Semantic Theory
Week 8 – Discourse Representation Theory

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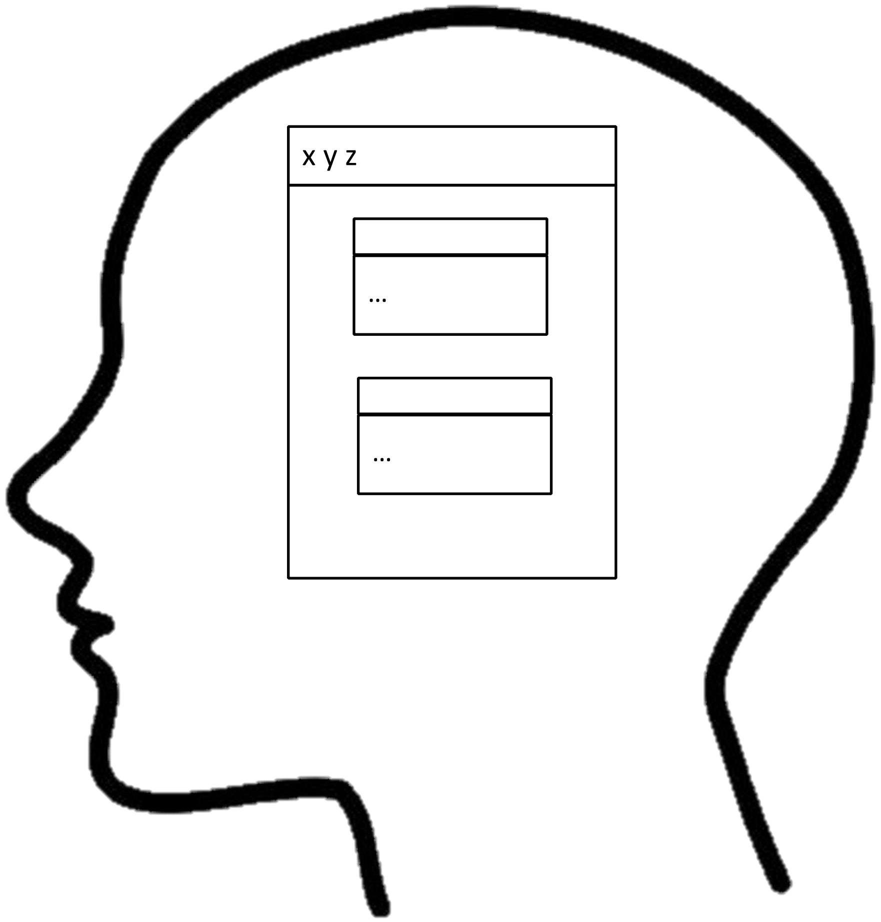
Discourse Representation Theory

Mentalist and representationalist theory of the interpretation of discourse

- Discourse Representation Structures
- Construction procedure for DRSs
- Model-theoretic interpretation at the discourse level



(Kamp, 1981; Kamp & Reyle, 1993)

DRS Syntax

A discourse representation structure (DRS) K is a pair $\langle U_K, C_K \rangle$, where:

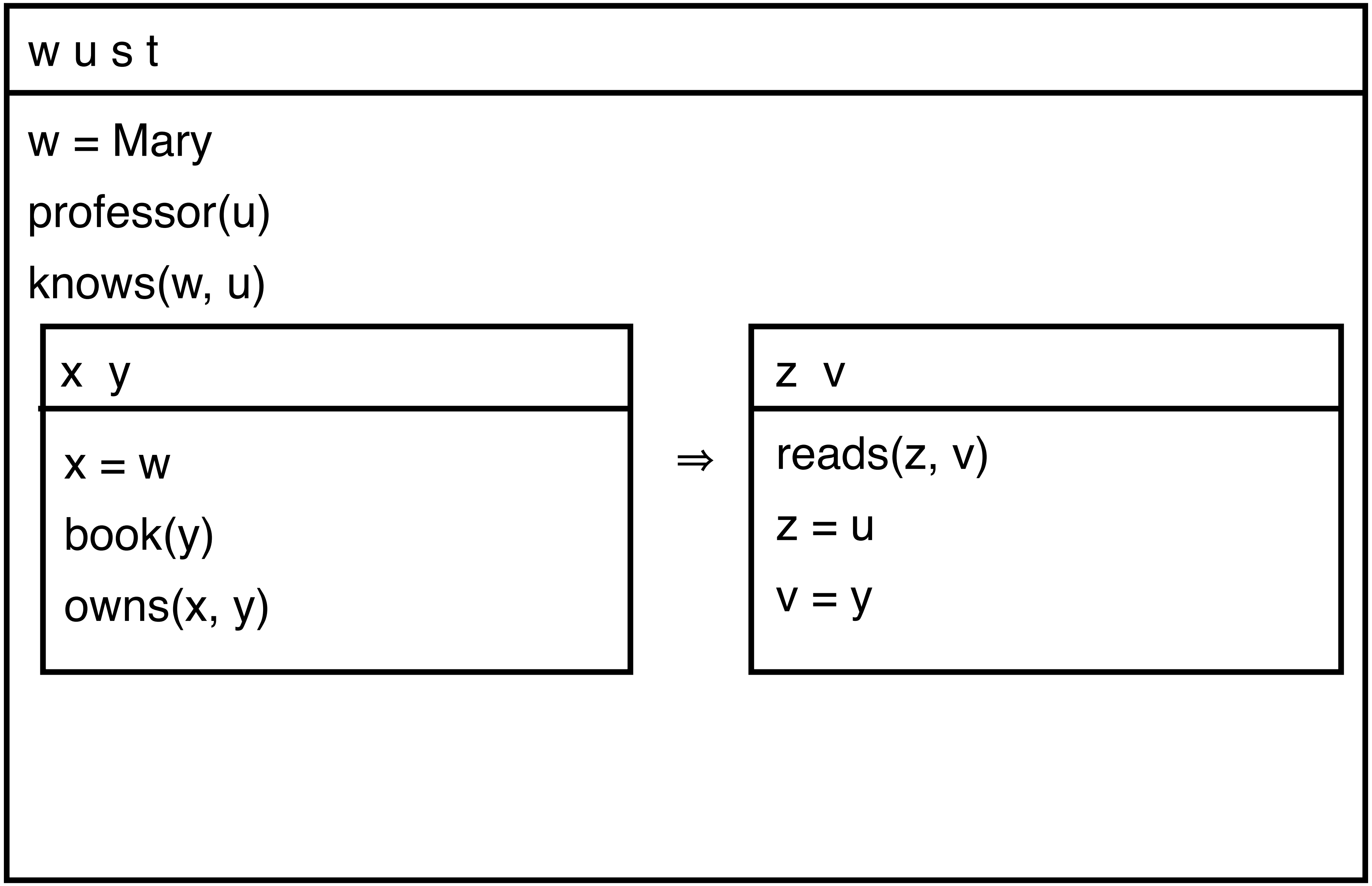
- $U_K \subseteq U_D$ and U_D is a set of discourse referents, and
- C_K is a set of well-formed DRS conditions

Well-formed DRS conditions:

• $R(u_1, \ldots, u_n)$	*where:*	R is an n-place relation, $u_i \in U_D$
• $u = v$		$u, v \in U_D$
• $u = a$		$u \in U_D$, a is a constant
• $\neg K_1$		K_1 is a DRS
• $K_1 \Rightarrow K_2$		K_1 and K_2 are DRSs
• $K_1 \vee K_2$		K_1 and K_2 are DRSs

Anaphora and accessibility

Mary knows a professor. If she owns a book, he reads it.



w u s t

w = Mary

professor(u)

knows(w, u)

x y

x = w

book(y)

owns(x, y)

