# Semantic Theory week 9 - DRT: Interpretation and Composition 

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## Subordination

A DRS $K_{1}$ is an immediate sub-DRS of a DRS $K=\left\langle U_{K}, C_{K}\right\rangle$ iff $C_{K}$ contains a condition of the form

- $\neg \mathrm{K}_{1}, \mathrm{~K}_{1} \Rightarrow \mathrm{~K}_{2}, \mathrm{~K}_{2} \Rightarrow \mathrm{~K}_{1}, \mathrm{~K}_{1} \vee \mathrm{~K}_{2}$ or $\mathrm{K}_{2} \vee \mathrm{~K}_{1}$.
$K_{1}$ is a sub-DRS of $K\left(\right.$ notation: $\left.K_{1} \leq K\right)$ iff
- $\mathrm{K}_{1}=\mathrm{K}$, or
- $\mathrm{K}_{1}$ is an immediate sub-DRS of K , or
- there is a DRS $\mathrm{K}_{2}$ such that $\mathrm{K}_{2} \leq \mathrm{K}$ and $\mathrm{K}_{1}$ is an immediate sub-DRS of $\mathrm{K}_{2}$ (i.e. reflexive, transitive closure)
$K_{1}$ is a proper sub-DRS of $K$ iff $K_{1} \leq K$ and $K_{1} \neq K$.


## Accessibility

Let $\mathrm{K}, \mathrm{K}_{1}, \mathrm{~K}_{2}$ be DRS such that $\mathrm{K}_{1}, \mathrm{~K}_{2} \leq \mathrm{K}, \mathrm{x} \in \mathrm{U}_{\mathrm{K}_{1}}, \gamma \in \mathrm{C}_{K_{2}}$ $x$ is accessible from $\gamma$ in $K$ iff

- $\mathrm{K}_{2} \leq \mathrm{K}_{1}$ or
- there are $K_{3}, K_{4} \leq K$ such that $K_{1} \Rightarrow K_{3} \in C_{K 4}$ and $K_{2} \leq K_{3}$



## Free and bound variables in DRT

A DRS variable $x$ introduced in condition $\gamma$ in DRS $K_{1} \leq K$, is free in global DRS $K$ iff there is no $K_{2} \leq K$ such that $x^{\prime} \in U\left(K_{2}\right)$, and $x^{\prime}=x$ and $x^{\prime}$ is accessible from $\gamma$ in $K$.

Properness: A DRS is proper iff it does not contain any free variables

Purity: A DRS is pure iff it does not contain any otiose declarations of variables

$$
x \in U\left(K_{1}\right) \text { and } x \in U\left(K_{2}\right) \text { and } K_{1} \leq K_{2}
$$

## From text to DRS

Text

$$
\Sigma=\left\langle S_{1}\right.
$$

$$
\downarrow
$$

$$
\begin{gathered}
S_{2} \\
\downarrow
\end{gathered}
$$

$$
S_{n}>
$$

Syntactic Analysis

DRS Construction

## DRS Construction Algorithm

Let the following be a well-formed, reducible DRS condition:

- Conditions of form a or $a(x 1, \ldots, x n)$, where $a$ is a context-free parse tree.

DRS construction algorithm:

- Given a text $\Sigma=\left\langle S_{1}, \ldots, S_{n}\right\rangle$, and a DRS $K_{0}(=\langle\varnothing, \varnothing\rangle$, by default)
- Repeat for $\mathrm{i}=1, \ldots, \mathrm{n}$ :
- Add parse tree $\mathrm{P}\left(\mathrm{S}_{\mathrm{i}}\right)$ to the conditions of $\mathrm{K}_{\mathrm{i}-1}$.
- Apply DRS construction rules to reducible conditions of $\mathrm{K}_{\mathrm{i}-1}$, until no reduction steps are possible any more.
- The resulting DRS is $K_{i}$, the discourse representation of text $\left\langle S_{1}, \ldots, S_{i}\right\rangle$.


## DRS Construction Example

- A farmer owns a donkey. He beats it.



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- A farmer owns a donkey. He beats it.

| x y |
| :--- |
| farmer(x) |
| donkey $(\mathrm{y})$ |
| owns $(x, y)$ |
|  |
|  |
|  |
|  |
|  |

## DRS Construction Example

- A farmer owns a donkey. He beats it.



