Semantic Theory

Week 6: Generalised Quantifiers

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Back to Noun Phrases

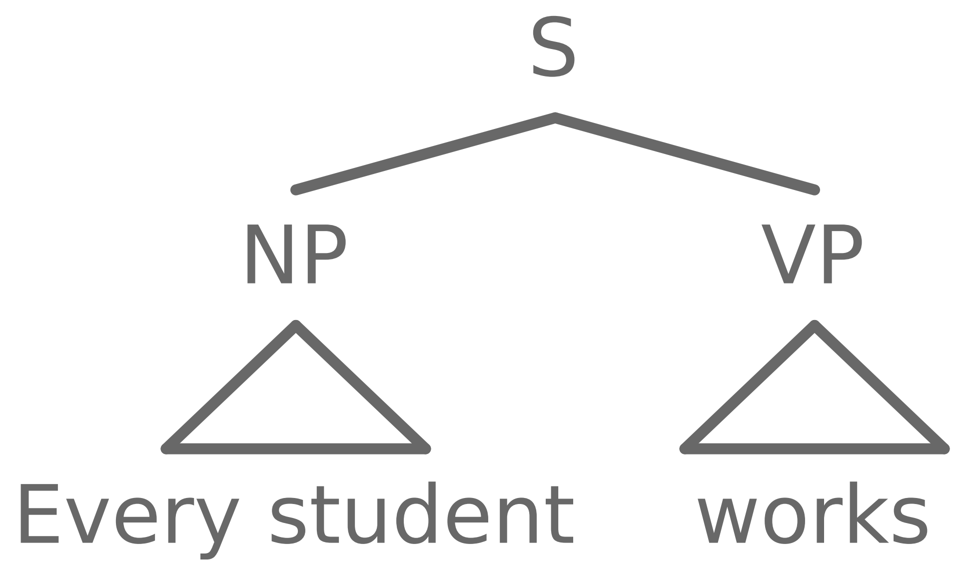
Natural language contains a wide variety of NPs, serving as quantifiers

all students, no woman, not every man, everything, nothing, three books, the ten professors, John, John and Mary, only John, firemen, at least five horses, most girls, all but ten marbles, less than half of the audience, John's car, some student's exercise, no student except Mary, more male than female cats, usually, each other.

Aristotle: "Quantifiers are second-order relations between sets"

Frege: "All quantifiers can be defined in terms of logical quantifiers (∀, ∃)"

NP interpretation



"*Every student*" $\mapsto \lambda P \forall x (\text{student}'(x) \rightarrow P(x))$

- $[\![\text{every student}]\!] \in D_{\langle\langle e,t\rangle,t\rangle}$
- $D_{\langle\langle e,t\rangle,t\rangle}$ is the set of functions from properties to truth values
- In other words: "Every student" denotes the *set of properties* that apply to every student (property = set of individuals).
  - $[\![\text{Every student}]\!]^M = \{ P \subseteq U_M \mid \text{every student has property } P \}$
    $= \{ P \subseteq U_M \mid [\![\text{student}]\!] \subseteq P \}$
  - $[\![\text{Every student works}]\!]^M = 1$ iff $[\![\text{work}]\!]^M \in [\![\text{every student}]\!]^M$

Generalised Quantifiers

Generalised quantifiers are sets of subsets of U_M (i.e., sets of properties)

every student $\mapsto \lambda P \forall x(\text{student'}(x) \rightarrow P(x))$

- $[\![\text{every student}]\!]^M = \{ P \subseteq U_M \mid [\![\text{student}]\!] \subseteq P\}$

"the set of properties P such that all students are P"

a student $\mapsto \lambda P \exists x(\text{student'}(x) \wedge P(x))$

- $[\![\text{a student}]\!]^M = \{ P \subseteq U_M \mid [\![\text{student}]\!] \cap P \neq \varnothing \}$

"the set of properties P such that at least one student is P"

