Semantic Theory
Week 10 – Distributional Formal Semantics

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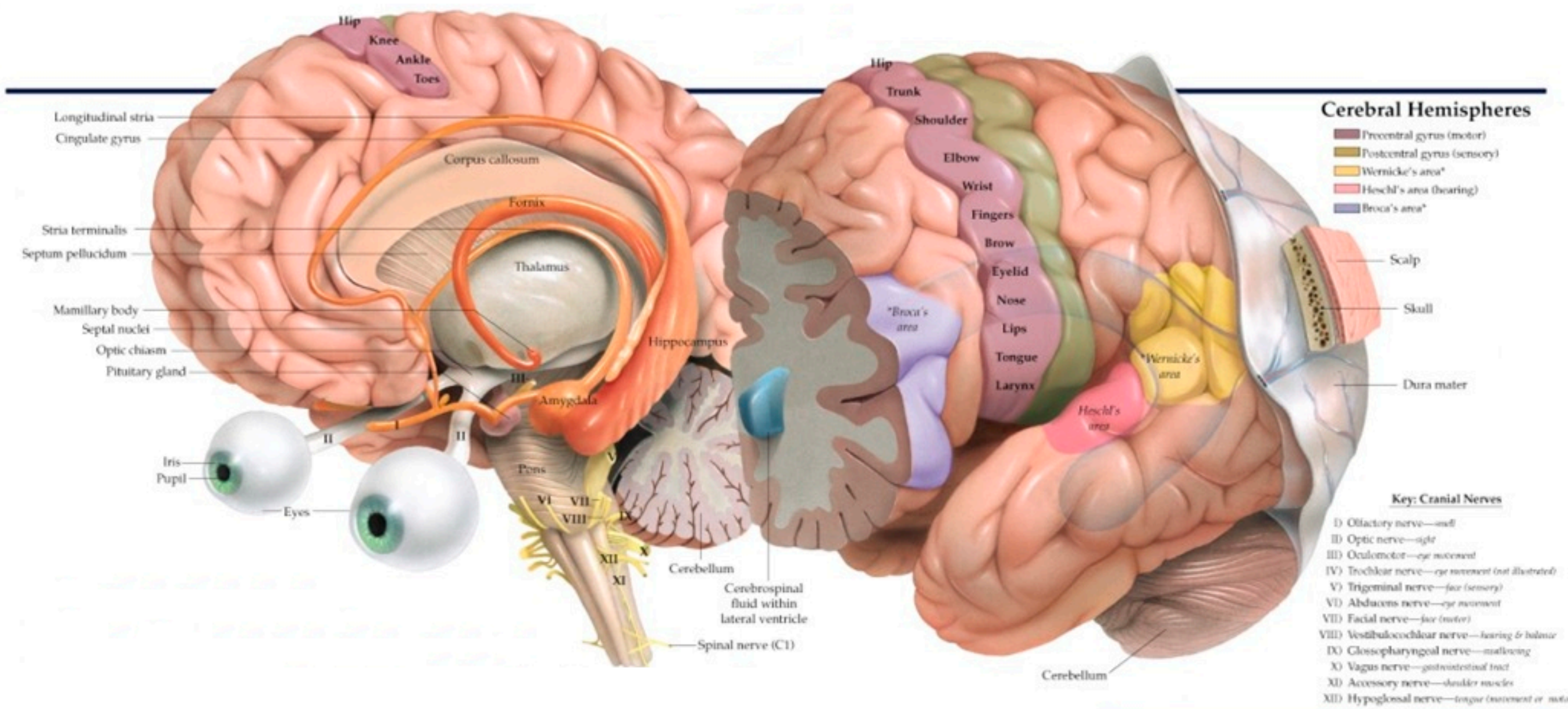
Summer 2022

The Greatest Semanticist of them all …



> Our **language comprehension system** is highly effective and accurate at attributing meaning to unfolding linguistic signal (~word-by-word)

>> This system's **representations** and **computational principles** are implemented in the **neural hardware** of the brain

>>> We should understand meaning construction and representation in terms of "brain-style computation"

A shopping list

Neural Plausibility: assumed representations and computational principles should be implementable at the neural level [cf. Rumelhart, 1989]

Expressivity: representations should capture necessary dimensions of meaning, such as negation, quantification, and modality [cf. Frege, 1892]

Compositionality: the meaning of complex expressions should be derivable from the meaning of its parts [cf. Partee, 1984]

Gradedness: meaning representations are probabilistic, rather than discrete in nature [cf. Spivey, 2008]

Inferential: The derivation of utterance meaning entails (direct) inferences that go beyond literal propositional content [cf. Johnson-Laird, 1983]

Incrementality: As natural language unfolds over time, representations should allow for incremental construction [cf. Tanenhaus et al., 1995]

Brouwer, Crocker & Venhuizen (2017)
In: *From Semantics to Dialectometry*

Distributional Semantics

"How much do we know at any time? Much more, or so I believe, than we know we know!"

— Agatha Christie, *The Moving Finger* (1942)

"You shall know a word by the company it keeps"

— J. R. Firth (1957)

Psychological Review
1997, Vol. 104, No. 2, 211–240

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0033-295X/97/$3.00

A Solution to Plato's Problem: The Latent Semantic Analysis Theory of Acquisition, Induction, and Representation of Knowledge

Thomas K Landauer
University of Colorado at Boulder

Susan T. Dumais
Bellcore

Distributional Semantics (cont'd)

How much wood would a woodchuck chuck ,
if a woodchuck could chuck wood ?
As much wood as a woodchuck would ,
if a woodchuck could chuck wood .

	a	as	chuck	could	how	if	much	wood	woodch.	would	,	.	?
a	0	5	9	6	1	10	4	8	18	9	10	0	0
as	5	4	2	1	0	0	7	10	3	2	1	0	5
chuck	9	2	0	8	0	5	1	9	11	2	4	3	3
could	6	1	8	0	0	4	0	6	8	0	2	2	2
how	1	0	0	0	0	0	4	3	0	2	0	0	0
if	10	0	5	4	0	0	0	0	10	3	8	0	0
much	4	7	1	0	4	0	0	10	2	3	0	0	3
wood	8	10	9	6	3	0	10	2	8	5	0	4	6
woodch.	18	3	11	8	0	10	2	8	0	8	10	1	1
would	9	2	2	0	2	3	3	5	8	0	5	0	0
,	10	1	4	2	0	8	0	0	10	5	0	0	0
.	0	0	3	2	0	0	0	4	1	0	0	0	0
?	0	5	3	2	0	0	3	6	1	0	0	0	0

(4-word ramped window: 1 2 3 4 [0] 4 3 2 1)

Rohde et al. (under revision)
Cogn. Sci.

