Semantic Theory
Week 5 – Typed Lambda Calculus

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Compositionality

The principle of compositionality: “The meaning of a complex expression is a function of the meanings of its parts and of the syntactic rules by which they are combined” (Partee et al.,1993)

Compositional semantics construction:

- compute meaning representations for sub-expressions
- combine them to obtain a meaning representation for a complex expression.

Problematic case: “Not smoking$_{\langle e,t\rangle}$ is healthy$_{\langle\langle e,t\rangle,t\rangle}$”

!

Lambda abstraction

λ-abstraction is the operation that transforms expressions of any type τ into a function $\langle\sigma,\tau\rangle$, where σ is the type of the λ-variable.

Formal definition:

If α is in WE_{τ}, and x is in VAR_{σ} then $\lambda x(\alpha)$ is in $WE_{\langle\sigma, \tau\rangle}$

- The scope of the λ-operator is the smallest WE to its right. Wider scope must be indicated by brackets.
- We often use the "dot notation" $\lambda x.\phi$ indicating that the λ-operator takes widest possible scope (over ϕ).

Interpretation of Lambda-expressions

If $\boldsymbol{\alpha} \in WE_\tau$ and $v \in VAR_\sigma$, then $[\![\lambda v \boldsymbol{\alpha}]\!]^{M,g}$ is that function $f : D_\sigma \rightarrow D_\tau$ such that for all $a \in D_\sigma$, $f(a) = [\![\boldsymbol{\alpha}]\!]^{M,g[v/a]}$

If the λ-expression is applied to some argument, we can simplify the interpretation:

- $[\![\lambda v \boldsymbol{\alpha}]\!]^{M,g}(x) = [\![\boldsymbol{\alpha}]\!]^{M,g[v/x]}$

Example: *"Bill is a non-smoker"*

$[\![\lambda x(\neg S(x))(b')]\!]^{M,g} = 1$

iff $[\![\lambda x(\neg S(x))]\!]^{M,g}([\![b']\!]^{M,g}) = 1$

iff $[\![\neg S(x)]\!]^{M,g[x/[\![b']\!]^{M,g}]} = 1$

iff $[\![S(x)]\!]^{M,g[x/[\![b']\!]^{M,g}]} = 0$

iff $[\![S]\!]^{M,g[x/[\![b']\!]^{M,g}]}([\![x]\!]^{M,g[x/[\![b']\!]^{M,g}]}) = 0$

iff $V_M(S)(V_M(b')) = 0$

β-Reduction

$[\![\lambda v(\boldsymbol{\alpha})(\boldsymbol{\beta})]\!]^{M,g} = [\![\boldsymbol{\alpha}]\!]^{M,g[v/[\![\boldsymbol{\beta}]\!]^{M,g}]}$

$\Rightarrow$ all (free) occurrences of the λ-variable in **α** get the interpretation of **β** as value.

This operation is called β-reduction

- $\lambda v(\boldsymbol{\alpha})(\boldsymbol{\beta}) \Leftrightarrow \boldsymbol{\alpha}[\boldsymbol{\beta}/v]$
- $\boldsymbol{\alpha}[\boldsymbol{\beta}/v]$ is the result of replacing all free occurrences of v in **α** with **β**

Achtung: The equivalence is not unconditionally valid!

Variable capturing

Q: Are $\lambda v(\boldsymbol{\alpha})(\boldsymbol{\beta})$ and $\boldsymbol{\alpha}[\boldsymbol{\beta}/v]$ always equivalent?

- $\lambda x(\text{drive'}(x) \wedge \text{drink'}(x))(j') \Leftrightarrow \text{drive'}(j') \wedge \text{drink'}(j')$
- $\lambda x(\text{drive'}(x) \wedge \text{drink'}(x))(y) \Leftrightarrow \text{drive'}(y) \wedge \text{drink'}(y)$
- $\lambda x(\forall y\ \text{know'}(x)(y))(j') \Leftrightarrow \forall y\ \text{know}(j')(y)$
- NOT: $\lambda x(\forall y\ \text{know'}(x)(y))(y) \Leftrightarrow \forall y\ \text{know}(y)(y)$

Let v, v' be variables of the same type, and let $\boldsymbol{\alpha}$ be any well-formed expression.