⇒

z v

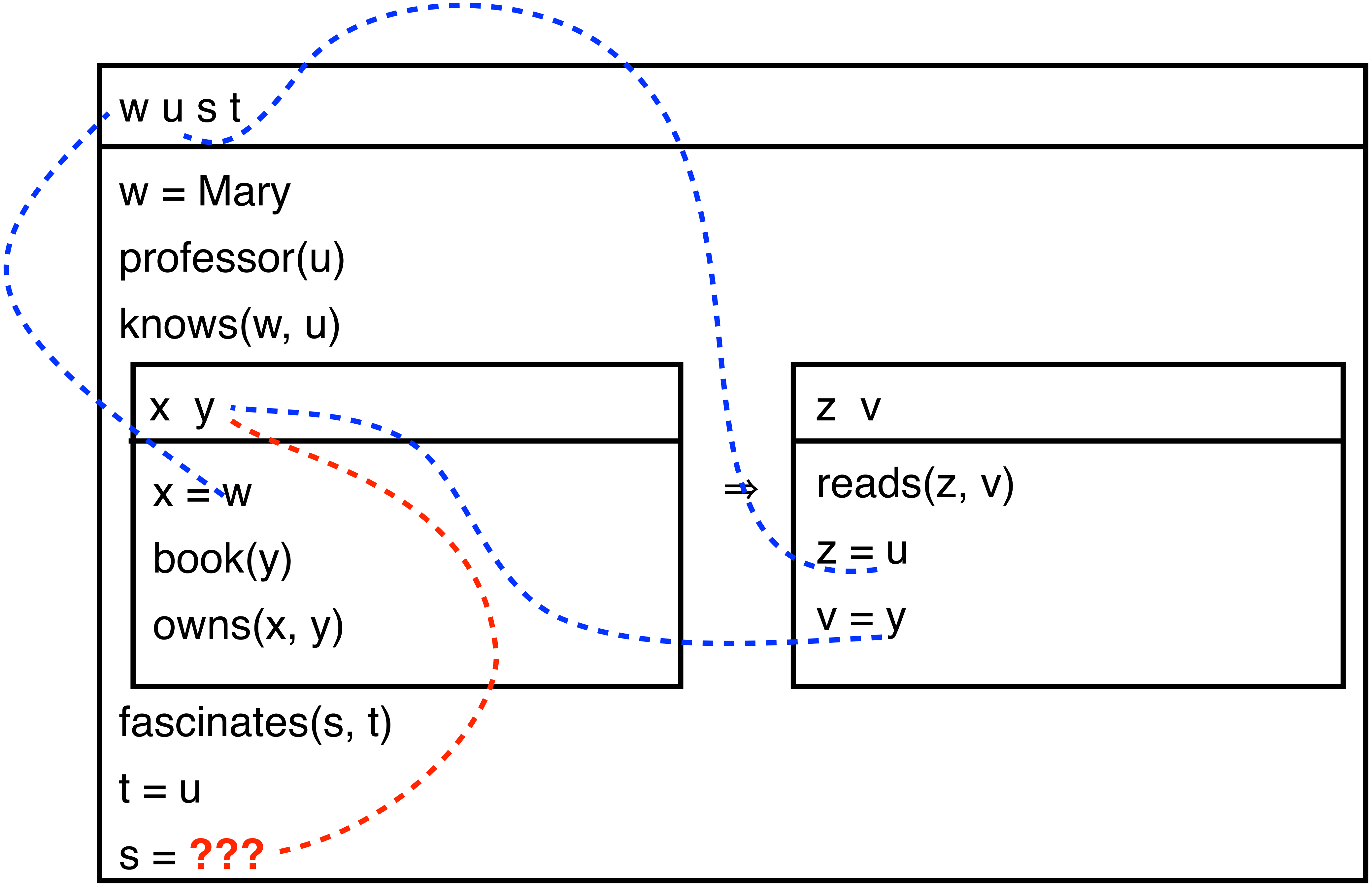
reads(z, v)

z = u

v = y

Anaphora and accessibility

Mary knows a professor. If she owns a book, he reads it. ?It fascinates him.



w u s t

w = Mary

professor(u)

knows(w, u)

x y

x = w

book(y)

owns(x, y)

⇒

z v

reads(z, v)

z = u

v = y

fascinates(s, t)

t = u

s = ???

Non-accessible discourse referents

Cases of non-accessibility:

(1) *If a professor owns a book, he reads it. It has 300 pages.*

(2) *It is not the case that a professor owns a book. He reads it.*

(3) *Every professor owns a book. He reads it.*

(4) *If every professor owns a book, he reads it.*

(5) *Peter owns a book, or Mary reads it.*

(6) *Peter reads a book, or Mary reads a newspaper article. It is interesting.*

Accessible discourse referents

The following discourse referents are accessible for a condition:

- DRs in the same local DRS
- DRs in a superordinate DRS
- DRs in the universe of an antecedent DRS, if the condition occurs in the consequent DRS.

We need a formal notion of DRS subordination

Subordination

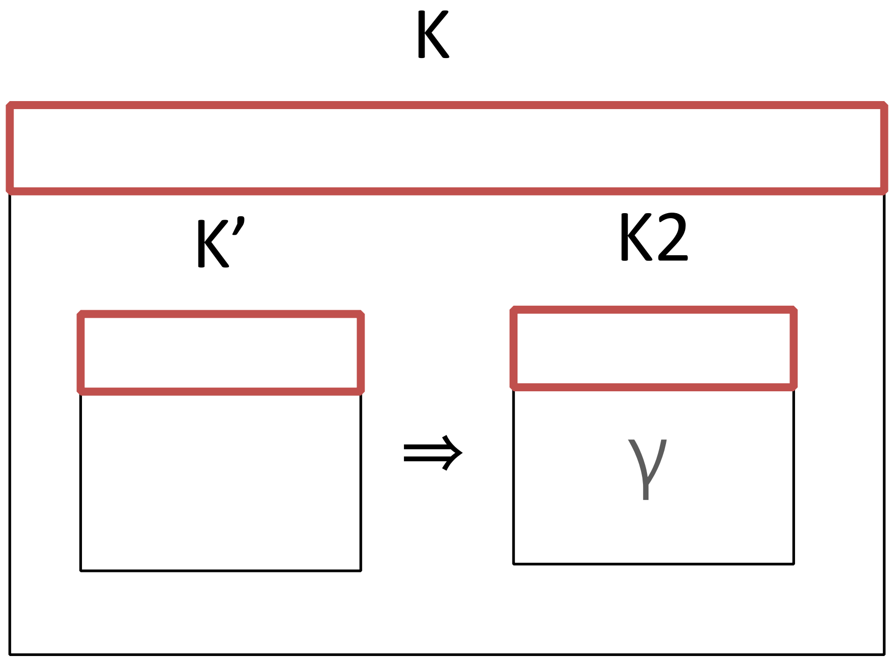
- DRS K_1 is an **immediate sub-DRS** of a DRS $K = \langle U_K, C_K \rangle$ *iff*
  - C_K contains a condition of the form: $\neg K_1$, $K_1 \Rightarrow K_2$, $K_2 \Rightarrow K_1$, $K_1 \vee K_2$ or $K_2 \vee K_1$.
- DRS K_1 is a **sub-DRS** of DRS K (notation: $K_1 \leq K$) *iff*
  - $K_1 = K$, or
  - K_1 is an immediate sub-DRS of K, or
  - there is a DRS K_2 such that $K_1 \leq K_2$ and K_2 is an immediate sub-DRS of K.
- DRS K_1 is a **proper sub-DRS** of DRS K *iff*
  - $K_1 \leq K$ and $K_1 \neq K$.