Distributional Semantics (cont'd)

$$\text{similarity} = \cos(\theta) = \frac{\mathbf{A} \cdot \mathbf{B}}{\|\mathbf{A}\| \|\mathbf{B}\|} = \frac{\sum_{i=1}^{n} A_i B_i}{\sqrt{\sum_{i=1}^{n} A_i^2} \sqrt{\sum_{i=1}^{n} B_i^2}}$$

Ranging from dissimilar (0) to similar (1) — e.g., similarity(wood, woodchuck) = .6

> **Neurally plausible** and **Graded** lexical representations

> But what about **Compositionality**, **Expressivity** and **Inference**?

Queen = King - Man?

X is queen ⊨ X is not a man

X is not a queen = ???

Some queens are rich = ???

→ Distributional Semantics lacks the logical capacity of Formal Semantics

(but is still highly suitable for modelling lexical semantic memory!)

Distributional Formal Semantics

Noortje Venhuizen
Petra Hendriks
Matthew Crocker
Harm Brouwer

NATURAL LANGUAGE SEMANTICS

Model-theoretic Semantics

- Truth-conditional meaning
- Logical entailment
- Compositionality

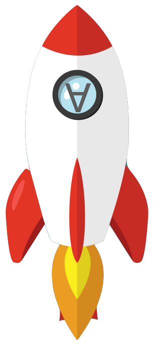
?

Distributional Semantics

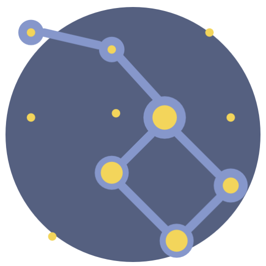
- Semantic similarity
- Empirically driven
- Cognitively inspired

E.g., Baroni et al. (2010,2014); Boleda & Herbelot (2016); Coecke et al. (2010); Grefenstette & Sadrzadeh (2011); Socher et al. (2012)

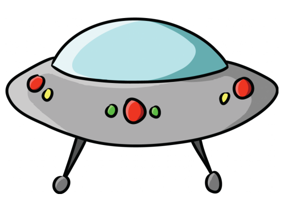
A FRAMEWORK FOR DISTRIBUTIONAL FORMAL SEMANTICS



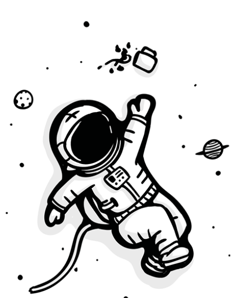
A meaning space for Distributional Formal Semantics



Formal properties of the meaning space

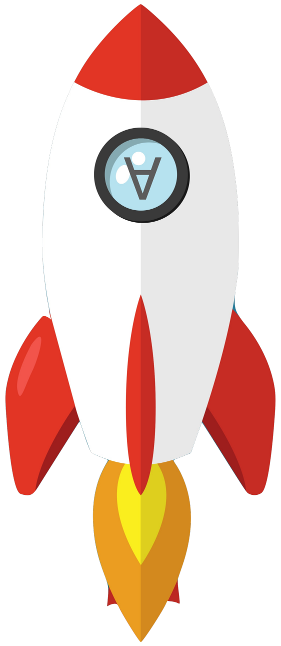


Incremental meaning construction



Semantic processing in the meaning space

A MEANING SPACE FOR DISTRIBUTIONAL FORMAL SEMANTICS



FROM MODELS TO MEANING SPACE

$M_1 = \langle U_1, V_1 \rangle$

$p_1 \wedge \neg p_2 \wedge p_3 \wedge \ldots$

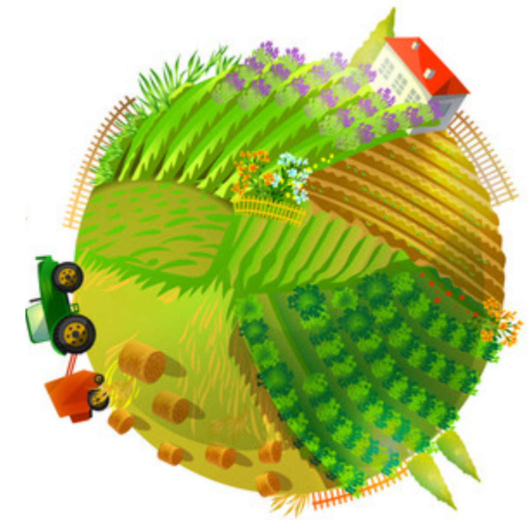
$M_2 = \langle U_2, V_2 \rangle$

$\neg p_1 \wedge p_2 \wedge p_3 \wedge \ldots$

$M_3 = \langle U_3, V_3 \rangle$

$\neg p_1 \wedge p_2 \wedge \neg p_3 \wedge \ldots$

...



$M_n = \langle U_n, V_n \rangle$

$\neg p_1 \wedge \neg p_2 \wedge \neg p_3 \wedge \ldots$

- The set of models $\mathcal{M}_{\mathcal{P}}$ — describing states-of-affairs over propositions in $\mathcal{P}$ — defines a meaning space
- Propositional meaning defined by co-occurrence across models

Venhuizen, Hendriks, Crocker, Brouwer (2019, WoLLIC; 2022, Inf. Comp)

CAPTURING THE STRUCTURE OF THE WORLD

"A boy rides a bike"

Boy is (likely) outside

Boy is not asleep

If it's evening, the light is on

The bike has wheels

etc.

