretation

- Formula A is true in the model structure M iff $[[A]]^{M,g} = 1$ for every variable assignment g.
- A model structure M satisfies a set of formulas Γ (or: M is a model of Γ) iff every formula $A \in \Gamma$ is true in M.

Central semantic concepts

- A formula A is valid ($\models A$) iff A is true in every model structure.
- A set of formulas Γ entails formula A ($\Gamma \models A$) iff A is true in in every model of Γ (i.e., in every model structure that satisfies Γ).
- A set of formulas Γ is satisfiable iff Γ has a model (i.e., there is a model structure that satisfies Γ).

Important Theorems

… actually, metatheorems (see below):

- Validity and entailment:
  A |=B iff |= A → B, more general:
  $\{A_1, \ldots, A_n\} \models B$ iff $\models A_1 \wedge \ldots \wedge A_n \rightarrow B$

- Entailment and satisfiability:
  $\Gamma \models A$ iff $\Gamma \cup \{\neg A\}$ is unsatisfiable.

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Deduction Calculi

- Computing entailment and other logical concepts through semantic interpretation inefficient and in many cases infeasible.
- Deduction calculi (or proof theoretic systems) provide a strictly syntactic way of checking logical concepts and relations, through symbol manipulation/ rewrite of logical formulas.

Axioms and Deduction Rules

- Deduction calculi are typically made up of (1) axioms and (2) deduction rules.
- Example for a frequently used axiom:
  - $A \vee \neg A$ (“Tertium non datur”)
- Example for a frequently used deduction rule (“Modus Ponens”)

$$\frac{A \rightarrow B,\ A}{B}$$

Semantic and Deductive Concepts

- There is a correspondence between basic semantic and deductive/ proof-theoretic concepts:

Validity	Provability
Entailment	Derivability/Deducibility
Satisfiability	Consistency

Central proof-theoretic concepts

- Formula A is derivable (deducible) from a set of formulas Γ (Γ |- A) iff there is a sequence of formulas $A_1, \ldots, A_n$ such that A_n = A and for all members A_i of the sequence: either
  - A_i is an (instantiation of an) axiom, or
  - A_i Γ, or
  - A_i is the result of the application of a deduction rule, whose conclusion is A_i ,and whose premisses all occur in the sequence before A_i
- A formula A is provable (|- A) iff ∅ |- A
- A set of formulas Γ is inconsistent iff there is a formula A such that Γ|- A and Γ|- ¬A
- A set of formulas Γ is consistent iff it is not inconsistent.

Soundness and Completeness

- Soundness: If Γ |- A, then Γ |- A.
- Completeness: If Γ |- A, then Γ |- A.

Important Metatheorems

- Derivability and Provability:
  $\{A_1, \ldots, A_n\}$ |- B iff |- $A_1 \wedge \ldots \wedge A_n \rightarrow B$
- Derivability and Consistency:
  Γ |= A iff Γ∪{¬A} is inconsistent.
- Validity and Provability:
  |= A iff |- A
- Satisfiability and Consistency:
  Γ Is satisfiable iff Γ is consistent.

Deduction Calculi

- There is one model-theoretic interpretation (for standard predicate logic).
- There is a wide variety of deduction calculi, e.g.:
  - Hilbert calculus
  - Semantic tableau calculus
  - Calculus of natural deduction (Gentzen calculus)
  - Resolution

Semantic Tableau Calculus

- Derivation and proofs through the generation of tableau trees via decomposition rules.
- Semantic Tableaus use rewrite on formulas, so it is a deduction calculus.
- They are called "semantic tableaus" because there is an affinity to semantics.

Semantic Tableau Rules

	Affirmed	Negated
$A \wedge B$	{A, B}	{¬A}, {¬B}
$A \vee B$	{A}, {B}	{¬A, ¬B}
$A \rightarrow B$	{¬A}, {B}	{A, ¬B}
$A \leftrightarrow B$	{A →B, B →A}	{¬(A →B)}, {¬(B →A)}
$\forall xA$	A[a/x] for arbitrary a	¬A[a/x] for a new a
$\exists xA$	A[a/x] for a new a	¬A[a/x] for arbitrary a

Semantic Tableau Calculus

- A subtableau is closed, iff it contains A and ¬A
- A tableau is closed iff all subtableaus are closed.
- $\Gamma \models A$ iff the decomposition rules result in a closed tableau for $\Gamma \cup \{\neg A\}$.

Refutation proof: To prove A from premisses Γ, add its negation and show that the result is inconsistent.

Deduction procedures

- Deduction procedure = deduction calculus + algorithm

Tractability:

- Propositional calculus is NP-complete (it requires exponential time)
- FOL is undecidable (provable /valid formulas are recursively enumerable)
- To arrive at efficient systems, heuristic knowledge and a lot of fine-tuning is required.

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Implemented deduction systems

We distinguish:

- Theorem provers, typically with
  - Refutation proofs
  - Resolution proof procedure
  - Input: Set of formulas (premisses)
  - Output: Yes, if proof sucessful.
  - Examples: Vampire, SPASS, BLIKSEM, OTTER
- Interactive theorem provers ("proof assistants")
  - Provide information about proof steps
  - Ask for guidance
  - Are typically based on mor intuitive calculi (e.g. Gentzen calculus)
  - Example: OMEGA
- Model generators
  - Check consistency
  - Using tableau techniques
  - Output is Yes, if the hypothesis is consistent with the premisses
  - Plus a model for $\Gamma \cup \{A\}$ as an important side effect.
  - Examples: MACE, KIMBA

Problems: Efficiency

- Combination of Theorem Provers (and Model Builders), Distributed Theorem Proving
- Optimization for specific tasks (e.g., mathematical vs. linguistic applications)
- Restriction to a FOL fragment, with
  - Horn Clause Logic (Prolog) and
  - Description Logics (e.g., RACER) as prominent examples

Problems: Resources& Coverage

- Required input is logical formulas
- Available linguistic input is text
  - Grammatical analysis, semantic construction
  - Disambiguation, Underspecification
  - Discourse analysis (e.g., coreference resolution)
- Additional input required comprises
  - Lexical semantic information (e.g., WordNet, FrameNet)
  - Extralinguistic Knowledge