Accessibility

Let K, K_1, K_2 be DRSs such that $K_1, K_2 \leq K$, $x \in U_{K_1}$, $\gamma \in C_{K_2}$

x is **accessible** from γ in K iff

- $K_2 \leq K_1$ or
- there are $K_3, K_4 \leq K$ such that $K_1 \Rightarrow K_3 \in C_{K_4}$ and $K_2 \leq K_3$



Free and bound variables in DRT

A DRS variable x, introduced in the conditions of DRS K_i, is **bound** in global DRS K iff

- there exists a DRS $K_j \leq K$, such that:
  - (i) $x \in U(K_j)$, and
  - (ii) K_j is accessible for K_i in K

Properness: A DRS is **proper** iff it does not contain any free variables

Purity: A DRS is **pure** iff it does not contain any *otiose declarations* of variables

$x \in U(K_1)$ and $x \in U(K_2)$ and $K_1 \leq K_2$

From text to DRS

$$\begin{tikzcd}
\text{Text} & \Sigma = \langle & S_1, \arrow[d] & S_2, \arrow[d] & \ldots, & S_n \rangle \arrow[d] \\
\text{Syntactic Analysis} & & P(S_1) \arrow[d] & P(S_2) \arrow[d] & \ldots & P(S_n) \arrow[d] \\
\text{DRS Construction} & & K_1 \arrow[r] & K_2 \arrow[r] & \ldots \arrow[r] & K_n
\end{tikzcd}$$

DRS Construction Algorithm

Let the following be a well-formed, *reducible* DRS condition:

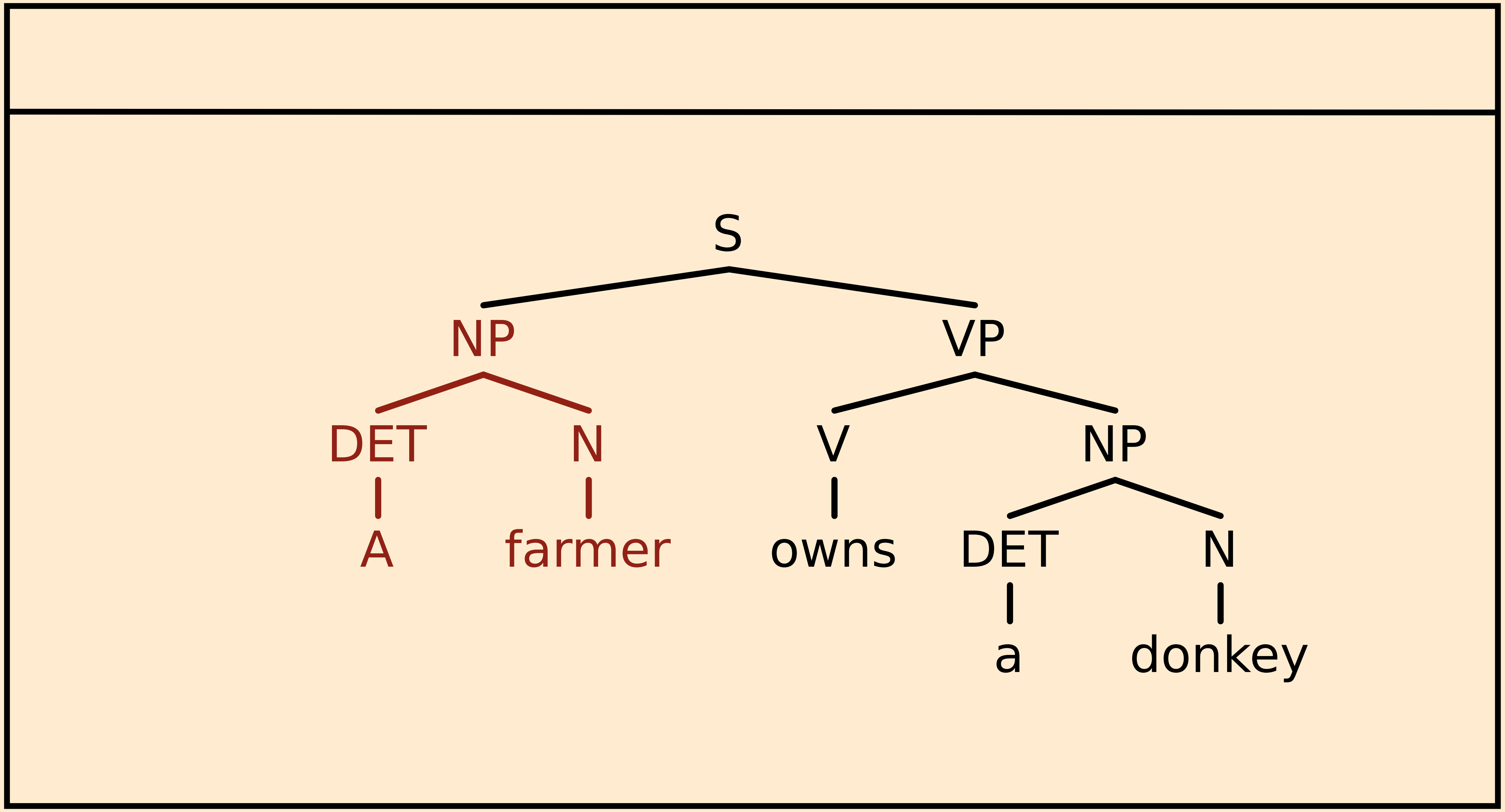
- Conditions of form α or α(x1, …, xn), where α is a context-free parse tree.

DRS construction algorithm:

- Given a text $\Sigma = \langle S_1, \ldots, S_n \rangle$, and a DRS K_0 (= $\langle \varnothing, \varnothing \rangle$, by default)
- Repeat for i = 1, …, n:
    - Add parse tree $P(S_i)$ to the conditions of K_{i-1}.
    - Apply DRS construction rules to reducible conditions of K_{i-1}, until no reduction steps are possible any more.
    - The resulting DRS K_i is the discourse representation of text $\langle S_1, \ldots, S_i \rangle$.

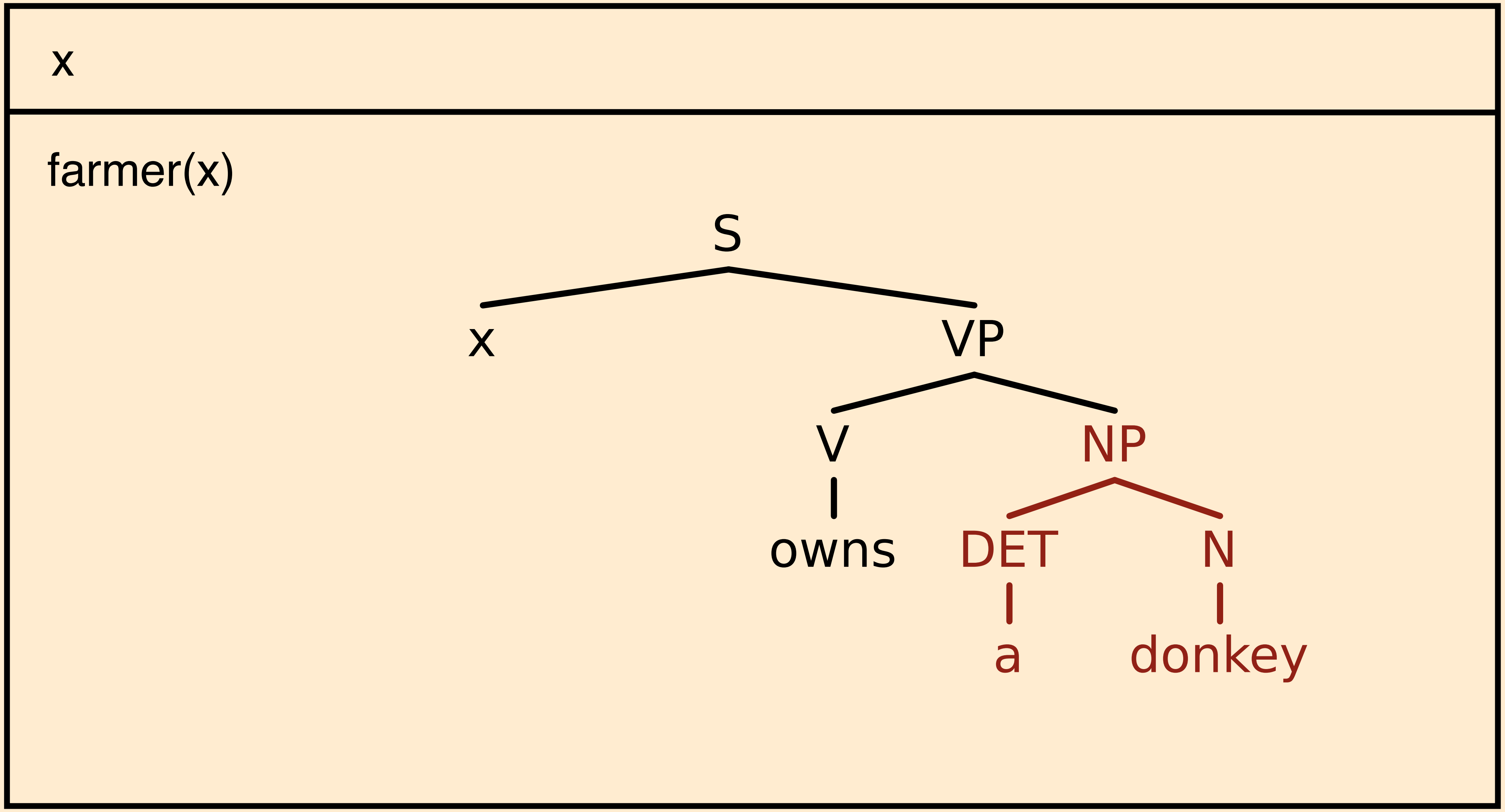
DRS Construction Example

- A farmer owns a donkey. He beats it.