World knowledge restricts propositional co-occurrence in the meaning space derived from the set of models $\mathcal{M}_{\mathcal{P}}$

- **Hard** world knowledge constraints restrict individual models
- **Probabilistic** constraints define probabilistic co-occurrences across the set of models $\mathcal{M}_{\mathcal{P}}$

Venhuizen, Hendriks, Crocker, Brouwer (2019, WoLLIC; 2022, Inf. Comp)

DFS MEANING SPACE $S_{\mathcal{M}\times\mathcal{P}}$

propositional meaning vectors (columns) / *formal models* (rows)

	p_1	p_2	p_3	p_4	…
M_1	1	1	0	0	…
M_2	1	0	0	1	…
M_3	0	1	0	1	…
M_4	1	1	1	1	…
M_5	0	1	0	0	…
…	…	…	…	…	…

$$[\![p_j]\!]^{\mathcal{M}} := v(p_j)$$

$$\text{where: } v_i(p_j) = 1 \text{ iff } M_i \vDash p_j$$

- **Incremental inference-based probabilistic sampling:** Based on a set of propositions $\mathcal{P}$, we sample a set of models $\mathcal{M}_{\mathcal{P}}$—taking into account hard and probabilistic world knowledge constraints
- **Co-occurrence defines meaning:** Propositions with related meanings are true in many of the same models, resulting in similar meaning vectors

Venhuizen, Hendriks, Crocker, Brouwer (2019, WoLLIC; 2022, Inf. Comp)

THE DISTRIBUTIONAL HYPOTHESIS REVISITED

> You shall know a word
> by the company it keeps

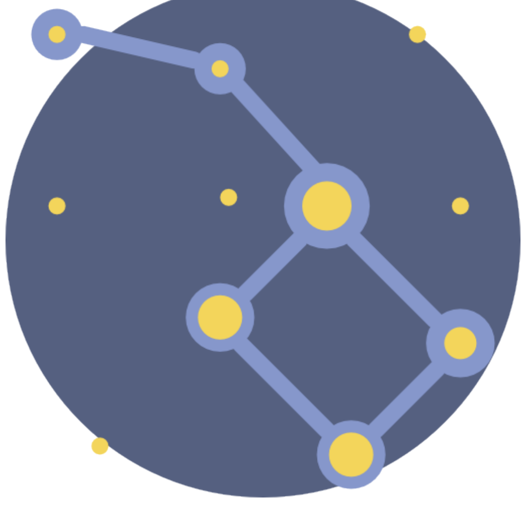
- J. R. Firth (1957)

THE DISTRIBUTIONAL HYPOTHESIS REVISITED

> You shall know a ~~word~~ *proposition*
> by the company it keeps

- J. R. Firth (1957)

FORMAL PROPERTIES OF THE MEANING SPACE



MEANING VECTOR COMPOSITION

Meaning vectors can be combined to define compositional meanings

- Standard logical operators interpreted as in model-theory

$$v_i(\neg p) = 1 \quad \textit{iff} \quad M_i \nvDash p$$

$$v_i(p \wedge q) = 1 \quad \textit{iff} \quad M_i \vDash p \textit{ and } M_i \vDash q$$

… *etc.*

- Quantification is defined relative to the combined universe of $\mathcal{M}_{\mathcal{P}}$: $\mathcal{U}_{\mathcal{M}} = \{e_{1} \ldots e_{m}\}$ (thereby preserving entailment in $\mathcal{M}_{\mathcal{P}}$)

$$v_i(\forall x \varphi) = 1 \quad \textit{iff} \quad M_i \vDash \varphi[x \backslash e_1] \wedge \ldots \wedge \varphi[x \backslash e_m]$$

$$v_i(\exists x \varphi) = 1 \quad \textit{iff} \quad M_i \vDash \varphi[x \backslash e_1] \vee \ldots \vee \varphi[x \backslash e_m]$$

Venhuizen, Hendriks, Crocker, Brouwer (2019, WoLLIC; 2022, Inf. Comp)

PROBABILITIES IN THE MEANING SPACE

All (sub-)propositional meaning vectors inherently encode (co-)occurrence probabilities

- Prior probability of meaning vector *a*

$$P(a) = \frac{1}{|\mathcal{M}|} \sum_i \vec{v}_i(a)$$

- Conjunction probability between *a* and *b*

$$P(a \wedge b) = \frac{1}{|\mathcal{M}|} \sum_i \vec{v}_i(a) \vec{v}_i(b)$$

- Conditional probability of *a* given *b*

$$P(a|b) = \frac{P(a \wedge b)}{P(b)}$$

	p_1	p_2	p_3	p_4	
M_1	1	1	0	0	...
M_2	1	0	0	1	...
M_3	0	1	0	1	...
M_4	1	1	1	1	...
	0	1	0	0	...
	...	...	...	...	...

Venhuizen, Hendriks, Crocker, Brouwer (2019, WoLLIC; 2022, Inf. Comp)

QUANTIFYING PROBABILISTIC INFERENCE

Probabilistic logical inference of meaning vector *a* given *b*

$$inference(a,b) = \begin{cases} [P(a|b) - P(a)] / [1 - P(a)] & \text{if } P(a|b) > P(a) \\ [P(a|b) - P(a)] / P(a) & \text{otherwise} \end{cases}$$

- $P(a|b) > P(a)$: Positive inference (b increases probability of a)

$$inference(a,b) = 1 \Leftrightarrow b \vDash a$$

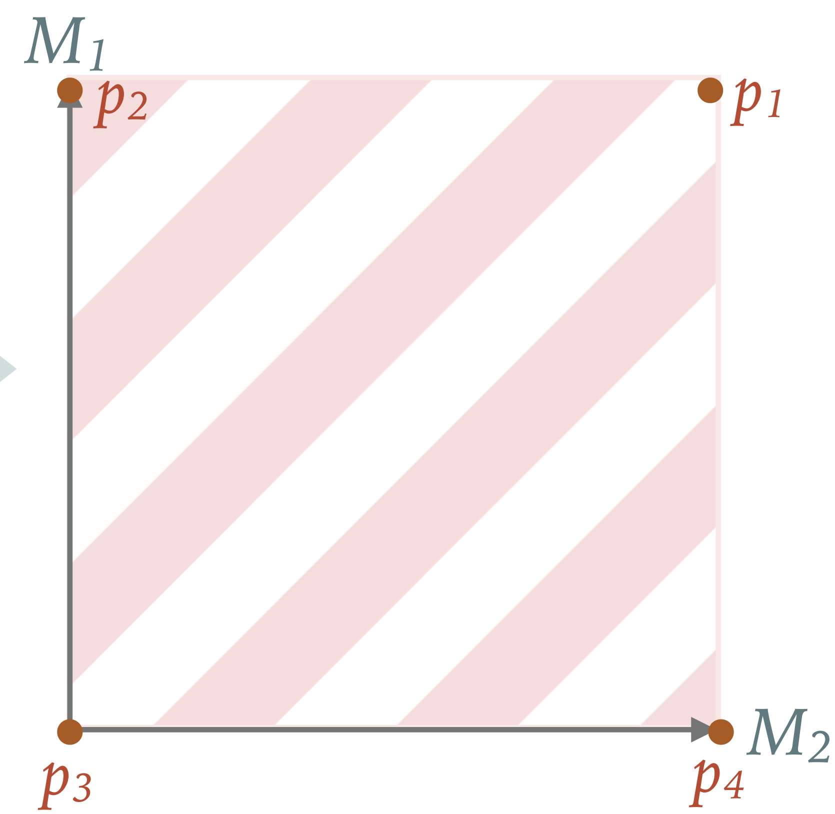
- $P(a|b) \leq P(a)$: Negative inference (b decreases probability of a)