Bill $\mapsto \lambda P.P(b^*)$

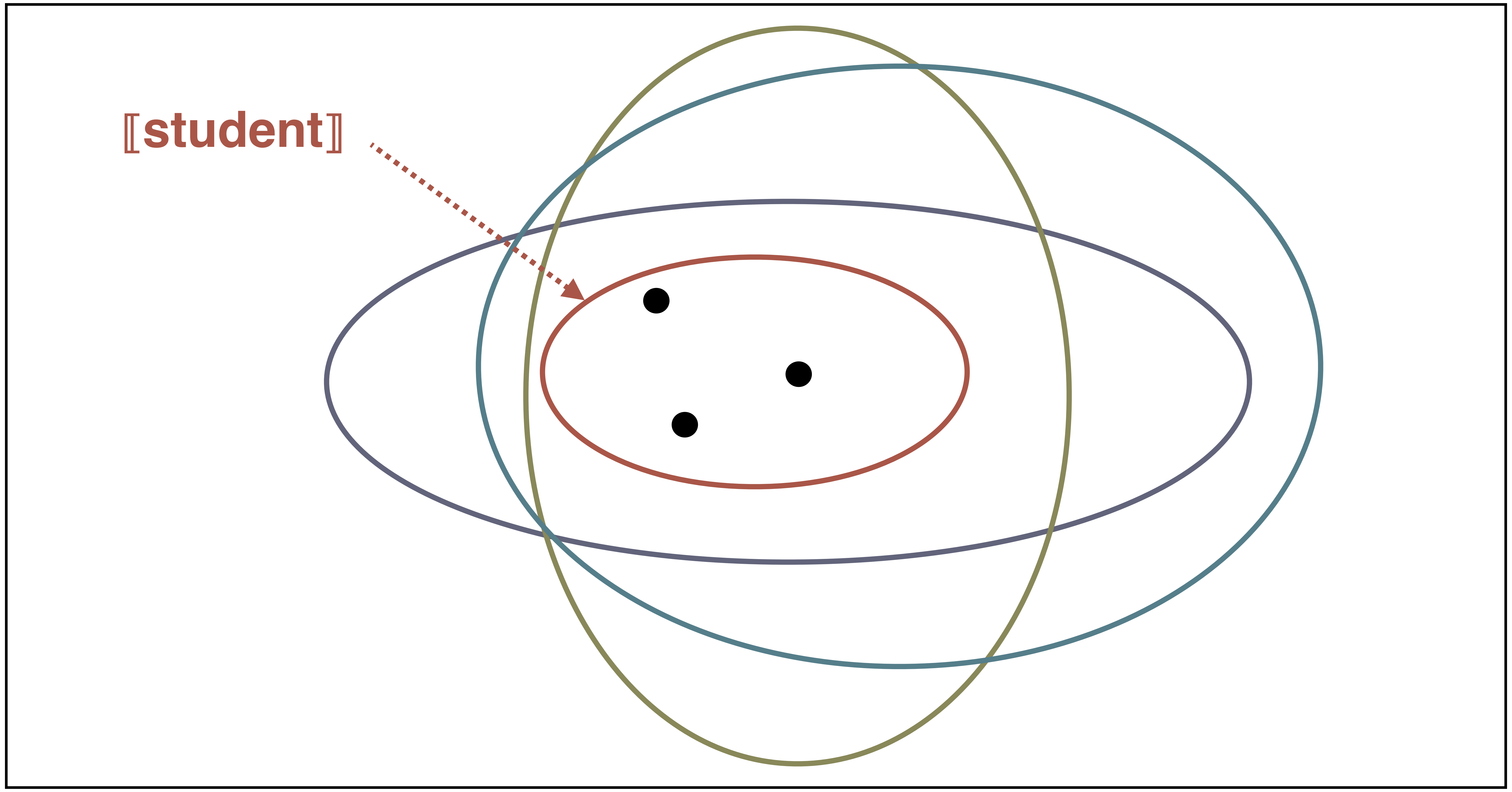
- $[\![\text{Bill}]\!]^M = \{ P \subseteq U_M \mid b^* \in P\}$

"the set of properties P, such that Bill has property P"