DRS Construction Example

- A farmer owns a donkey. He beats it.



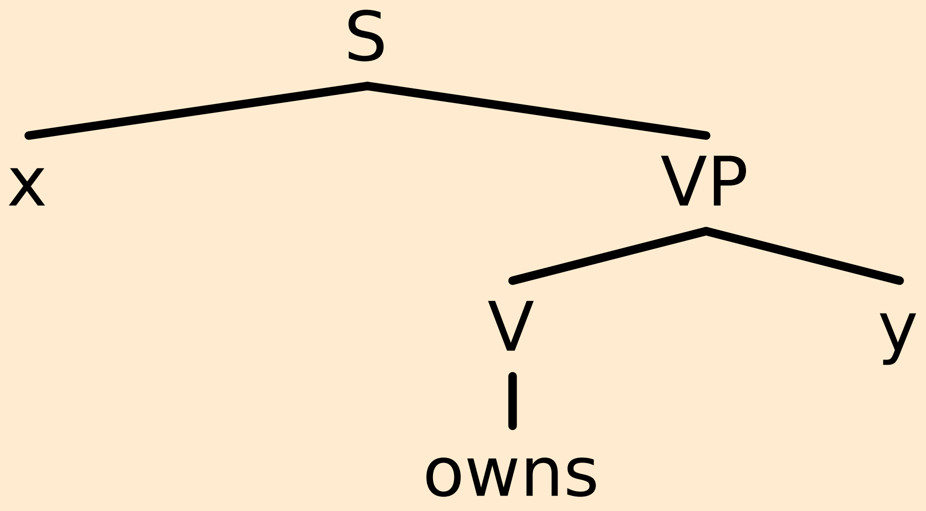
DRS Construction Example

- A farmer owns a donkey. He beats it.

x y

farmer(x)

donkey(y)



DRS Construction Example

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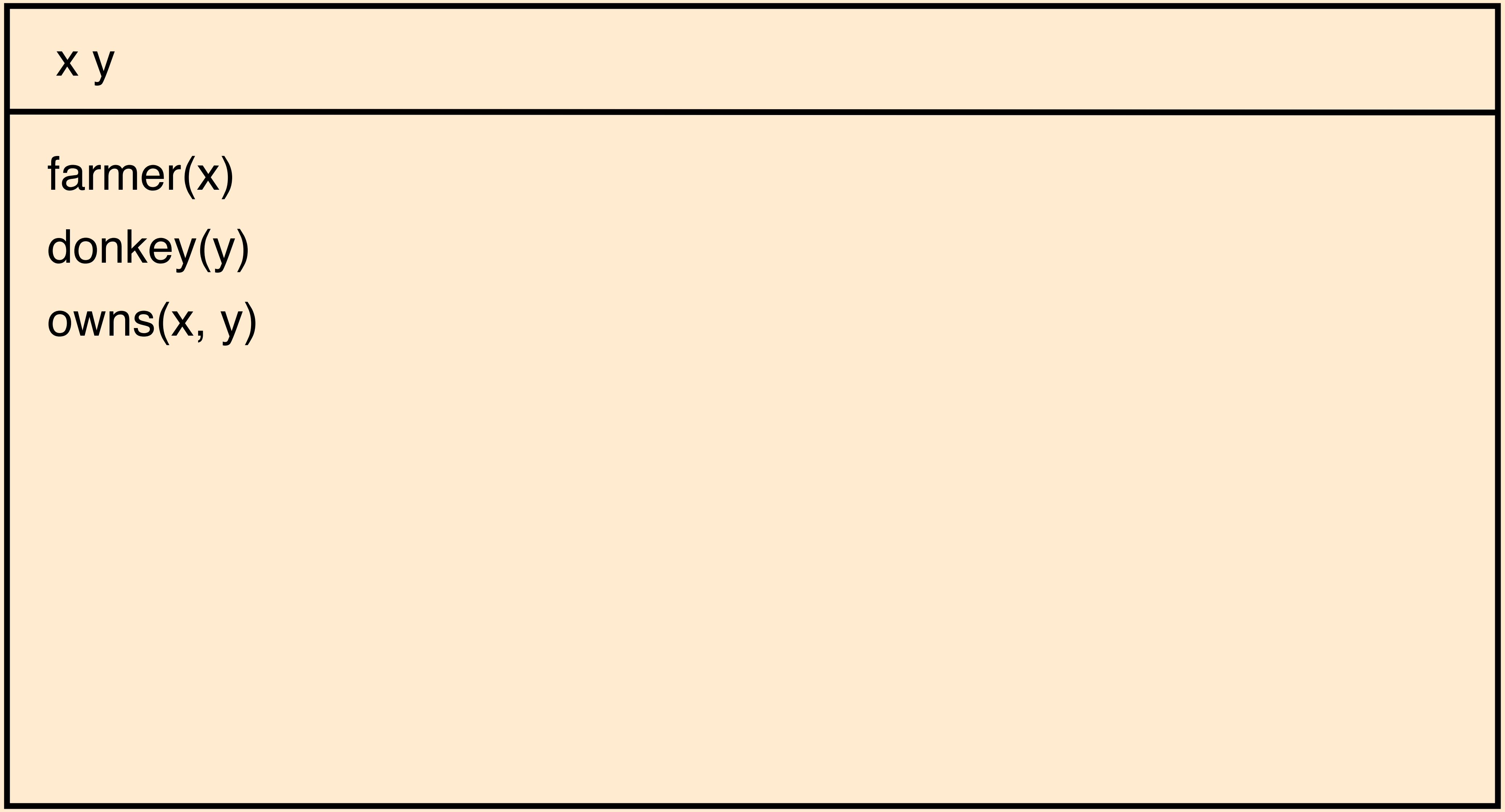