$$inference(a,b) = -1 \Leftrightarrow b \vDash \neg a$$

Frank et al. (2009,Cognition)

CONTINUOUS NATURE OF THE MEANING SPACE

	p_1	p_2	p_3	p_4	
M_1	1	1	0	0	…
M_2	1	0	0	1	…
M_3	0	1	0	1	…
M_4	1	1	1	1	…
M_5	0	1	0	0	…
	…	…	…	…	…

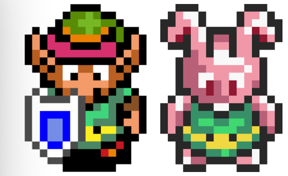


- Each point in the meaning space can be interpreted relative to $\mathcal{M}_{\mathcal{P}}$
  - Binary vectors: propositional meanings (simple or complex)
  - Real-valued vectors: sub-propositional meanings
- Sub-propositional meaning derives from incremental mapping from (sequences of) words to proposition-level meanings

SAMPLI

Incremen … **set of**
propositi … **a model**
(*Light Wo* … ***rld*)**

- *a propos* … *orld*
  - *p is* …
  - *addi* …
- *a propos* … *orld*
  - *p is* …
  - *addi* … *ark World*
- *if p cann* … *ed to the Light W* …



```
⌥⌘1                                tmux
/*
 * Copyright 2017-2020 Harm Brouwer <me@hbrouwer.eu>
 *     and Noortje Venhuizen <njvenhuizen@gmail.com>
 *
 * Licensed under the Apache License, Version 2.0 (the "License");
 * you may not use this file except in compliance with the License.
 * You may obtain a copy of the License at
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 *     http://www.apache.org/licenses/LICENSE-2.0
 *
 * Unless required by applicable law or agreed to in writing, software
 * distributed under the License is distributed on an "AS IS" BASIS,
 * WITHOUT WARRANTIES OR CONDITIONS OF ANY KIND, either express or implied.
 * See the License for the specific language governing permissions and
 * limitations under the License.
 */

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% "Link, it is I, Sahasrahla. I am communicating to you across the void   %
% through telepathy... The place where you now stand was the Golden Land, %
% but evil power turned it into the Dark World. The wizard has broken the %
% wise men's seal and opened a gate to link the worlds at Hyrule Castle.  %
% In order to save this half of the world, the Light World, you must win  %
% back the Golden Power."                                                  %
%       — Sahasrahla                                                       %
%                                                                          %
% From: The Legend of Zelda: A Link to the Past (1992)                     %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

:- module(dfs_sampling,
        [
                op(900,fx, @+),         %% constant
                op(900,fx, @*),         %% property
                op(900,fx, @#),         %% constraint
                op(900,xfx,<-),         %% probability

                dfs_sample_models/2,
                dfs_sample_model/1,

                dfs_sample_models_mt/3
        ]).

:- use_module(library(debug)). % topic: dfs_sampling
:- use_module(library(lists)).
:- use_module(library(ordsets)).
:- use_module(library(random)).

:- use_module(dfs_interpretation).
:- use_module(dfs_io).
:- use_module(dfs_logic).

:- public
        (@+)/1,
NORMAL  +0 ~0 -0 ᚠ master  dfs_sampling.pl        prolog  utf-8[unix]  0% ☰  1/558 ln :  1  ☰ [91]trailing
[0] 1:nvim* 2:zsh- 3:zsh  4:zsh                    "Harms-MacBook-Pro.loc" 09:02 30-Jun-20
```


Venhuizen, Hendriks, Crocker, Brouwer (2019, WoLLIC; 2022, Inf. Comp)

