Semantic Theory
week 7 – Discussion of exercises

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Exercise sheet 4: Exercise 1b

Beta-reduction and Type-raising

$\lambda y \lambda e \exists u[s'(u) \wedge call(e, y, u)] \quad :: \langle e,\langle e,t\rangle\rangle$

$\lambda y \lambda e\ [\exists u[s'(u) \wedge (\lambda x[call(e, y, x)])](u)]$

$\lambda y \lambda e[\lambda R \exists u[s'(u) \wedge R(u)](\lambda x[call(e, y, x)])]$

$\lambda Q \lambda y \lambda e\ [Q(\lambda x[call(e, y, x)])]$

$:: \langle\langle\langle e,t\rangle,t\rangle,\langle e,\langle e,t\rangle\rangle$

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$\lambda y \lambda x \lambda e\ [\ call(e, x, y)] :: \langle e,\langle e,\langle e,t\rangle\rangle$

call

$\lambda R\ \exists u\ [s'(u) \wedge R(u)]$

$:: \langle\langle e,t\rangle,t\rangle$

$\lambda P \lambda R\ \exists u\ [P(u) \wedge R(u)]$

$:: \langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$

a

$s' = \lambda x(s'(x))$

$:: \langle e,t\rangle$

senator

Exercise sheet 3: Exercise 3

A formal proof

The external negation of an upward monotonic quantifier is a downward monotonic quantifier.

Proof: We first collect the three relevant definitions:

(1) The external negation $\neg Q$ of a quantifier Q is defined as $\{P \subseteq U_M | P \notin Q\}$.

(2) Q is an upward monotonic quantifier iff for all $X, Y \subseteq U_M$: if $X \in Q$ and $X \subseteq Y$, then $Y \in Q$

(3) Q' is a downward monotonic quantifier iff for all $X, Y \subseteq U_M$: if $X \in Q'$ and $Y \subseteq X$, then $Y \in Q'$

Take an arbitrary $A, B \subseteq U_M$ and suppose $A \subseteq B$ and $B \in \neg Q$. To prove: $A \in \neg Q$.

Suppose that $A \notin \neg Q$, then it follows from (1) that $A \in Q$. Given that Q is an upward monotonic quantifier, it follows from (2) that $B \in Q$. But this is in conflict with the assumption $B \in \neg Q$, which means that $B \notin Q$ (by (1)). Therefore, it must hold that $A \in \neg Q$. Now it follows from (3) that $\neg Q$ is downward monotonic.