x y

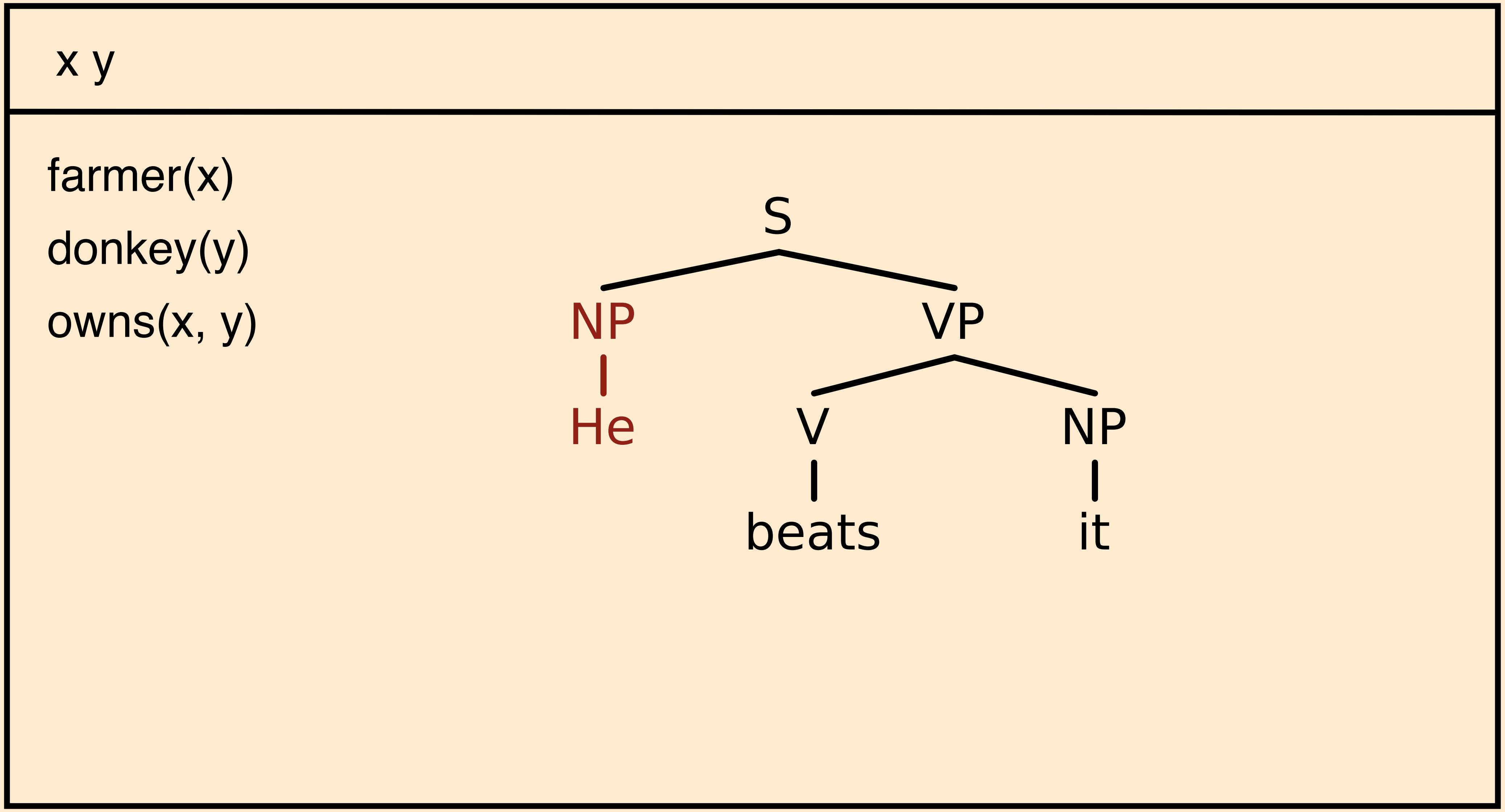
farmer(x)

donkey(y)

owns(x, y)

DRS Construction Example

- A farmer owns a donkey. He beats it.



x y

farmer(x)

donkey(y)

owns(x, y)

DRS Construction Example

- A farmer owns a donkey. He beats it.

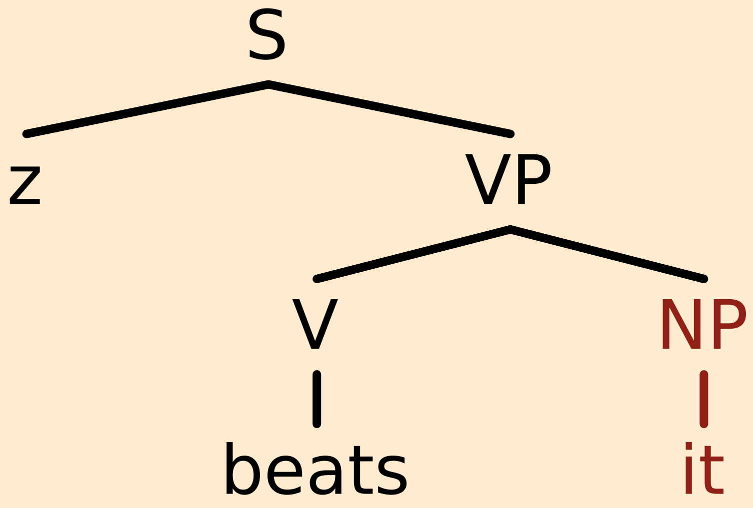
x y z

farmer(x)

donkey(y)

owns(x, y)

z = x



DRS Construction Example

- A farmer owns a donkey. He beats it.

x y z u

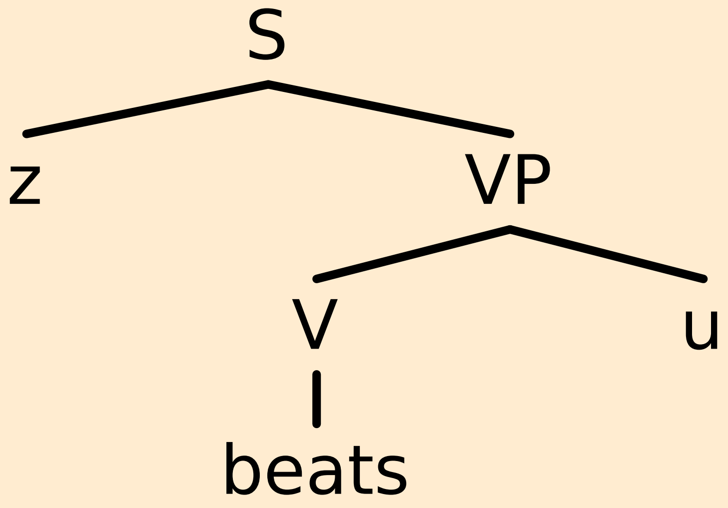
farmer(x)

donkey(y)

owns(x, y)

z = x

u = y



DRS Construction Example

- A farmer owns a donkey. He beats it.

x y z u

farmer(x)

donkey(y)

owns(x, y)

z = x

u = y

beat(z, u)

Construction Rules: Examples

Indefinite NPs

- **Trigger:** a reducible condition α in DRS K that has a substructure [NP β], such that β is εδ, where ε is an indefinite article
- **Action:** Add new DR x to U_K; Replace β in α by x; Add δ(x) to C_K

Personal Pronouns

- **Trigger:** a global DRS K*, and some $K \leq K^*$, with a reducible condition α in K that has substructure [NP β], such that β is a personal pronoun
- **Action:** Add a new DR x to U_K; Replace β in α by x; Select an appropriate DR y that is accessible from α in K*; Add $x = y$ to C_K

A constraint on DRS construction

Problem: The basic DRS construction algorithm can derive DRSs for both of the following sentences, with the indicated anaphoric binding:

(1) [A professor]$_i$ recommends a book that she$_i$ likes

(2) She$_i$ recommends a book that [a professor]$_i$ likes

Solution: If two different triggering configurations occur in a reducible condition, then first apply the construction rule to the highest triggering configuration.

- *The highest triggering configuration* is the one whose top node dominates the top nodes of all other triggering configurations.

From text to DRS to models

$$\begin{tikzcd}
\text{Text} & \Sigma = \langle & S_1, \arrow[d] & S_2, \arrow[d] & \ldots, & S_n \rangle \arrow[d] \\
\text{Syntactic Analysis} & & P(S_1) \arrow[d] & P(S_2) \arrow[d] & \ldots & P(S_n) \arrow[d] \\
\text{DRS Construction} & & K_1 \arrow[r] & K_2 \arrow[r] & \ldots \arrow[r] & K_n \arrow[d] \\
 & & & & & \text{Interpretation by model embedding: Truth-conditions of } \Sigma
\end{tikzcd}$$

DRS Interpretation

Given a DRS $K = \langle U_K, C_K \rangle$, with $U_K \subseteq U_D$

Let $M = \langle U_M, V_M \rangle$ be a FOL model structure appropriate for K, i.e. a model structure that provides interpretations for all predicates and relations occurring in K

DRS K is **true** in model M *iff*

- there is an embedding function for K in M which verifies all conditions in K

… where: an embedding of K into M is a (partial) function **f** from U_D to U_M such that $U_K \subseteq \mathrm{Dom}(\mathbf{f})$.