SAMPLING A MEANING SPACE: EXAMPLE

Truth constraint: LW ⊨ *All boys*$_{\{dustin,lucas,mike\}}$ *ride a bicycle*

? Mike rides a bicycle

Dustin rides a bicycle

Mike rides a bicycle

Mike rides a bicycle

Falsehood constraint: DW ⊨ *There is a boy that rides a bicycle*

SAMPLING A MEANING SPACE: EXAMPLE

Truth constraint: LW ⊨ *All boys*$_{\{dustin, lucas, mike\}}$ *ride a bicycle*

✓ *Mike rides a bicycle*

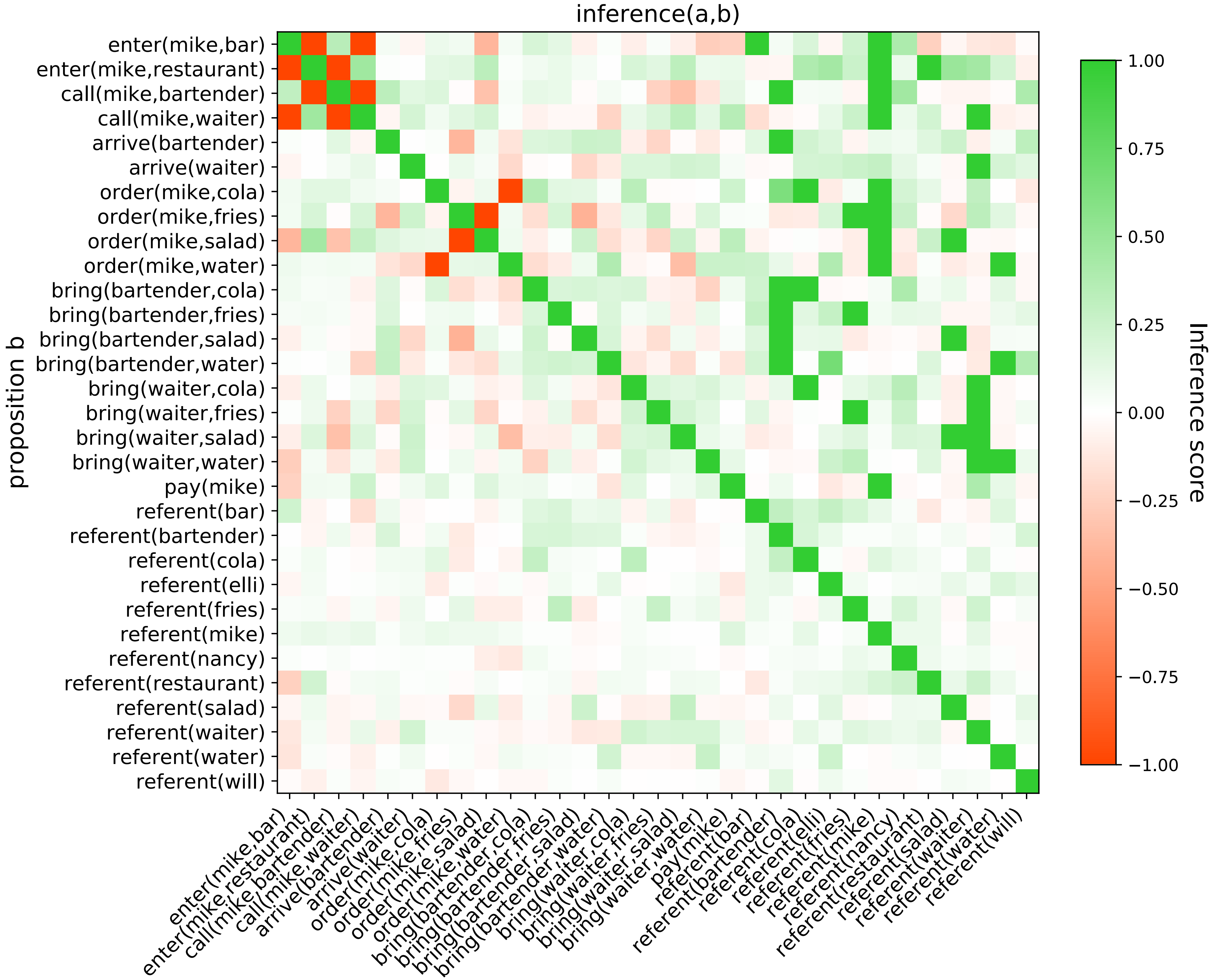
Dustin rides a bicycle

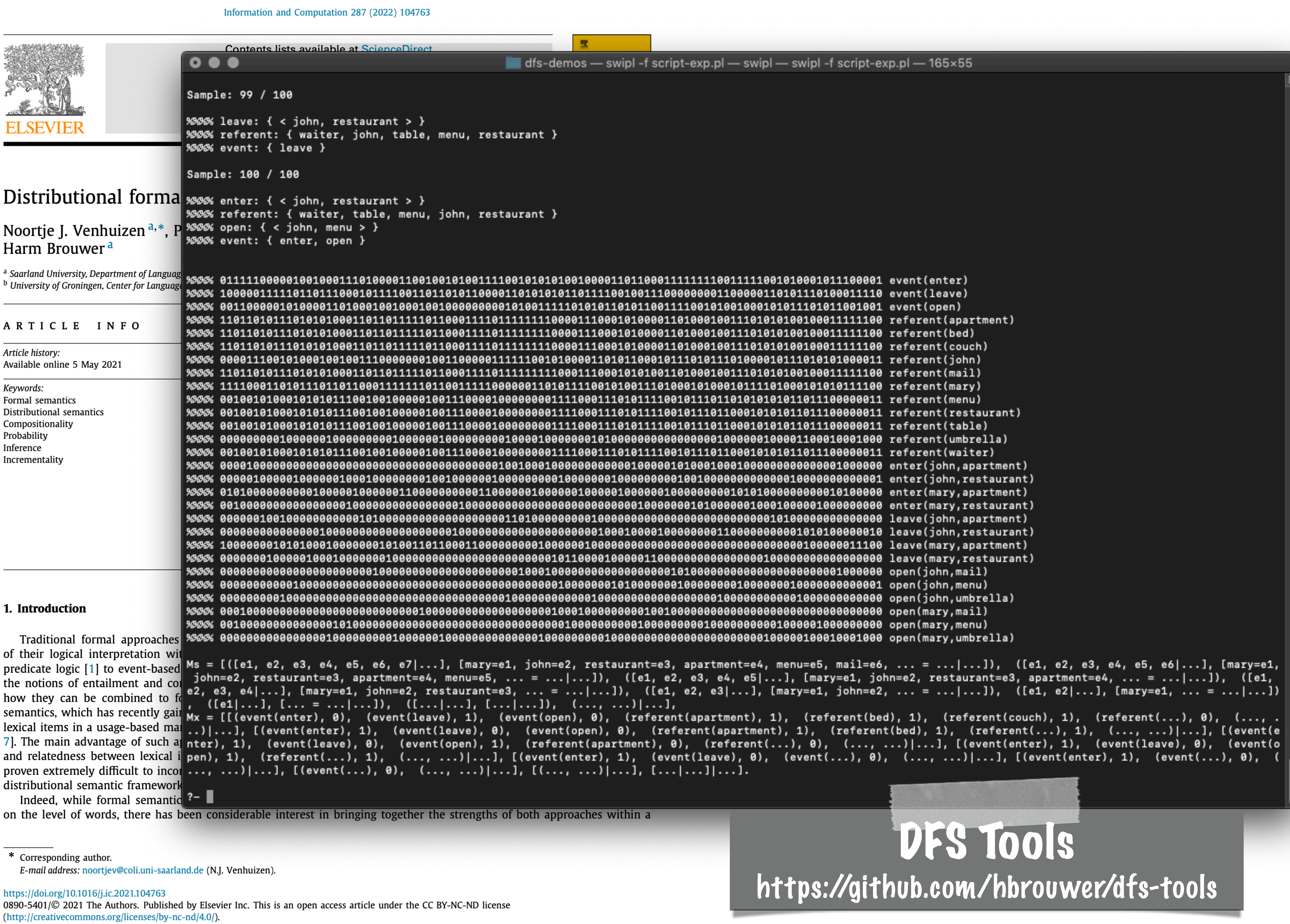
Mike rides a bicycle

Falsehood constraint: DW ⊨ *There is a boy that rides a bicycle*

CONSTRUCTING THE MODEL: MEANING SPACE

We sampled a meaning space of 150 models describing 51 propositions

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ELSEVIER
Distributional forma
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1. Introduction
Traditional formal approaches
of their logical interpretation wi
predicate logic [1] to event-based
the notions of entailment and co
how they can be combined to f
semantics, which has recently gai
lexical items in a usage-based ma
7]. The main advantage of such a
and relatedness between lexical i
proven extremely difficult to inco
distributional semantic framework
Indeed, while formal semanti
on the level of words, there has been considerable interest in bringing together the strengths of both approaches within a
* Corresponding author.
E-mail address: noortjev@coli.uni-saarland.de (N.J. Venhuizen).
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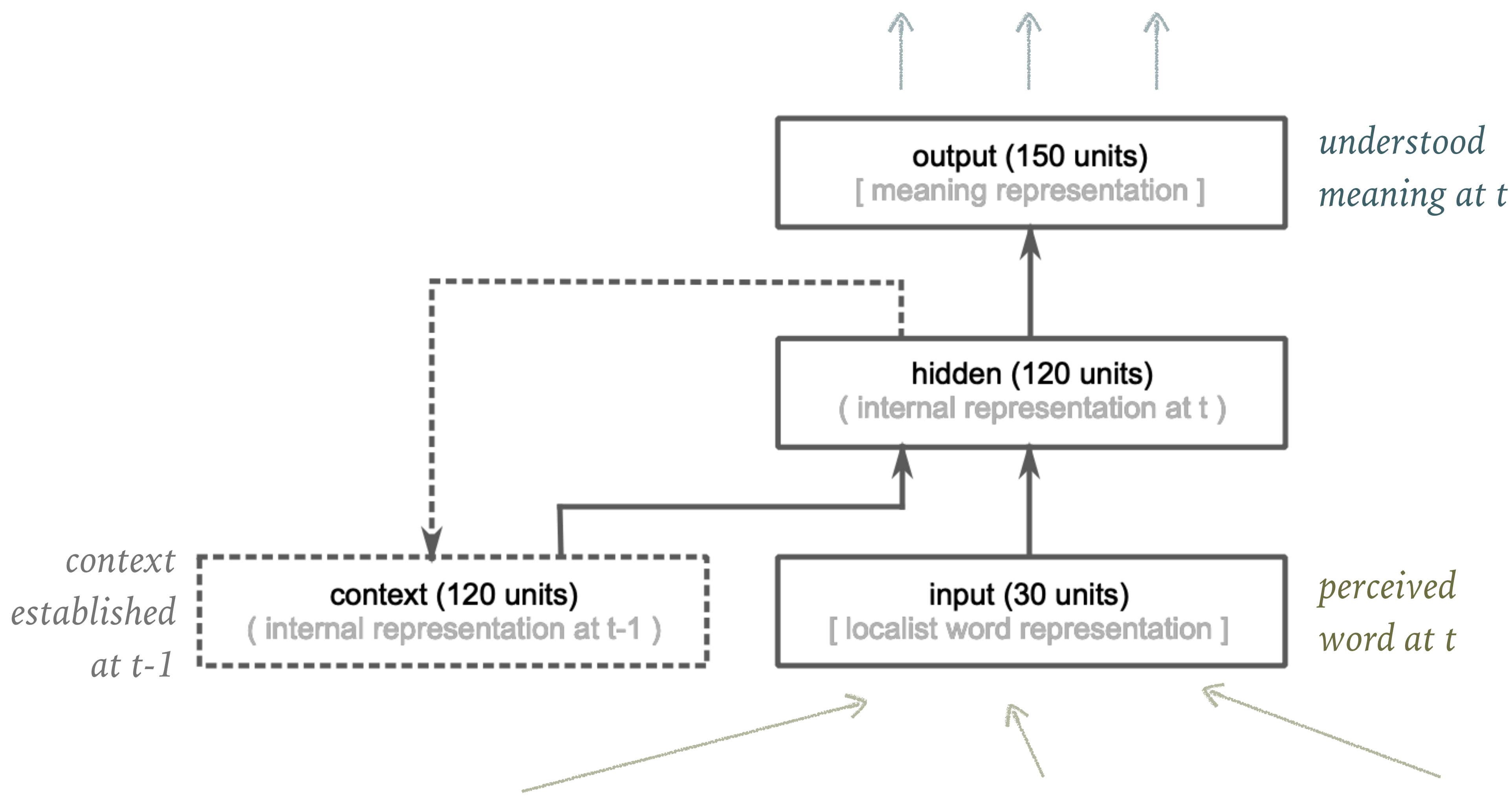
(http://creativecommons.org/licenses/by-nc-nd/4.0/).
dfs-demos — swipl -f script-exp.pl — swipl — swipl -f script-exp.pl — 165×55
Sample: 99 / 100
%%%% leave: { < john, restaurant > }
%%%% referent: { waiter, john, table, menu, restaurant }
%%%% event: { leave }
Sample: 100 / 100
%%%% enter: { < john, restaurant > }
%%%% referent: { waiter, table, menu, john, restaurant }
%%%% open: { < john, menu > }
%%%% event: { enter, open }
%%%% 0111110000010010001110100001100100101001111001010101001000011011000111111110011111001010001011100001 event(enter)
%%%% 1000001111101101110001011110011011010110000110101010110111100100111000000001100000110101110100011110 event(leave)
%%%% 0011000001010000110100010010001001000000000101001111010101101011001111001010010001010111010110010 01 event(open)