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## DRS Construction Example

- A farmer owns a donkey. He beats it.

| x y z u |  |
| :---: | :---: |
| $\begin{aligned} & \text { farmer(x) } \\ & \text { donkey(y) } \\ & \text { owns }(x, y) \\ & z=x \\ & u=y \end{aligned}$ |  |

## DRS Construction Example

- A farmer owns a donkey. He beats it.
$x y z u$
farmer(x)
donkey(y)
owns(x, y)
$z=x$
$u=y$
beat(z, u)


## Construction Rules: Examples

## Indefinite NPs

- Given a reducible condition a in DRS K, with [S [NP $\beta$ ] [VP $\gamma$ ]] or [VP [V $\gamma$ ] [NP $\beta$ ]] as a substructure, and $\beta$ is $\varepsilon \delta$, where $\varepsilon$ is an indefinite article
- Action: (i) Add a new DR $x$ to $U_{K}$; (ii) Replace $\beta$ in a by $x$; (iii) $\operatorname{Add} \delta(x)$ to $C_{k}$.


## Personal Pronouns

- Given a global DRS $K^{*}$, and some $K \leq K^{*}$, such that a is a reducible condition in DRS K, with [S [NP $\beta$ ] [VP $\gamma$ ]] or [VP [V $\gamma$ ] [NP $\beta$ ]] as a substructure, and $\beta$ is a personal pronoun
- Action: (i) Add a new DR $x$ to Uk; (ii) Replace $\beta$ in a by $x$; (iii) Select an appropriate DR y that is accessible from $a$ in $K^{*}$ and add $x=y$ to $\mathrm{C}_{\mathrm{k}}$


## From text to DRS

Text
$\Sigma=\left\langle S_{1}\right.$,
1
$P\left(S_{1}\right)$
$\downarrow \downarrow$
$\begin{aligned} \mathrm{K}_{1} & \longrightarrow \mathrm{~K}_{2} \longrightarrow \ldots\end{aligned} \begin{array}{ll} & \mathrm{K}_{\mathrm{n}} \\ & \\ & \downarrow\end{array}$
Interpretation by
model embedding:
Truth-conditions of $\Sigma$

## DRS Interpretation

Given a DRS $K=\left\langle U_{K}, C_{K}\right\rangle$, with $U_{K} \subseteq U_{D}$

Let $M=\left\langle U_{M}, V_{M}\right\rangle$ be a $F O L$ model structure appropriate for $K$, i.e. a model structure that provides interpretations for all predicates and relations occurring in K

DRS K is true in model M iff

- there is an embedding function for K in M which verifies all conditions in K
... where: an embedding of K into M is a (partial) function $\mathbf{f}$ from $U_{D}$ to $U_{M}$ such that $U_{K} \subseteq \operatorname{Dom}(\mathbf{f})$.


## Verifying embedding

An embedding $\mathbf{f}$ of $K$ in $M$ verifies $K$ in $M\left(f \models_{M} K\right)$ iff $\mathbf{f}$ verifies every condition $a \in \mathrm{C}_{\mathrm{K}}$

- $\mathbf{f} \models_{M} R\left(x_{1}, \ldots, x_{n}\right) \quad$ iff $\left\langle f\left(x_{1}\right), \ldots, f\left(x_{n}\right)\right\rangle \in V_{M}(R)$
- $\mathbf{f} \vDash_{\mathrm{M}} \mathrm{X}=\mathrm{y}$ iff $\mathrm{f}(\mathrm{x})=\mathbf{f}(\mathrm{y})$
- $\mathbf{f} \vDash \mathrm{m} X=\mathrm{a}$ iff $\mathbf{f}(\mathrm{X})=V_{M}(\mathrm{a})$
- $\mathbf{f} \models_{\mathrm{m}} \neg \mathrm{K}_{1} \quad$ iff $\quad$ there is $n o \mathbf{g} \supseteq \cup_{K 1} \mathbf{f}$ such that $g \models_{\mathrm{m}} K_{1}$
- $\mathbf{f} \models m K_{1} \Rightarrow K_{2} \quad$ iff $\quad$ for all $\mathbf{g} \supseteq \cup_{K 1} f$ such that $\mathbf{g} \models_{M} K_{1}$ there is a $\mathbf{h} \supseteq \cup_{K 2} \mathbf{g}$ such that $\mathbf{h} \models_{M} K_{2}$
- $\mathbf{f} \models_{\mathrm{M}} K_{1} \vee K_{2} \quad$ iff there is a $\mathbf{g}_{1} \supseteq \cup_{K 1} \mathbf{f}$ such that $\mathbf{g}_{1} \models_{\mathrm{M}} K_{1}$ or there is a $\mathbf{g}_{2} \supseteq \cup_{\mathrm{K} 2} \mathbf{f}$ such that $\mathbf{g}_{2} \models \mathrm{M} \mathrm{K}_{2}$