Generalised Quantifier Denotation

⟦every student⟧

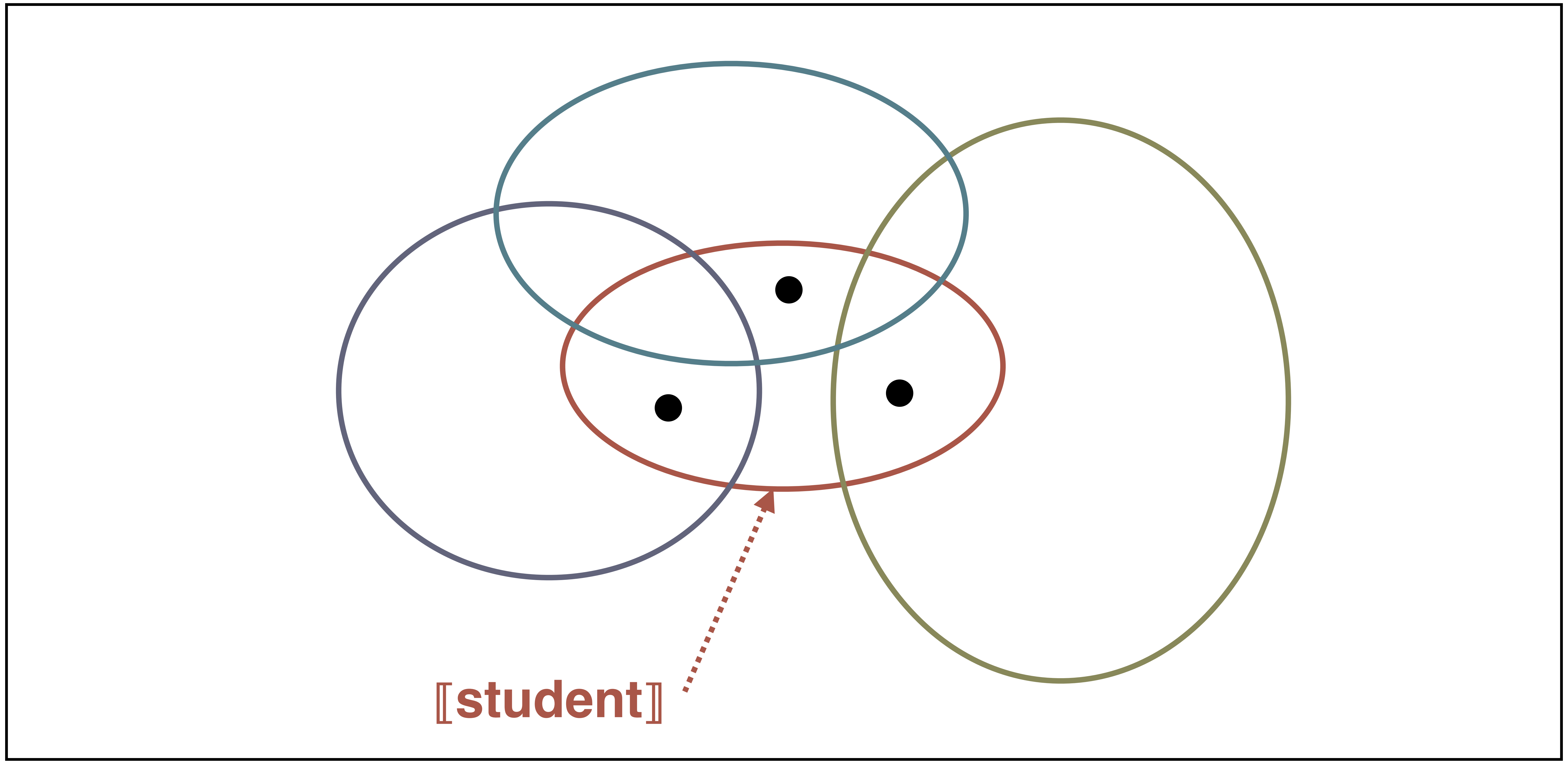
- "every student" denotes the set of properties that apply to every student (i.e., all supersets of ⟦student⟧)