- v is free for v' in $\boldsymbol{\alpha}$ iff no free occurrence of v' in $\boldsymbol{\alpha}$ is in the scope of a quantifier or a λ-operator that binds v.

Conversion rules

- β-conversion: $\lambda v(\boldsymbol{\alpha})(\boldsymbol{\beta}) \Leftrightarrow \boldsymbol{\alpha}[\boldsymbol{\beta}/v]$

  (if all free variables in $\boldsymbol{\beta}$ are free for v in $\boldsymbol{\alpha}$)

- α-conversion: $\lambda v.\boldsymbol{\alpha} \Leftrightarrow \lambda w.\boldsymbol{\alpha}[w/v]$

  (if w is free for v in $\boldsymbol{\alpha}$)

- η-conversion: $\lambda v.\boldsymbol{\alpha}(v) \Leftrightarrow \boldsymbol{\alpha}$

Determiners as lambda-expressions

- a student works ➔ $\exists x(\text{student}'(x) \wedge \text{work}'(x)) :: t$
    - a student ➔ $\lambda P \exists x(\text{student}'(x) \wedge P(x)) :: \langle\langle e,t\rangle,t\rangle$
    - a, some ➔ $\lambda Q \lambda P \exists x(Q(x) \wedge P(x)) :: \langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$
- every student ➔ $\lambda P \forall x(\text{student}'(x) \rightarrow P(x)) :: \langle\langle e,t\rangle,t\rangle$
    - every ➔ $\lambda Q \lambda P \forall x(Q(x) \rightarrow P(x)) :: \langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$
- no student ➔ $\lambda P \neg \exists x(\text{student}(x) \wedge P(x)) :: \langle\langle e,t\rangle,t\rangle$
    - no ➔ $\lambda Q \lambda P \neg \exists x(Q(x) \wedge P(x)) :: \langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$
- someone ➔ $\lambda F \exists x F(x) :: \langle\langle e,t\rangle,t\rangle$

NL Quantifier Expressions: Interpretation

- someone’$\in$ CON$_{\langle\langle e,t\rangle,t\rangle}$, so V_M(someone’) $\in D_{\langle\langle e,t\rangle,t\rangle}$
- $D_{\langle\langle e,t\rangle,t\rangle}$ is the set of functions from $D_{\langle e,t\rangle}$ to D_t , i.e.,

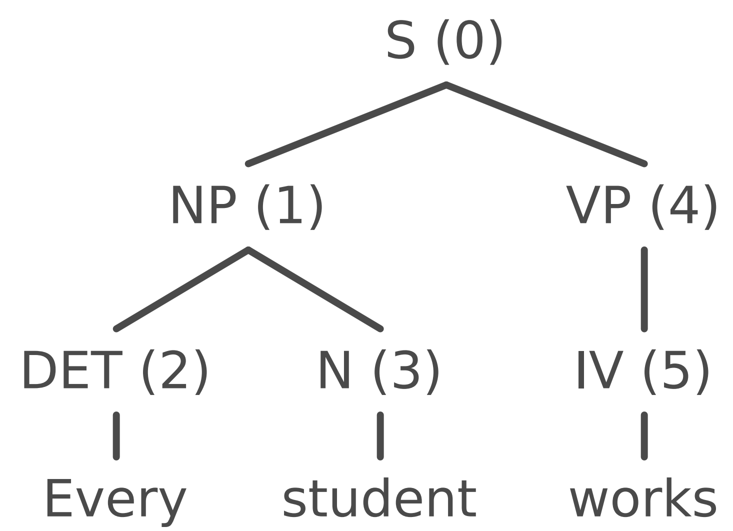
  the set of functions from $\mathcal{P}(U_M)$ to $\{0,1\}$,

  which in turn is equivalent to $\mathcal{P}(\mathcal{P}(U_M))$
- Thus, V_M(someone’) $\subseteq \mathcal{P}(U_M)$. More specifically:
- V_M(someone’) $= \{S \subseteq U_M \mid S \neq \varnothing\}$, if U_M is a domain of persons

⇒ More on Natural Language Quantifiers next week!