Verifying embedding

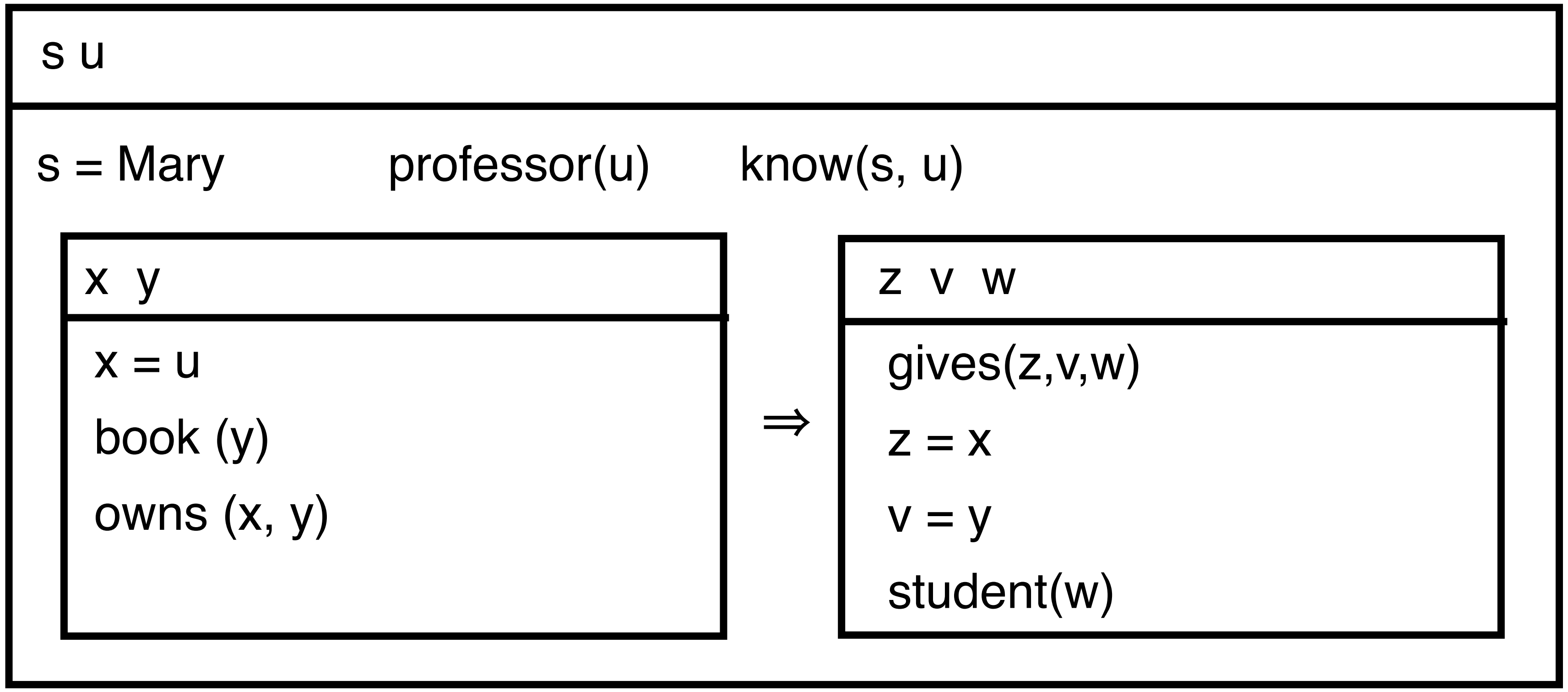
An embedding **f** of K in M verifies K in M (**f** $\vDash_M$ K) iff **f** verifies every condition $\alpha \in C_K$

- **f** $\vDash_M R(x_1, \ldots, x_n)$ iff $\langle \mathbf{f}(x_1), \ldots, \mathbf{f}(x_n) \rangle \in V_M(R)$
- **f** $\vDash_M x = y$ iff $\mathbf{f}(x) = \mathbf{f}(y)$
- **f** $\vDash_M x = a$ iff $\mathbf{f}(x) = V_M(a)$
- **f** $\vDash_M \neg K_1$ iff there is no **g** $\supseteq_{U_{K1}}$ **f** such that g $\vDash_M K_1$
- **f** $\vDash_M K_1 \Rightarrow K_2$ iff for all **g** $\supseteq_{U_{K1}}$ **f** such that **g** $\vDash_M K_1$

  there is a **h** $\supseteq_{U_{K2}}$ **g** such that **h** $\vDash_M K_2$
- **f** $\vDash_M K_1 \vee K_2$ iff there is a $\mathbf{g_1} \supseteq_{U_{K1}}$ **f** such that $\mathbf{g_1} \vDash_M K_1$
  or there is a $\mathbf{g_2} \supseteq_{U_{K2}}$ **f** such that $\mathbf{g_2} \vDash_M K_2$

Verifying embedding: example

Mary knows a professor. If he owns a book, he gives it to a student.



s u

s = Mary

professor(u)

know(s, u)

x y

x = u

book (y)

owns (x, y)

⇒

z v w

gives(z,v,w)

z = x

v = y

student(w)

…is **true** in $M = \langle U_M, V_M \rangle$ *iff* there is an **f** :: $U_D \rightarrow U_M$, (with $\{s,u\} \subseteq \mathrm{Dom}(\mathbf{f})$) such that:

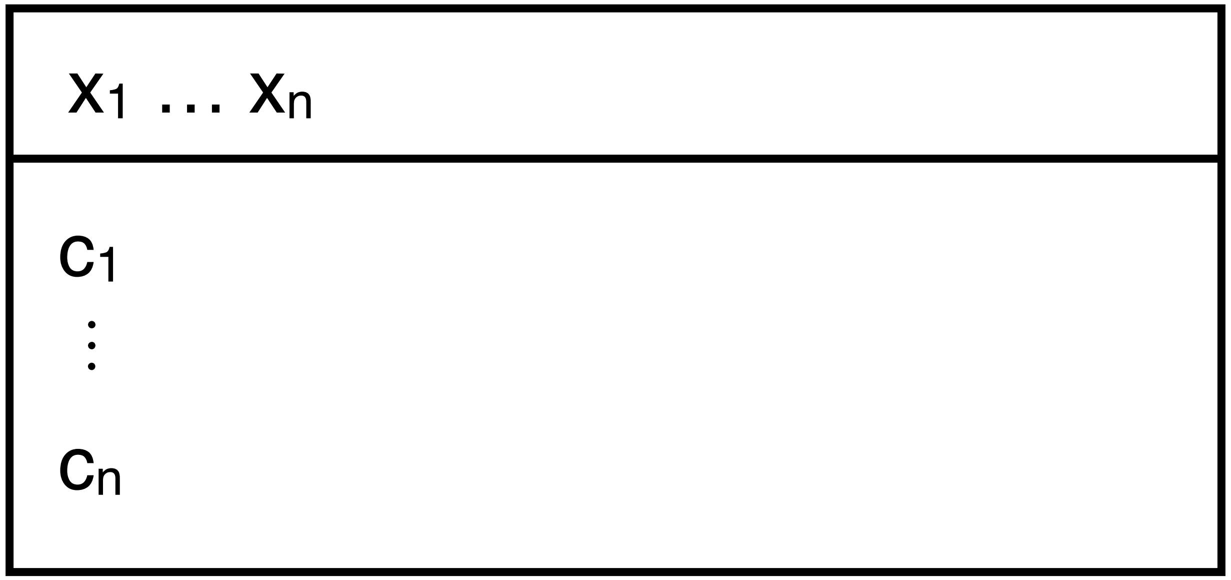
$\mathbf{f}(s) = V_M(\mathrm{Mary})$ & $\mathbf{f}(u) \in V_M(\mathrm{prof'})$ & $\langle \mathbf{f}(s), \mathbf{f}(u) \rangle \in V_M(\mathrm{know})$,

and for all $\mathbf{g} \supseteq_{\{x,y\}} \mathbf{f}$ s.t. $\mathbf{g}(x) = \mathbf{g}(u)$ $(=\mathbf{f}(u))$ & $\mathbf{g}(y) \in V_M(\mathrm{book})$ & $\langle \mathbf{g}(x), \mathbf{g}(y) \rangle \in V_M(\mathrm{own})$,

there is a $\mathbf{h} \supseteq_{\{z,\, v,\, w\}} \mathbf{g}$ s.t. $\langle \mathbf{h}(z), \mathbf{h}(v), \mathbf{h}(w) \rangle \in V_M(\mathrm{give})$ & $\mathbf{h}(z) = \mathbf{h}(x)$ $(=\mathbf{g}(x))$ & … etc.

Translation of DRSs to FOL

Consider DRS $K = \langle\{x_1, \ldots, x_n\}, \{c_1, \ldots, c_k\}\rangle$


$x_1 \ldots x_n$

c_1

$\vdots$

c_n

K is truth-conditionally equivalent to the following FOL formula:

$$\exists x_1 \ldots \exists x_n [c_1 \wedge \ldots \wedge c_k]$$

DRT and compositionality

- DRT is non-compositional on truth conditions: The difference in discourse-semantic status of the text pairs is not predictable through the (identical) truth conditions of its component sentences.