Generalised Quantifier Denotation

⟦a student⟧

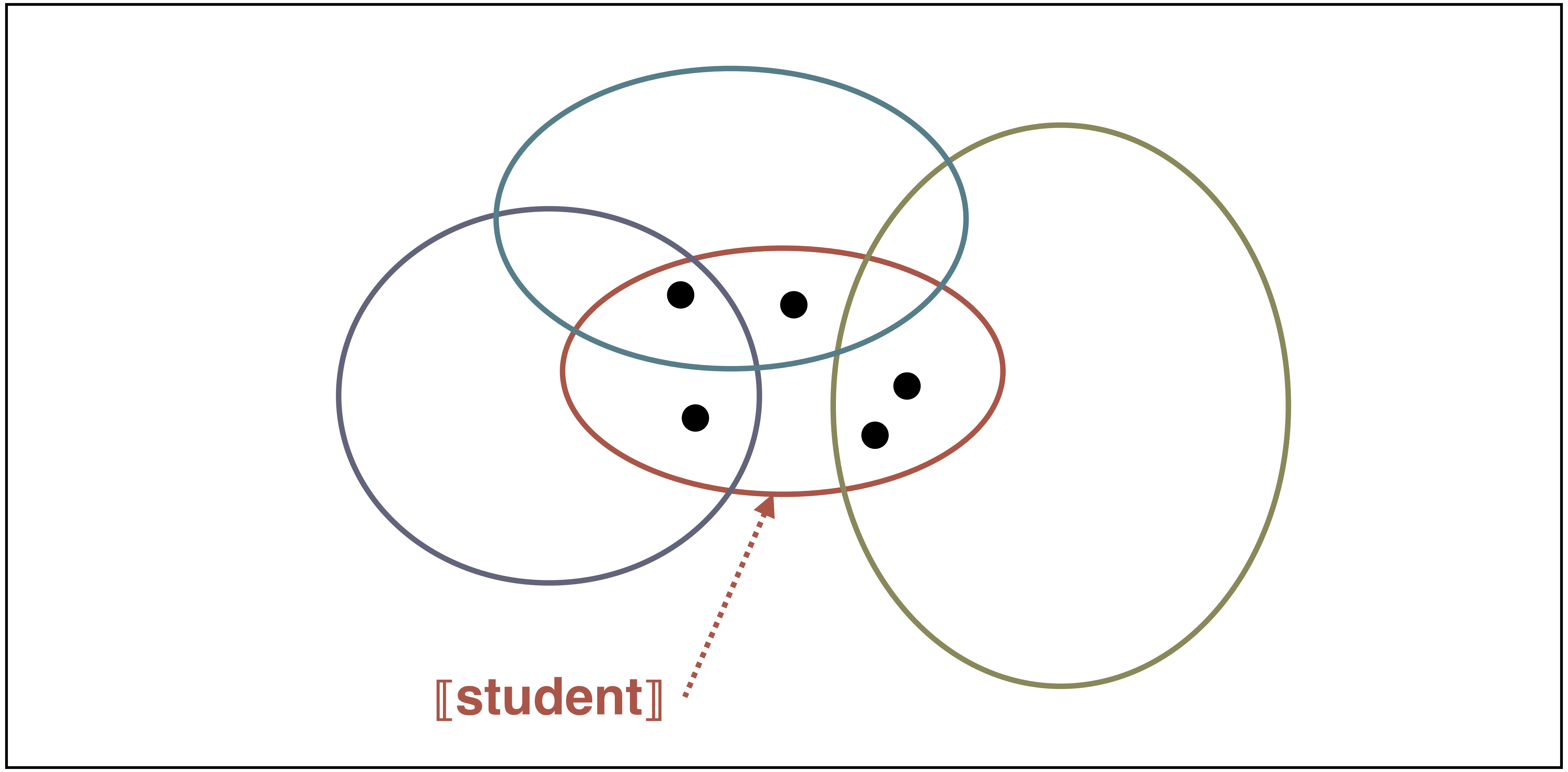
- “a student” denotes the set of properties that apply to at least one student.



Generalised Quantifier Denotation

⟦two students⟧

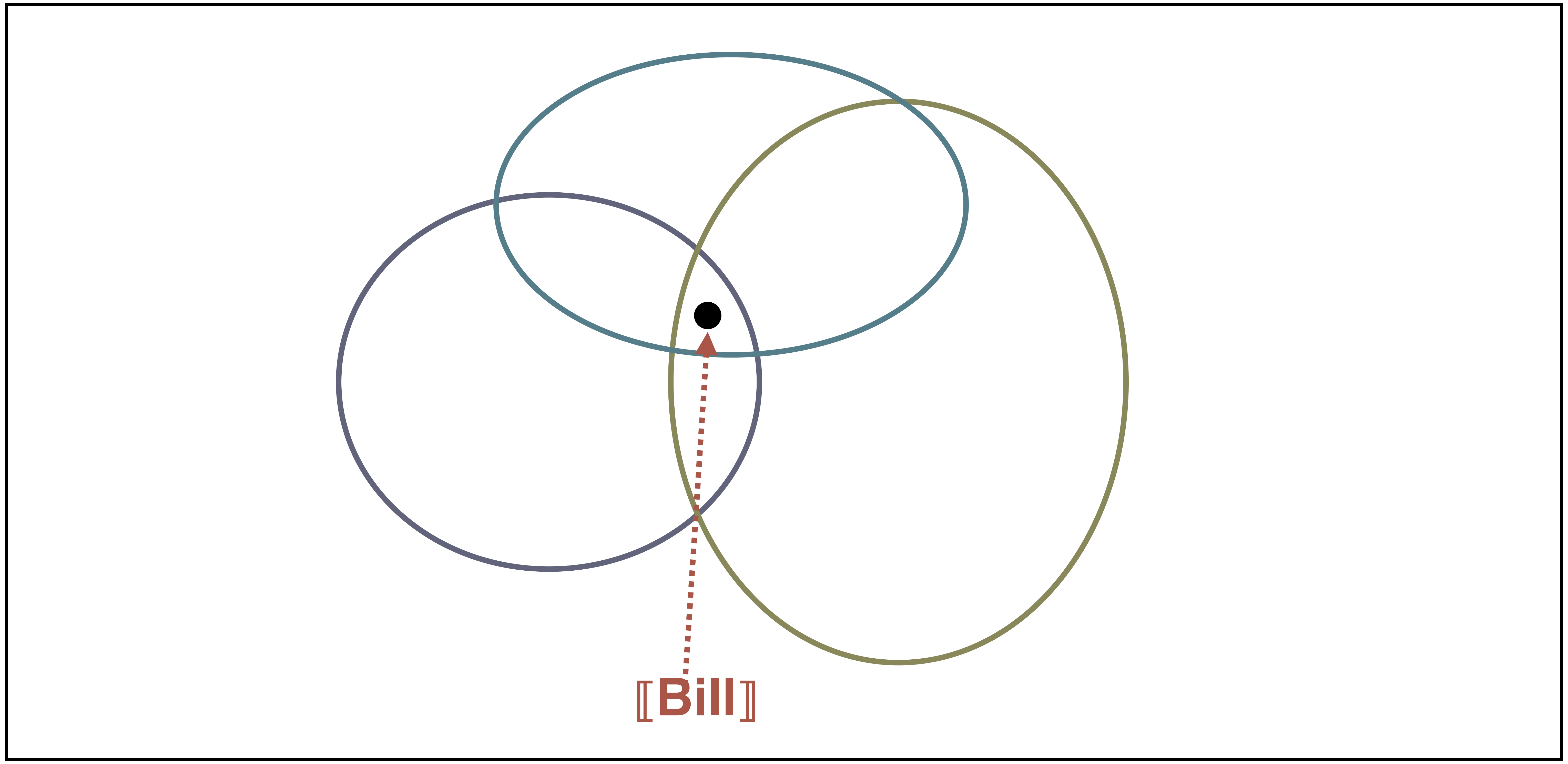
- "two students" denotes the set of properties that apply to at least (or: exactly) two students.