β-Reduction Example

Every student works.



(2) $\lambda P \lambda Q \forall x(P(x) \rightarrow Q(x))$ :: $\langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle$

(3) $\lambda x.\text{student}'(x) \Leftrightarrow^{\eta} \text{student}'$ :: $\langle e, t\rangle$

(1) $\lambda P \lambda Q \forall x(P(x) \rightarrow Q(x))(\text{student}')$

$\Leftrightarrow^{\beta} \lambda Q \forall x(\text{student}'(x) \rightarrow Q(x))$ :: $\langle\langle e, t\rangle, t\rangle$

(4)/(5) $\lambda x.\text{work}'(x) \Leftrightarrow^{\eta} \text{work}'$ :: $\langle e, t\rangle$

(0) $\lambda Q \forall x(\text{student}'(x) \rightarrow Q(x))(\text{work}') \Leftrightarrow^{\beta} \forall x(\text{student}'(x) \rightarrow \text{work}'(x))$ :: t

Transitive Verbs: Type Clash

- Someone reads a book

$$\dfrac{\text{someone} :: \langle\langle e, t\rangle, t\rangle \qquad \dfrac{\text{read} :: \langle e, \langle e, t\rangle\rangle \qquad \text{a book} :: \langle\langle e, t\rangle, t\rangle}{?? :: ??}}{?? :: t}$$

Solution: reverse functor-argument relation (again)

$\text{read}_{\langle\langle\langle e,\, t\rangle, t\rangle, \langle e,\, t\rangle\rangle}$ (*Type Raising*)

Type Raising

It's not enough to just change the type of the transitive verb:

- read ➔ read' $\in CON_{\langle\langle\langle e,t\rangle, t\rangle, \langle e, t\rangle\rangle}$

  someone reads a book:
  $\lambda F \exists x F(x)(read'(\lambda P \exists y(book'(y) \wedge P(y)))$
  $\Leftrightarrow^{\beta} \exists x(read'(\lambda P \exists y(book'(y) \wedge P(y)))(x)$

  …but this does not support the following entailment:
  someone reads a book ⊨ *there exists a book*

We need a more explicit λ-term:

- read ➔ $\lambda Q \lambda z.Q(\lambda x(read^*(x)(z))) \in WE_{\langle\langle\langle e,t\rangle, t\rangle, \langle e, t\rangle\rangle}$
  where: read* $\in WE_{\langle e, \langle e, t\rangle\rangle}$ is the "underlying" first-order relation

Transitive Verbs: example

someone reads a book

$\lambda F \exists x F(x)(\lambda Q \lambda z.Q(\lambda x(read^*(x)(z)))(\lambda R \lambda P.\exists y(R(y) \wedge P(y))\ (book')))$

$\Leftrightarrow_\beta\ \lambda F \exists x F(x)(\lambda Q \lambda z.Q(\lambda x(read^*(x)(z)))(\lambda P.\exists y(book'(y) \wedge P(y))))$

$\Leftrightarrow_\beta\ \lambda F \exists x F(x)(\lambda z.(\lambda P.\exists y(book'(y) \wedge P(y)))(\lambda x(read^*(x)(z))))$

$\Leftrightarrow_\beta\ \lambda F \exists x F(x)(\lambda z.\exists y(book'(y) \wedge \lambda x(read^*(x)(z))(y)))$

$\Leftrightarrow_\beta\ \lambda F \exists x F(x)(\lambda z.\exists y(book'(y) \wedge read^*(y)(z)))$

$\Leftrightarrow_\beta\ \exists x(\lambda z.\exists y(book'(y) \wedge read^*(y)(z)))(x)$

$\Leftrightarrow_\beta\ \exists x \exists y(book'(y) \wedge read^*(y)(x))$

Background reading material

- Gamut: Logic, Language, and Meaning Vol II
  (Chapter 4, minus 4.3)

- Winter: Elements of Formal Semantics (Chapter 3)
  http://www.phil.uu.nl/~yoad/efs/main.html