## Verifying embedding: example

Mary knows a professor. If he owns a book, he gives it to a student.

...is true in $M=\left\langle U_{M}, V_{M}\right\rangle$ iff there is an $\mathbf{f}:: U_{D} \rightarrow U_{M}$, (with $\{s, u\} \subseteq \operatorname{Dom}(\mathbf{f})$ ) such that:
$\mathbf{f}(\mathrm{s})=\mathrm{V}_{\mathrm{M}}($ Mary $) \& \mathbf{f}(\mathrm{u}) \in \mathrm{V}_{\mathrm{M}}\left(\right.$ prof $\left.{ }^{\prime}\right) \&\langle\mathbf{f}(\mathrm{~s}), \mathbf{f}(\mathrm{u})\rangle \in \mathrm{V}_{\mathrm{M}}($ know $)$,
and for all $\mathbf{g} \supseteq_{\{x, y\}} \mathbf{f}$ s.t. $\mathbf{g}(x)=\mathbf{g}(u)(=\mathbf{f}(u)) \& \mathbf{g}(y) \in V_{M}($ book $) \&\langle\mathbf{g}(x), \mathbf{g}(y)\rangle \in V_{M}(o w n)$, there is $\mathbf{a} \mathbf{h} \supseteq_{\{z, v, w} \mathbf{g}$ s.t. $\langle\mathbf{h}(z), \mathbf{h}(\mathrm{v}), \mathbf{h}(\mathrm{w})\rangle \in \mathrm{V}_{\mathrm{M}}(\mathrm{give}) \& \mathbf{h}(\mathrm{z})=\mathbf{h}(\mathrm{x})(=\mathbf{g}(\mathrm{x})) \& \ldots$ etc.

## Translation of DRSs to FOL

Consider DRS K $=\left\langle\left\{\mathrm{x}_{1}, \ldots, \mathrm{x}_{n}\right\},\left\{\mathrm{C}_{1}, \ldots, \mathrm{C}_{\mathrm{k}}\right\}\right\rangle$

| $\mathrm{X}_{1} \ldots \mathrm{X}_{\mathrm{n}}$ |
| :--- |
| $\mathrm{C}_{1}$ |
| $\vdots$ |
| $\mathrm{C}_{\mathrm{n}}$ |

K is truth-conditionally equivalent to the following FOL formula:

$$
\exists x_{1} \ldots \exists x_{n}\left[c_{1} \wedge \ldots \wedge c_{k}\right]
$$

## DRT is non-compositional

- DRT is non-compositional on truth conditions: The difference in discourse-semantic status of the text pairs is not predictable through the (identical) truth conditions of its component sentences.
- Since structural information which cannot be reduced to truth conditions is required to compute the semantic value of texts, DRT is called a representational theory of meaning.

However...

## Wait a minute ...

- Why can't we just combine type theoretic semantics and DRT?
- Use $\lambda$-abstraction and reduction as we did before, but:
- Assume that the target representations which we want to arrive at are not First-Order Logic formulas, but DRSs.
- The result is called $\lambda$-DRT.


## $\lambda$-DRSs

An expression in $\lambda$-DRT consists of a lambda prefix and a partially instantiated DRS.

- every student :: $\langle\langle e, t\rangle, t\rangle \mapsto \quad \lambda G$.