Generalised Quantifier Denotation

⟦Bill⟧

- "Bill" denotes the set of properties that apply to Bill



Noun Phrase Interpretations

$[\![\text{all N}]\!]^M = \{ P \subseteq U_M \mid [\![N]\!] \cap P = [\![N]\!] \}$

$[\![\text{a(n) N}]\!]^M = \{ P \subseteq U_M \mid [\![N]\!] \cap P \neq \varnothing \}$

$[\![\text{bill}]\!]^M = \{ P \subseteq U_M \mid b^* \in P \}$

$[\![\text{not all N}]\!]^M = \{ P \subseteq U_M \mid [\![N]\!] \cap P \neq [\![N]\!] \}$

$[\![\text{no N}]\!]^M = \{ P \subseteq U_M \mid [\![N]\!] \cap P = \varnothing \}$

$[\![\text{exactly n N}]\!]^M = \{ P \subseteq U_M \mid \text{card}([\![N]\!] \cap P) = n \}$

$[\![\text{at most n N}]\!]^M = \{ P \subseteq U_M \mid \text{card}([\![N]\!] \cap P) \leq n \}$

$[\![\text{at least n N}]\!]^M = \{ P \subseteq U_M \mid \text{card}([\![N]\!] \cap P) \geq n \}$

Generalised Quantifier Theory

Central questions

I. How do generalised quantifiers differ in terms of their formal properties?

II. What universal regularities govern the meaning of terms?

III. Which subclasses represent meanings of natural language noun phrases?

Observation 1

Inference Patterns

(1) *All men walked rapidly* ⊨ *All men walked*

(2) *A girl smoked a cigar* ⊨ *A girl smoked*

(3) *No man walked* ⊨ *No man walked rapidly*

(4) *Few girls smoked* ⊨ *Few girls smoked a cigar*

How to explain the different inference patterns for quantifiers?

Observation 2

Negative Polarity Items

NPIs (*need*, *any*, *ever*, …) typically occur only in contexts with negation

(1) a. *John needn't go there.*
b. **John need go there.*

(2) a. *Nobody saw anything.*
b. **Somebody saw anything.*

(3) a. *No student has ever been in Saarbrücken.*
b. **Some student has ever been in Saarbrücken.*

What formally licenses Negative Polarity Items?

Observation 3

Coordination

(1) *No man and few women walked.*

(2) *None of the girls and at most three boys walked.*

(3) **A man and few women walked.*

(4) **John and no woman saw Jane.*

Which noun phrases can be coordinated?

Explaining Observation 1

Subsets and Supersets

(1) *All men walked rapidly* ⊨ *All men walked*

- ⟦to walk rapidly⟧ ⊆ ⟦to walk⟧

(2) *A girl smoked a cigar* ⊨ *A girl smoked*

- ⟦to smoke a cigar⟧ ⊆ ⟦to smoke⟧

Intuitively: For the given quantifiers, the sentence [$_S$ NP VP] remains true if the denotation of the VP is made "larger"

Upward Monotonicity

A quantifier Q is **upward monotonic** (or: *monotone increasing*) in $M = \langle U, V \rangle$ iff Q is "closed under supersets", i.e.:

- for all $X, Y \subseteq U$: if $X \in Q$ and $X \subseteq Y$, then $Y \in Q$

A noun phrase is upward monotonic if it denotes an upward monotonic quantifier.