%%%% 1101101011101010100011011011111011000111101111111110000111000101000011010001001110101010010001111110 0 referent(apartment)
%%%% 110110101110101010001101101111101100011110111111111000011100010100001101000100111010101001000111111 00 referent(bed)
%%%% 110110101110101010001101101111101100011110111111111000011100010100001101000100111010101001000111111 00 referent(couch)
%%%% 0000111001010001001001110000000100110000011111100101000011010110001011101011101000010111010101000011 referent(john)
%%%% 1101101011101010100011011011111011000111101111111110001110001010100110100010011101010100100011111100 referent(mail)
%%%% 1111000110101110110110001111111011001111100000011010111100101001110100010100010111101000101010111100 referent(mary)
%%%% 0010010100010101011100100100000100111000010000000011110001110101111001011101101010101011011100000011 referent(menu)
%%%% 0010010100010101011100100100000100111000010000000011110001110101111001011101100010101011011100000011 referent(restaurant)
%%%% 0010010100010101011100100100000100111000010000000011110001110101111001011101100010101011011100000011 referent(table)
%%%% 0000000001000000100000000010000001000000000100001000000001010000000000000000100000010000110001000100 0 referent(umbrella)
%%%% 0010010100010101011100100100000100111000010000000011110001110101111001011101100010101011011100000011 referent(waiter)
%%%% 0000100000000000000000000000000000000001001000100000000000000100000101000100010000000000000000010000 00 enter(john,apartment)
%%%% 0000010000010000000100010000000001001000000010000000000100000001000000000010010000000000000001000000000000 1 enter(john,restaurant)
%%%% 0101000000000001000001000000110000000000110000001000000100000100000001000000000010101000000000000101 00000 enter(mary,apartment)
%%%% 0010000000000000000010000000000000000010000000000000000000000000000000100000000101000000100010000010 00000000 enter(mary,restaurant)
%%%% 0000001001000000000000101000000000000000000000011010000000001000000000000000000000000000001010000000 00000000 leave(john,apartment)
%%%% 0000000000000001000000000000000000000010000000000000000000000010001000010000000011000000000001010100 000010 leave(john,restaurant)
%%%% 1000000010101000100000000101001101100011000000000100000010000000000000000000000000000000000000001000 0011100 leave(mary,apartment)