- Since structural information which cannot be reduced to truth conditions is required to compute the semantic value of texts, DRT is called a *representational* theory of meaning.

However…

Wait a minute …

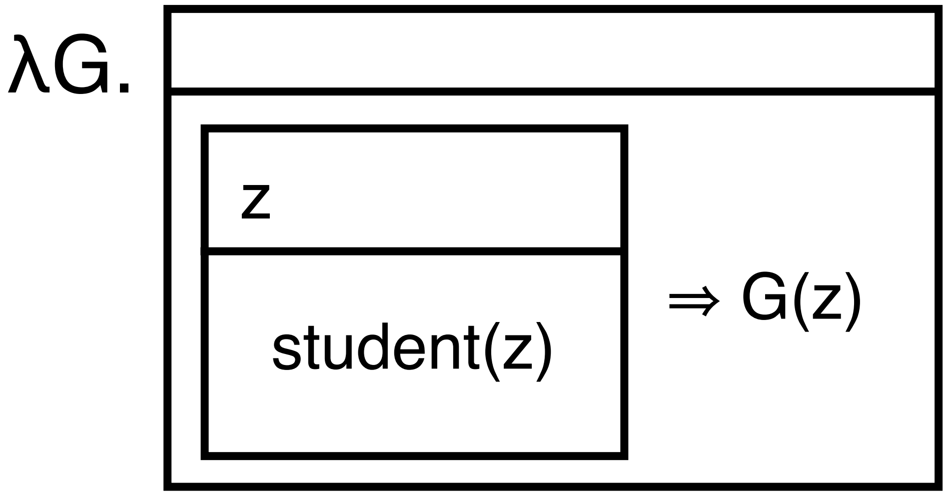
- Why can’t we just combine type theoretic semantics and DRT?
- Use λ-abstraction and reduction as we did before, but:
- Assume that the target representations which we want to arrive at are not First-Order Logic formulas, but DRSs.
- The result is called λ-DRT.

λ-DRSs

Muskens (1996), *Ling. and phil.*

An expression in λ-DRT consists of a lambda prefix and a partially instantiated DRS.

- *every student :: ⟨⟨e, t⟩, t⟩* ↦



λG.

z

student(z)

⇒ G(z)

Alternative notation: λG [∅ | [z | student(z)] ⇒ G(z)]

- *works :: ⟨e, t⟩* ↦ λx [∅ | work(x)]

λ-DRT: β-reduction

Every student works

$\mapsto \lambda G[\ \varnothing \mid [\ z \mid \text{student}(z)\ ] \Rightarrow G(z)\ ]](\lambda x\ [\ \varnothing \mid \text{work}(x)\ ])$

$\Rightarrow^{\beta} [\ \varnothing \mid [\ z \mid \text{student}(z)\ ] \Rightarrow (\lambda x\ [\ \varnothing \mid \text{work}(x)\ ])(z)\ ]$

$\Rightarrow^{\beta} [\ \varnothing \mid [\ z \mid \text{student}(z)\ ] \Rightarrow [\ \varnothing \mid \text{work}(z)\ ]]$

How do we define conjunction on DRSs?

(Naïve) Merge

The “merge” operation on DRSs combines two DRSs (conditions and universes).

- Let $K_1 = [\ U_1 \mid C_1\ ]$ and $K_2 = [\ U_2 \mid C_2\ ]$.

Merge: $K_1 + K_2 = [\ U_1 \cup U_2 \mid C_1 \cup C_2\ ]$

Merge: An example

- *a student* ↦ $\lambda G\,([\,z \mid \text{student}(z)\,] + G(z))$
- *works* ↦ $\lambda x\,[\,\varnothing \mid \text{work}(x)\,]$

A student works ↦ $\lambda G\,([\,z \mid \text{student}(z)\,] + G(z))\,(\lambda x[\,\varnothing \mid \text{work}(x)])$

$\Rightarrow^{\beta}$ $[\,z \mid \text{student}(z)\,] + \lambda x[\,\varnothing \mid \text{work}(x)](z)$

$\Rightarrow^{\beta}$ $[\,z \mid \text{student}(z)\,] + [\,\varnothing \mid \text{work}(z)]$

$\Rightarrow^{\beta}$ $[\,z \mid \text{student}(z), \text{work}(z)]$

Compositional analysis

- *Mary* $\mapsto$ $\lambda G\,([\, z \mid z = \text{Mary}\,] + G(z))$
- she $\mapsto$ $\lambda G.G(z)$

Mary works. She is successful.

$$\mapsto \lambda K\, \lambda K'(K + K')([\, z \mid z = \text{Mary, work}(z)])([\ \mid \text{successful}(z)])$$

$$\Rightarrow^{\beta} \lambda K'([\, z \mid z = \text{Mary, work}(z)] + K')([\ \mid \text{successful}(z)])$$

$$\Rightarrow^{\beta} [\, z \mid z = \text{Mary, work}(z)] + ([\ \mid \text{successful}(z)])$$

$$\Rightarrow^{\beta} [z \mid z = \text{Mary, work}(z), \text{successful}(z)]$$

Merge again

The “merge” operation on DRSs combines two DRSs (conditions and universes).

- Let $K_1 = [\ U_1 \mid C_1\ ]$ and $K_2 = [\ U_2 \mid C_2\ ]$.

Merge: $K_1 + K_2 \Rightarrow [\ U_1 \cup U_2 \mid C_1 \cup C_2\ ]$
under the assumption that no discourse referent $u \in U_2$ occurs free in a condition $\gamma \in C_1$.

Note that under this definition Merge is directional:
$K_1 + K_2 \nLeftrightarrow K_2 + K_1$

Variable capturing

In λ-DRT, discourse referents are captured via the interaction of β-reduction and DRS-binding:

λK'([z | student(z), work(z)] + K')([| successful(z)])

$\Rightarrow^{\beta}$ [z | student(z), work(z)] + [| successful(z)]

$\Rightarrow^{\beta}$ [z | student(z), work(z), successful(z)]

- But the β-reduced DRS must be *equivalent* to the original DRS!
- This means that the potential for capturing discourse referents must be captured in the interpretation of λ-DRSs.

➡ Possible, but tricky.