Upward Monotonicity

Entailment tests

If $[\![VP_1]\!] \subseteq [\![VP_2]\!]$, then $NP\ VP_1 \vDash NP\ VP_2$

- $[\![$to walk rapidly$]\!] \subseteq [\![$to walk$]\!]$
- All men walked rapidly $\vDash$ All men walked 👍
- No man walked rapidly $\nvDash$ No man walked 👎

$NP\ VP_1$ and $VP_2 \vDash NP\ VP_1$ and $NP\ VP_2$ (where: $[\![VP_1 \text{ and } VP_2]\!] = [\![VP_1]\!] \cap [\![VP_2]\!]$)

- All men smoked and drank $\vDash$ All men smoked and all men drank 👍
- No man smoked and drank $\nvDash$ No man smoked and no man drank 👎

Downward Monotonicity

(3) *No man walked* ⊨ *No man walked rapidly*
⟦*to walk rapidly*⟧ ⊆ ⟦*to walk*⟧

(4) *Few girls smoked* ⊨ *Few girls smoked a cigar*
⟦*to smoke a cigar*⟧ ⊆ ⟦*to smoke*⟧

A quantifier Q is **downward monotonic** (or: *monotone decreasing*) in M = ⟨U, V⟩ iff Q is closed under inclusion:

- for all X, Y ⊆ U: if X ∈ Q and Y ⊆ X, then Y ∈ Q

A noun phrase is downward monotonic if it denotes a downward monotonic quantifier.

Downward Monotonicity

Entailment tests

If $[\![VP_2]\!] \subseteq [\![VP_1]\!]$, then $NP\ VP_1 \vDash NP\ VP_2$

- $[\![$to walk rapidly$]\!] \subseteq [\![$to walk$]\!]$
- No man walked ⊨ No man walked rapidly 👍
- All men walked ⊭ All men walked rapidly 👎

$NP\ VP_1$ or $VP_2 \vDash NP\ VP_1$ and $NP\ VP_2$
(where: $[\![$VP1 or VP2$]\!] = [\![$VP1$]\!] \cup [\![$VP2$]\!]$ and $[\![$VP1 and VP2$]\!] = [\![$VP1$]\!] \cap [\![$VP2$]\!]$)

- Neither girl drank or smoked ⊨ Neither girl drank and neither girl smoked. 👍
- All boys sing or dance ⊭ All boys sing and all boys dance. 👎

Explaining Observation 2

Negative Polarity Items

(1) a. *John needn't go there.*
b. **John need go there.*

(2) a. *Nobody saw anything.*
b. **Somebody saw anything.*

(3) a. *No student has ever been in Saarbrücken.*
b. **Some student has ever been in Saarbrücken.*

NPIs are licensed only in downward monotonic contexts.

Explaining Observation 3

Coordination

(1) *No man and few women walked.*

(2) *None of the girls and at most three boys walked.*

(3) **A man and few women walked.*

(4) **John and no woman saw Jane.*

(Non-comparative) NPs can be coordinated iff they have the same direction of monotonicity.

(3') *A man but few women walked.*

(4') *John but no woman saw Jane.*

Coordination with the connective "but" requires NPs with a different direction of monotonicity.

Monotonicity and logical operators

Monotonic quantifiers are *closed under* conjunction and disjunction:

- *<u>All boys and a girl</u> walked rapidly* ⊨ *<u>All boys and a girl</u> walked*
- *<u>John or a student</u> arrived late* ⊨ *<u>John or a student</u> arrived*
- where: $[\![NP_1 \text{ and } NP_2]\!] = [\![NP_1]\!] \cap [\![NP_2]\!]$
  $[\![NP_1 \text{ or } NP_2]\!] = [\![NP_1]\!] \cup [\![NP_2]\!]$

The intersection/union of two monotonic quantifiers is a quantifier with the same direction of monotonicity.