%%%% 0000000100000100010000000010000000000000000000000000000010110000100000110000000000000000000010000000 000000000 leave(mary,restaurant)
%%%% 0000000000000000000000001000000000000000000000001000100000000000000000000010100000000000000000000000 001000000 open(john,mail)
%%%% 0000000000010000000000000000000000000000000000000100000000101000000001000000000100000000100000000000 00001 open(john,menu)
%%%% 0000000001000000000000000000000000000000000000010000000000000100000000000000000000100000000000001000 000000000 open(john,umbrella)
%%%% 0001000000000000000000000000000100000000000000000000001000100000000000100100000000000000000000000000 000000000 open(mary,mail)
%%%% 0010000000000000010100000000000000000000000000000000001000000000000100000000000100000000000010000010 00000000 open(mary,menu)
%%%% 0000000000000000100000000001000000010000000000000000000001000000000010000000000000000000000000000001 000001000 open(mary,umbrella)
Ms = [([e1, e2, e3, e4, e5, e6, e7|...], [mary=e1, john=e2, restaurant=e3, apartment=e4, menu=e5, mail=e6, ... = ...|...]), ([e1, e2, e3, e4, e5, e6|...], [mary=e1, john=e2, restaurant=e3, apartment=e4, menu=e5, ... = ...|...]), ([e1, e2, e3, e4, e5|...], [mary=e1, john=e2, restaurant=e3, apartment=e4, ... = ...|...]), ([e1, e2, e3, e4|...], [mary=e1, john=e2, restaurant=e3, ... = ...|...]), ([e1, e2, e3|...], [mary=e1, john=e2, ... = ...|...]), ([e1, e2|...], [mary=e1, ... = ...|...]), ([e1|...], [... = ...|...]), ([...|...], [...|...]), (..., ...)|...],
Mx = [[(event(enter), 0), (event(leave), 1), (event(open), 0), (referent(apartment), 1), (referent(bed), 1), (referent(couch), 1), (referent(...), 0), (..., ...)|...], [(event(enter), 1), (event(leave), 0), (event(open), 0), (referent(apartment), 1), (referent(bed), 1), (referent(...), 1), (..., ...)|...], [(event(enter), 1), (event(leave), 0), (event(open), 1), (referent(apartment), 0), (referent(...), 0), (..., ...)|...], [(event(enter), 1), (event(leave), 0), (event(open), 1), (referent(...), 1), (..., ...)|...], [(event(enter), 1), (event(leave), 0), (event(...), 0), (..., ...)|...], [(event(enter), 1), (event(...), 0), (..., ...)|...], [(event(...), 0), (..., ...)|...], [(..., ...)|...], [...|...]|...].
?-
DFS Tools
https://github.com/hbrouwer/dfs-tools