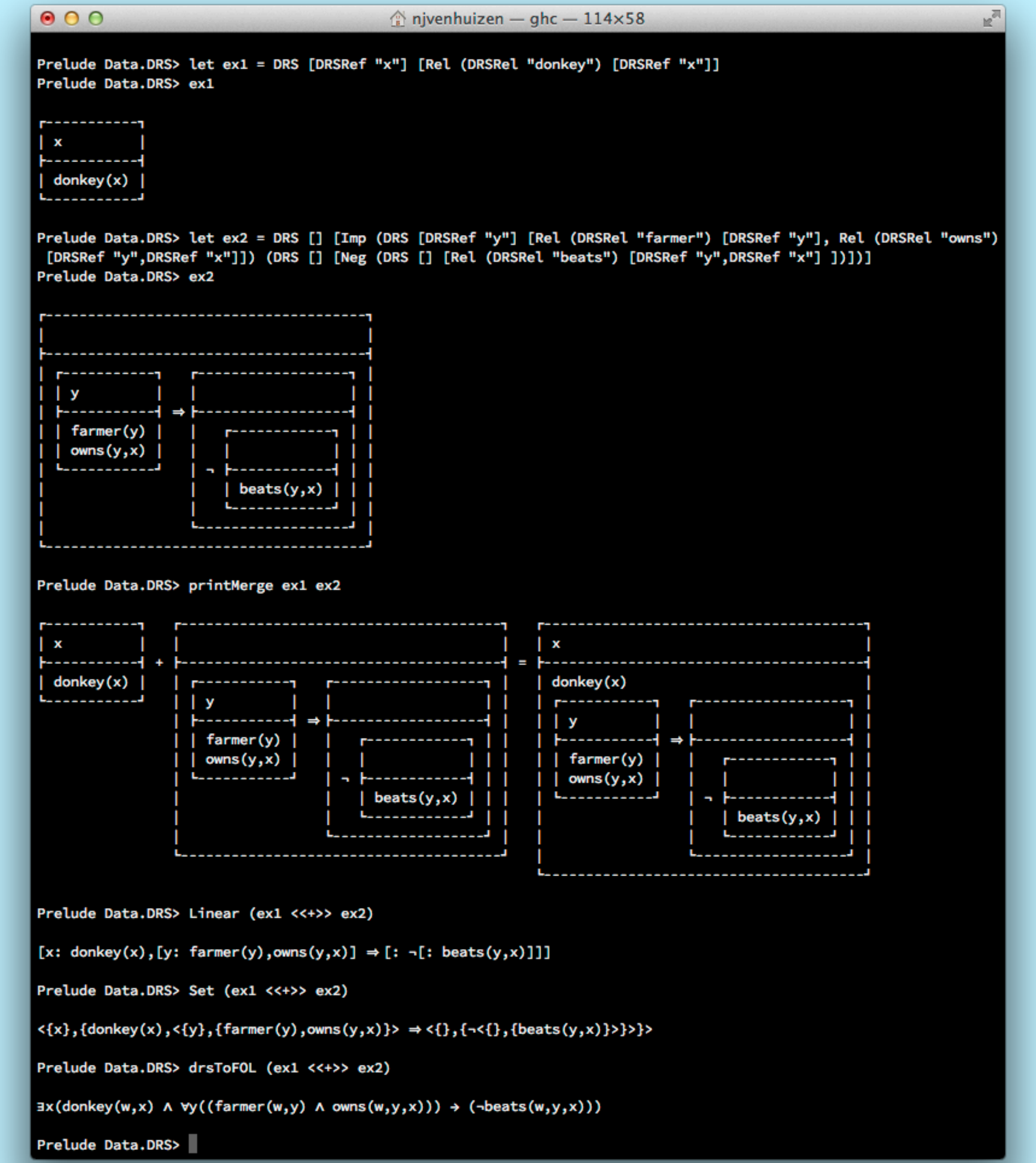
Playing in the sandbox

PDRT-SANDBOX is a Haskell library that implements Discourse Representation Theory (and the extension Projective DRT)

http://hbrouwer.github.io/pdrt-sandbox/
also available via: login.coli.uni-saarland.de:/proj/courses/semantics-19

- Define your own DRSs, using the internal syntax or the set-theoretic notation
- Show the DRSs in different output formats (boxes, linear, set-theoretic, internal syntax)
- Composition of DRSs (using lambda's)
- Translate DRSs to FOL formulas

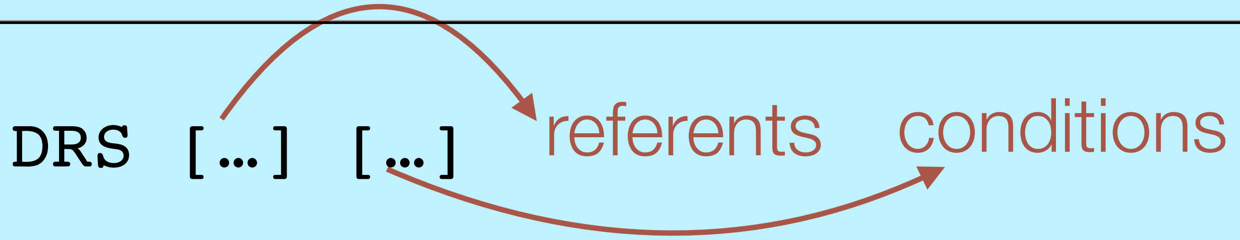
Playing in the sandbox



```
Prelude Data.DRS> let ex1 = DRS [DRSRef "x"] [Rel (DRSRel "donkey") [DRSRef "x"]]
Prelude Data.DRS> ex1

┌-----------┐
| x         |
├-----------┤
| donkey(x) |
└-----------┘

Prelude Data.DRS> let ex2 = DRS [] [Imp (DRS [DRSRef "y"] [Rel (DRSRel "farmer") [DRSRef "y"], Rel (DRSRel "owns")
 [DRSRef "y",DRSRef "x"]]) (DRS [] [Neg (DRS [] [Rel (DRSRel "beats") [DRSRef "y",DRSRef "x"] ])])]
Prelude Data.DRS> ex2

┌--------------------------------------┐
|                                      |
├--------------------------------------┤
| ┌-----------┐   ┌------------------┐ |
| | y         |   |                  | |
| ├-----------┤ ⇒ ├------------------┤ |
| | farmer(y) |   |   ┌------------┐ | |
| | owns(y,x) |   |   |            | | |
| └-----------┘   | ¬ ├------------┤ | |
|                 |   | beats(y,x) | | |
|                 |   └------------┘ | |
|                 └------------------┘ |
└--------------------------------------┘

Prelude Data.DRS> printMerge ex1 ex2

┌-----------┐   ┌--------------------------------------┐   ┌--------------------------------------┐
| x         |   |                                      |   | x                                    |
├-----------┤ + ├--------------------------------------┤ = ├--------------------------------------┤
| donkey(x) |   | ┌-----------┐   ┌------------------┐ |   | donkey(x)                            |
└-----------┘   | | y         |   |                  | |   | ┌-----------┐   ┌------------------┐ |
                | ├-----------┤ ⇒ ├------------------┤ |   | | y         |   |                  | |
                | | farmer(y) |   |   ┌------------┐ | |   | ├-----------┤ ⇒ ├------------------┤ |
                | | owns(y,x) |   |   |            | | |   | | farmer(y) |   |   ┌------------┐ | |
                | └-----------┘   | ¬ ├------------┤ | |   | | owns(y,x) |   |   |            | | |
                |                 |   | beats(y,x) | | |   | └-----------┘   | ¬ ├------------┤ | |
                |                 |   └------------┘ | |   |                 |   | beats(y,x) | | |
                |                 └------------------┘ |   |                 |   └------------┘ | |
                └--------------------------------------┘   |                 └------------------┘ |
                                                           └--------------------------------------┘

Prelude Data.DRS> Linear (ex1 <<+>> ex2)

[x: donkey(x),[y: farmer(y),owns(y,x)] ⇒ [: ¬[: beats(y,x)]]]

Prelude Data.DRS> Set (ex1 <<+>> ex2)

<{x},{donkey(x),<{y},{farmer(y),owns(y,x)}> ⇒ <{},{¬<{},{beats(y,x)}>}>}>

Prelude Data.DRS> drsToFOL (ex1 <<+>> ex2)

∃x(donkey(w,x) ∧ ∀y((farmer(w,y) ∧ owns(w,y,x))) → (¬beats(w,y,x)))

Prelude Data.DRS> 
```

PDRT
SANDBOX

DRS Syntax in PDRT-SANDBOX



DRS: `DRS […] […]`

Referents: `DRSRef “x”, DRSRef “Mary”`

Conditions:

- **Relation:** `Rel (DRSRel "man") [DRSRef “x"]`
- **Identity:** `Rel (DRSRel "=") [DRSRef “x”,DRSRef “y"]`
- **Negation:** `Neg (DRS […] […])`
- **Implication:** `Imp (DRS […] […]) (DRS […] […])`
- **Disjunction:** `Or (DRS […] […]) (DRS […] […])`

Properties: `isPure(DRS […] […]), isProper(DRS […] […])`

Using PDRT-SANDBOX on **coli**

```
~$ cp -r /proj/courses/semantics-19/pdrt-sandbox/ .
~$ cp /proj/courses/semantics-19/ghci .ghci
~$ cd pdrt-sandbox/
~/pdrt-sandbox$ make
[…]
~/pdrt-sandbox$ cd tutorials/
~/pdrt-sandbox/tutorials$ ghci DRSTutorial.hs
GHCi, version 7.10.3: http://www.haskell.org/ghc/  :? for help
[1 of 1] Compiling Main             ( DRSTutorial.hs, interpreted )
Ok, modules loaded: Main.
*Main>
```

Literature

References:

- Hans Kamp and Uwe Reyle. From Discourse to Logic, Kluwer: Dordrecht 1993.
- Reinhard Muskens. "Combining Montague semantics and discourse representation." *Linguistics and philosophy* (1996): 143-186.

Background reading:

- https://plato.stanford.edu/entries/discourse-representation-theory/