Quantifier Negation

External negation: $\neg Q = \{ P \subseteq U_M \mid P \notin Q \}$

- $\neg[\![\text{all N}]\!] = \{ P \subseteq U_M \mid P \notin [\![\text{all N}]\!] \}$
  $= \{ P \subseteq U_M \mid [\![N]\!] \cap P \neq [\![N]\!] \} = [\![\text{not all N}]\!]$

Internal negation: $Q\neg = \{ P \subseteq U_M \mid (U_M - P) \in Q \}$

- $[\![\text{all N}]\!]\neg = \{ P \subseteq U_M \mid (U_M - P) \in [\![\text{all N}]\!]\}$
  $= \{ P \subseteq U_M \mid [\![N]\!] \cap (U_M - P) = [\![N]\!] \}$
  $= \{ P \subseteq U_M \mid [\![N]\!] \cap (U_M - P) \neq \varnothing \}$
  $= \{ P \subseteq U_M \mid [\![N]\!] \cap P = \varnothing \} = [\![\text{no N}]\!]$

Internal and external negation of a quantifier both flip the direction of monotonicity (*upward* → *downward* or *downward* → *upward*)

Quantifier Negation

Duals

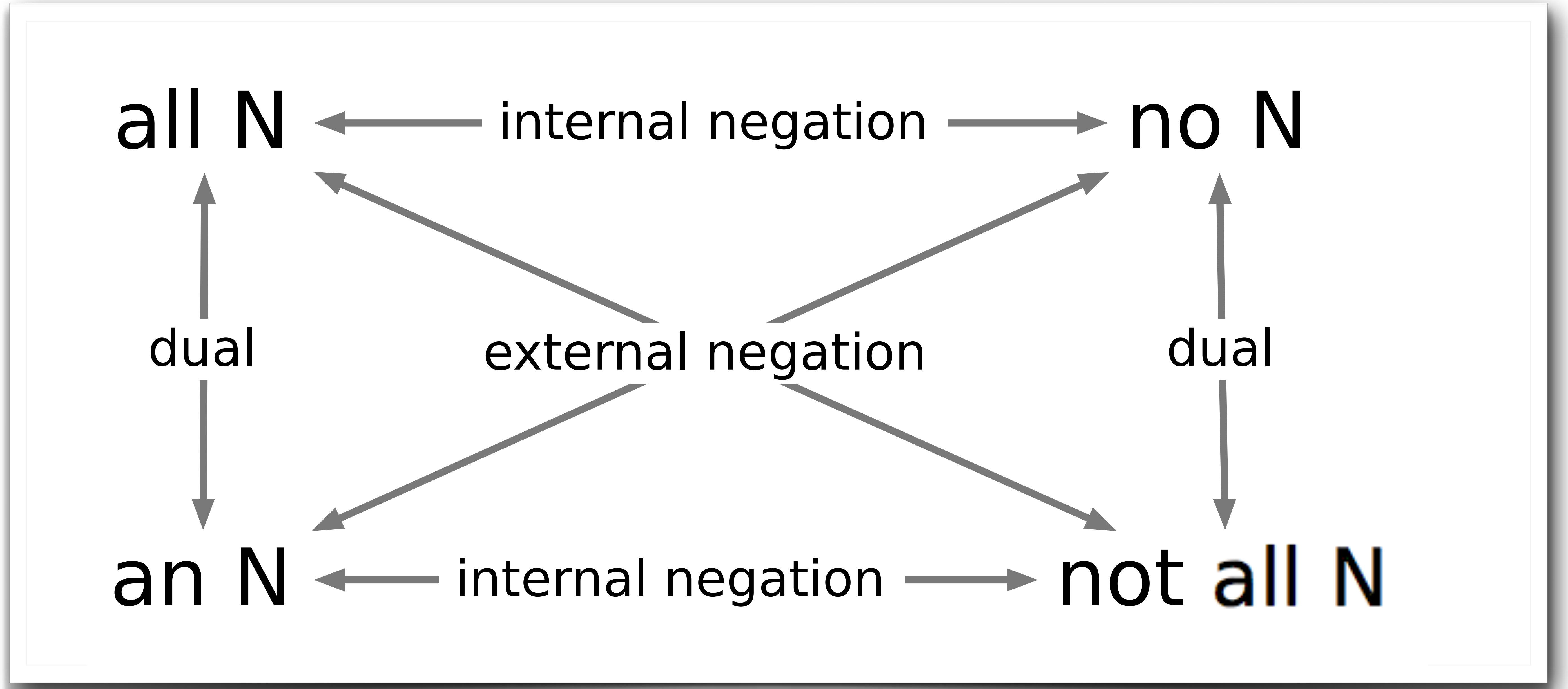
The dual Q^* of a quantifier Q in M is defined as the external and internal negation of Q:

$$Q^* = \neg Q \neg$$
$$= \{ P \subseteq U_M \mid (U_M - P) \in \neg Q \}$$
$$= \{ P \subseteq U_M \mid (U_M - P) \notin Q \}.$$

If Q is *upward monotonic*, then Q* is *upward monotonic.*

If Q is *downward monotonic*, then Q* is *downward monotonic*.

