

# Semantic Theory 2022 – Practice Exam

## *Supplementary materials*

### 1 Type Theory: Semantics

Let  $U$  is a non-empty set of entities. For every type  $\tau$ , the domain of possible denotations  $D_\tau$  is given by:

- $D_e = U$
- $D_t = \{0, 1\}$
- $D_{\langle\sigma,\tau\rangle}$  is the set of functions from  $D_\sigma$  to  $D_\tau$ .

A model structure is a pair  $M = \langle U_M, V_M \rangle$  such that  $U_M$  is a non-empty set of individuals, and  $V_M$  is a function assigning every non-logical constant of type  $\tau$  a member of  $D_\tau$ .

Interpretation:

- $\llbracket \alpha \rrbracket^{M,g} = V_M(\alpha)$  if  $\alpha$  is a constant
- $\llbracket \alpha \rrbracket^{M,g} = g(\alpha)$  if  $\alpha$  is a variable
- $\llbracket \alpha(\beta) \rrbracket^{M,g} = \llbracket \alpha \rrbracket^{M,g}(\llbracket \beta \rrbracket^{M,g})$
- $\llbracket \lambda v \alpha \rrbracket^{M,g} =$  that function  $f : D_\sigma \rightarrow D_\tau$  such that for all  $a \in D_\sigma$ ,  $f(a) = \llbracket \alpha \rrbracket^{M,g[v/a]}$  (for  $v$  a variable of type  $\sigma$ )
- $\llbracket \alpha = \beta \rrbracket^{M,g} = 1$  iff  $\llbracket \alpha \rrbracket^{M,g} = \llbracket \beta \rrbracket^{M,g}$
- $\llbracket \neg \varphi \rrbracket^{M,g} = 1$  iff  $\llbracket \varphi \rrbracket^{M,g} = 0$
- $\llbracket \varphi \wedge \psi \rrbracket^{M,g} = 1$  iff  $\llbracket \varphi \rrbracket^{M,g} = 1$  and  $\llbracket \psi \rrbracket^{M,g} = 1$
- $\llbracket \varphi \vee \psi \rrbracket^{M,g} = 1$  iff  $\llbracket \varphi \rrbracket^{M,g} = 1$  or  $\llbracket \psi \rrbracket^{M,g} = 1$
- $\llbracket \varphi \rightarrow \psi \rrbracket^{M,g} = 1$  iff  $\llbracket \varphi \rrbracket^{M,g} = 0$  or  $\llbracket \psi \rrbracket^{M,g} = 1$
- $\llbracket \exists v \varphi \rrbracket^{M,g} = 1$  iff there is an  $a \in D_\tau$  such that  $\llbracket \varphi \rrbracket^{M,g[v/a]} = 1$  (for  $v$  a variable of type  $\tau$ )
- $\llbracket \forall v \varphi \rrbracket^{M,g} = 1$  iff for all  $a \in D_\tau$ ,  $\llbracket \varphi \rrbracket^{M,g[v/a]} = 1$  (for  $v$  a variable of type  $\tau$ )

## 2 Generalized Quantifiers

**Definition 1** (Persistence:  $\uparrow mon$ ). A determiner  $D$  is persistent in  $M$  iff: for all  $X, Y, Z$ : if  $D(X, Z)$  and  $X \subseteq_M Y$ , then  $D(Y, Z)$

**Definition 2** (Antipersistence:  $\downarrow mon$ ). A determiner  $D$  is antipersistent in  $M$  iff: for all  $X, Y, Z$ : if  $D(X, Z)$  and  $Y \subseteq_M X$ , then  $D(Y, Z)$

**Definition 3** (Upward Monotonicity:  $mon \uparrow$ ). A quantifier  $Q$  is upward monotonic (or: monotone increasing) in  $M = \langle U, V \rangle$  iff  $Q$  is “closed under supersets”, i.e.: for all  $X, Y \subseteq U$ : if  $X \in Q$  and  $X \subseteq Y$ , then  $Y \in Q$

**Definition 4** (Downward Monotonicity:  $mon \downarrow$ ). A quantifier  $Q$  is downward monotonic (or: monotone decreasing) in  $M = \langle U, V \rangle$  iff  $Q$  is closed under inclusion: for all  $X, Y \subseteq U$ : if  $X \in Q$  and  $Y \subseteq X$ , then  $Y \in Q$

**Definition 5** (Conservativity). A quantifier  $Q$  is conservative in  $M = \langle U, V \rangle$  iff for every  $A, B \subseteq U$ :  $D(A, B) \Leftrightarrow D(A, A \cap B)$

## 3 DRT: Syntax

A discourse representation structure (DRS)  $K$  is a pair  $\langle U_K, C_K \rangle$  where  $U_K$  is a set of discourse referents, and  $C_K$  is a set of conditions.

The set of well-formed conditions is defined as follows:

- $R(u_1, \dots, u_n)$ , where  $R$  is an  $n$ -place relation and  $u_i \in U_K$
- $u = v$ , with  $u, v \in U_K$
- $u = a$ , with  $u \in U_K$  and  $a$  is a proper name
- $K_1 \Rightarrow K_2$ , where  $K_1$  and  $K_2$  are DRSs
- $K_1 \vee K_2$ , where  $K_1$  and  $K_2$  are DRSs
- $\neg K_1$ , where  $K_1$  is a DRS

## 4 DRT: Embedding, verifying embedding

Let  $U_D$  be a set of discourse referents,  $K = \langle U_K, C_K \rangle$  a DRS with  $U_K \subseteq U_D$ , and  $M = \langle U_M, V_M \rangle$  a model structure of first-order predicate logic that is suitable for  $K$ . An embedding of  $U_D$  into  $M$  is a (partial) function from  $U_D$  to  $U_M$  that assigns individuals from  $U_M$  to discourse referents.

An embedding  $f$  verifies the DRS  $K$  in  $M$  ( $f \models_M K$ ) iff

1.  $U_K \subseteq \text{Dom}(f)$ , and
2.  $f$  verifies each condition  $\alpha \in C_K$ .

$f$  verifies a condition  $\alpha$  in  $M$  ( $f \models_M \alpha$ ) in the following cases:

- $f \models_M R(u_1, \dots, u_n)$  iff  $\langle f(u_1), \dots, f(u_n) \rangle \in V_M(R)$
- $f \models_M u = v$  iff  $f(u) = f(v)$
- $f \models_M u = a$  iff  $f(u) = V_M(a)$
- $f \models_M K_1 \Rightarrow K_2$  iff for all  $g \supseteq_{U_{K_1}} f$  such that  $g \models_M K_1$ , there is a  $h \supseteq_{U_{K_2}} g$  such that  $h \models_M K_2$
- $f \models_M \neg K_1$  iff there is no  $g \supseteq_{U_{K_1}} f$  such that  $g \models_M K_1$
- $f \models_M K_1 \vee K_2$  iff there is a  $g_1 \supseteq_{U_{K_1}} f$  such that  $g_1 \models_M K_1$ , or there is a  $g_2 \supseteq_{U_{K_2}} f$  such that  $g_2 \models_M K_2$ .