Alternative notation: $\lambda \mathrm{G}[\varnothing \mid[\mathrm{z} \mid$ student $(\mathrm{z})] \Rightarrow \mathrm{G}(\mathrm{z})$ ]

- works :: $\langle e, t\rangle \mapsto \lambda \times[\varnothing \mid \operatorname{work}(x)]$


## $\lambda$-DRT: $\beta$-reduction

Every student works

$$
\begin{aligned}
& \mapsto \lambda G[\varnothing \mid[z \mid \operatorname{student}(z)] \Rightarrow G(z)]](\lambda x[\varnothing \mid \operatorname{work}(x)]) \\
& \Rightarrow^{\beta}[\varnothing \mid[z \mid \operatorname{student}(z)] \Rightarrow(\lambda x[\varnothing \mid \operatorname{work}(x)])(z)] \\
& \Rightarrow^{\beta}[\varnothing \mid[z \mid \operatorname{student}(z)] \Rightarrow[\varnothing \mid \operatorname{work}(z)]]
\end{aligned}
$$

## (Naïve) Merge

The "merge" operation on DRSs combines two DRSs (conditions and universes).

- Let $\mathrm{K}_{1}=\left[\mathrm{U}_{1} \mid \mathrm{C}_{1}\right]$ and $\mathrm{K}_{2}=\left[\mathrm{U}_{2} \mid \mathrm{C}_{2}\right]$.

Merge: $\quad K_{1}+K_{2}=\left[U_{1} \cup U_{2} \mid C_{1} \cup C_{2}\right]$

## Merge: An example

- a student $\mapsto \lambda G([z \mid$ student $(z)]+G(z))$
- works $\mapsto \lambda \times[\varnothing \mid$ work $(x)]$

A student works $\quad \mapsto \lambda G([z \mid$ student(z) ] $+G(z))(\lambda x[\varnothing \mid$ work(x)])
$\Rightarrow{ }^{\beta}[z \mid \operatorname{student}(z)]+\lambda x[\varnothing \mid \operatorname{work}(x)](z)$
$\Rightarrow{ }^{\beta}[z \mid \operatorname{student}(z)]+[\varnothing \mid \operatorname{work}(z)]$
$\Rightarrow{ }^{\beta}[z \mid$ student(z), work(z)]

## Compositional analysis

- Mary $\mapsto \lambda G([z \mid z=$ Mary ] + G(z))
- she $\quad \mapsto \lambda G . G(z)$

Mary works. She is successful.

$$
\begin{aligned}
& \mapsto \lambda K \lambda K^{\prime}\left(K+K^{\prime}\right)([z \mid z=\operatorname{Mary}, \text { work(z)])([ |successful(z)]]) } \\
& \Rightarrow^{\beta} \lambda K^{\prime}\left(\left[z \mid z=\operatorname{Mary}, \text { work(z)] }+K^{\prime}\right)([\mid \text { successful(z)]) }\right. \\
& \Rightarrow^{\beta}[z \mid z=\operatorname{Mary}, \text { work(z) }]+([\mid \operatorname{successful(z)])} \\
& \Rightarrow^{\beta}[z \mid z=\operatorname{Mary}, \text { work(z), successful(z)] }
\end{aligned}
$$

## Merge again

The "merge" operation on DRSs combines two DRSs (conditions and universes).

- Let $\mathrm{K}_{1}=\left[\mathrm{U}_{1} \mid \mathrm{C}_{1}\right]$ and $\mathrm{K}_{2}=\left[\mathrm{U}_{2} \mid \mathrm{C}_{2}\right]$.

Merge: $K_{1}+K_{2} \Rightarrow\left[U_{1} \cup U_{2} \mid C_{1} \cup C_{2}\right]$
under the assumption that no discourse referent $u \in U_{2}$ occurs free in a condition $v \in C_{1}$.

## Variable capturing

In $\lambda$-DRT, discourse referents are captured via the interaction of $\beta$-reduction and DRS-binding:

- $\lambda K^{\prime}\left(\left[z \mid\right.\right.$ student(z), work(z)] $\left.+K^{\prime}\right)([\mid$ successful(z)])
$\Rightarrow^{\beta}[\mathrm{z} \mathrm{\mid} \mathrm{student(z)}, \mathrm{work(z)]}+[\mid$ successful(z)]
$\Rightarrow{ }^{\beta}[z \mid \operatorname{student}(z)$, work(z), successful(z)]

But the $\beta$-reduced DRS must still be equivalent to the original DRS!

So, the potential for capturing discourse referents must be captured into the interpretation of a $\lambda$-DRS. Possible, but tricky.