The “Square of Opposition”



Looking for Universals I

Monotonicity Constraint

"The simple noun phrases of any natural language express monotone quantifiers or conjunctions of monotone quantifiers."
(Barwise & Cooper 1981)

Simple noun phrase: Proper names or NPs of the form [NP ← DET N]

Monotone quantifiers: Quantifiers that are upward or downward monotonic

BEHAVIORAL AND BRAIN SCIENCES (2009) **32**, 429–492
doi:10.1017/S0140525X0999094X

The myth
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Universal grammar is dead

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Abstract: The idea of a biologically evolved, universal grammar with linguistic content is a myth, perpetuated by three spurious explanatory strategies of generative linguists. To make progress in understanding human linguistic competence, cognitive scientists must abandon the idea of an innate universal grammar and instead try to build theories that explain both linguistic universals and diversity and how they emerge.

Universal grammar is, and has been for some time, a completely empty concept. Ask yourself: what exactly is in universal grammar? Oh, you don't know – but you are sure that the experts (generative linguists) do. Wrong; they don't. And not only that, they have no method for finding out. If there is a method, it would be looking carefully at all the world's thousands of languages to discern universals. But that is what linguistic typologists have been doing for the past several decades, and, as Evans & Levinson (E&L) report, they find no universal grammar.

ersals: Abstract but not

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From NPs to Determiners

Every man walked $\mapsto \forall x(\text{man}'(x) \rightarrow \text{walk}'(x))$

- $[\![\textit{Every}]\!] = [\![\lambda P \lambda Q \forall x(P(x) \rightarrow Q(x))]\!] \in D_{\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle}$
- $[\![\text{Every}]\!](A)(B) = 1$ iff $A \subseteq B$

- **Syntactically** determiners are expressions that take a noun and a verb phrase to form a sentence.

- **Semantically** the interpretation of a determiner can be seen as:
  - a *function* from sets of entities to sets of properties: $\langle\langle e, t\rangle,\langle\langle e, t\rangle, t\rangle\rangle$
  - a *relation* between two sets A and B, denoted by the NP and VP, respectively

Persistence

Upward monotonicity of the first argument of the determiner

A determiner D is **persistent** in M iff: for all X, Y, Z:

- if D(X, Z) and X ⊆ Y, then D(Y, Z)

Persistence test: If ⟦N_1⟧ ⊆ ⟦N_2⟧, then DET N_1 VP ⊨ DET N_2 VP

- ⟦man⟧ ⊆ ⟦human being⟧
  - Some men walked ⊨ Some human beings walked
- ⟦girl⟧ ⊆ ⟦female⟧
  - At least four girls were smoking ⊨ At least four females were smoking.

Antipersistence

Downward monotonicity of the first argument of the determiner

A determiner D is **antipersistent** in M iff: for all X, Y, Z:

- if D(X, Z) and Y ⊆ X, then D(Y, Z)

Antipersistence test: If ⟦N_2⟧ ⊆ ⟦N_1⟧, then DET N_1 VP ⊨ DET N_2 VP

- ⟦toddler⟧ ⊆ ⟦children⟧
- All children walked ⊨ All toddlers walked
- ⟦girl⟧ ⊆ ⟦female⟧
- No female was smoking ⊨ No girl was smoking.

Determiner Persistence and Monotonicity of NPs

Left vs. right monotonicity

Persistence/ Antipersistence	⇔ upward/ downward monotonicity of the first argument of the determiner
	left-monotonicity (↑mon and ↓mon)
Upward/ Downward monotonicity of noun phrases	⇔ upward/ downward monotonicity of the second argument of the determiner
	right-monotonicity (mon↑ and mon↓)

Left and Right Monotonicity of Determiners

Some examples

↑mon↑ *some*

↓mon↑ *all*

↓mon↓ *no*

↑mon↓ *not all*

Looking for Universals II

Conservativity constraint

In every natural language, simple determiners together with an N yield an NP which "lives on ⟦N⟧". **(Barwise & Cooper 1981)**

A determiner D is conservative iff *"D lives on A"*:

- for every A, B ⊆ U: D(A, B) ⇔ D(A, A ∩ B)
  - *All students work* ⇔ *All students are students that work*
  - *Some girls are dancing* ⇔ *Some girls are girls that are dancing*
  - But: *Only men smoke cigars* ⇎ *Only men are men that smoke cigars*

Corollary: "only" is not a determiner?

Literature

Background reading material & references

- Generalized Quantifiers (Stanford Encyclopedia of Philosophy): https://plato.stanford.edu/entries/generalized-quantifiers/
- L.T.F. Gamut. Logic, Language, and Meaning. Vol 2. Chapter 7.
- Jon Barwise & Robin Cooper. Generalized Quantifiers. Linguistics and Philosophy. 1981.