A MODEL OF INCREMENTAL MEANING CONSTRUCTION

enter(mike,restaurant) ∧ order(mike,salad)

[1,0,1,0,0,1,1,0,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,1,0,…]



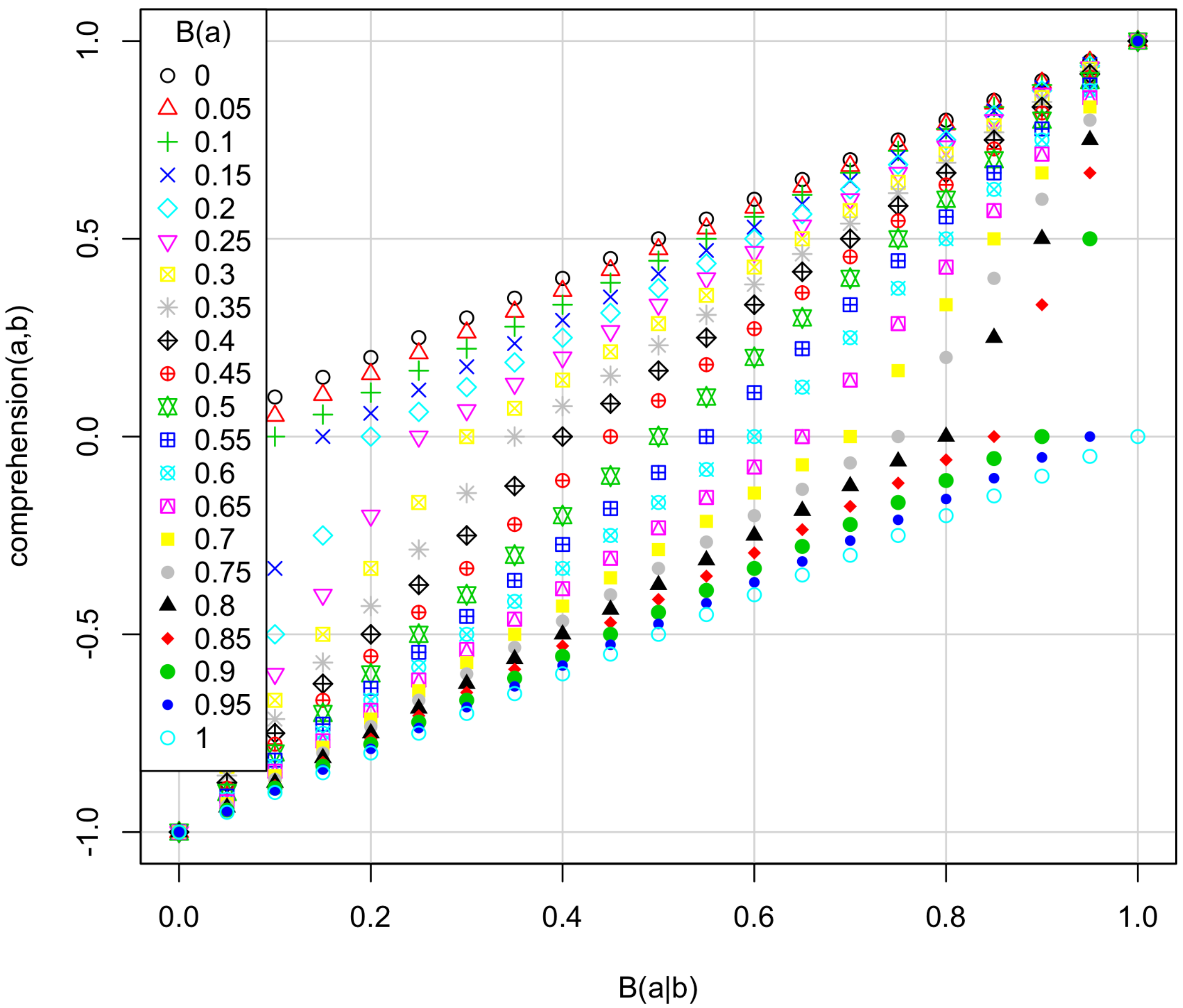
< [0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0], [0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0], […] >

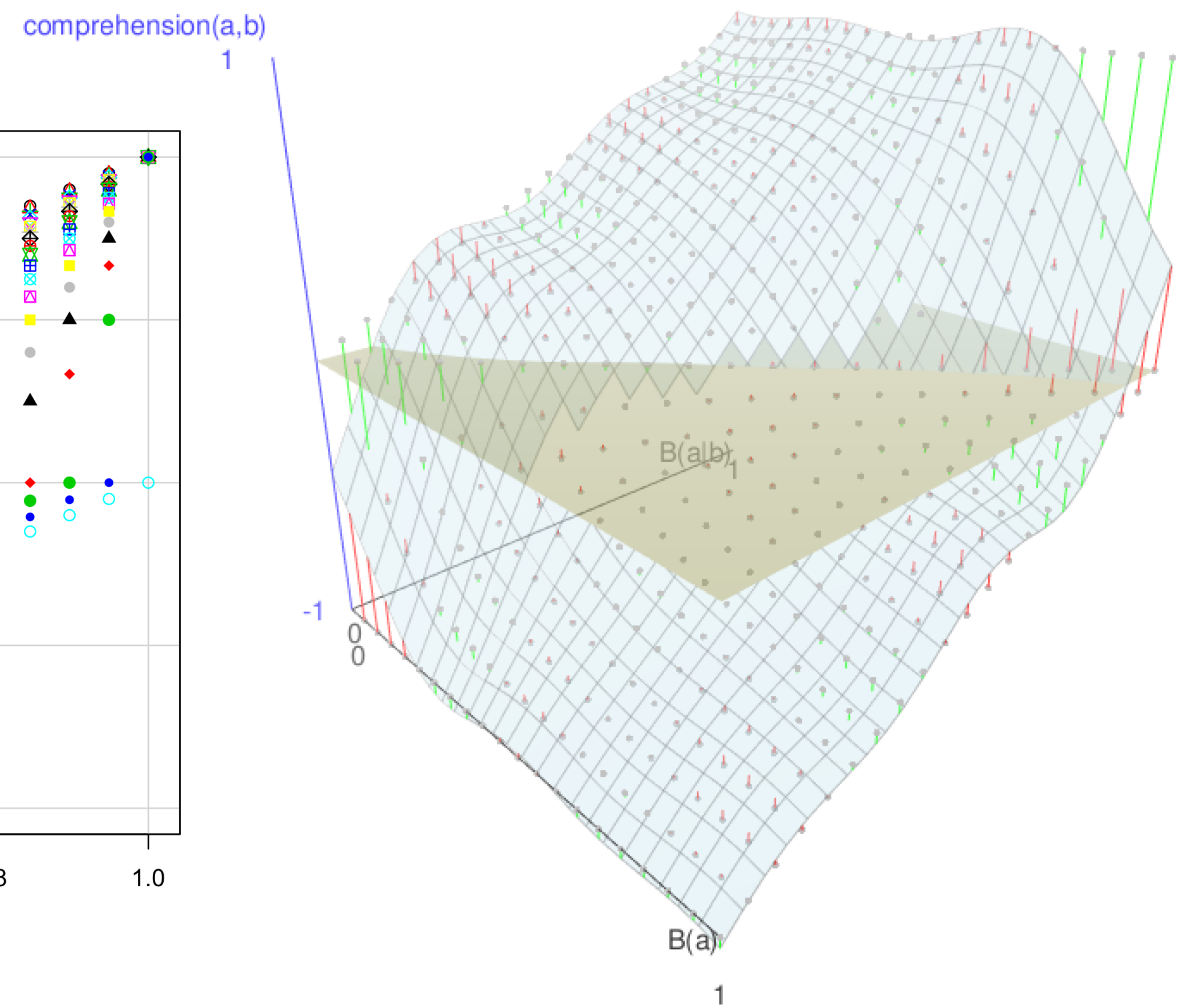
<“mike”, “entered”, “the”, “restaurant”, “and” “ordered”, “salad”>

“Mike entered the restaurant and ordered salad”

— to be continued —

Comprehension scores





The higher *B(a)* the more difficult it is to *increase certainty* in *a*, and the lower *B(a)* the more difficult it is to *increase uncertainty* in *a*