Semantic Theory
week 5 – Generalised Quantifiers

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Summer 2015

Back to Noun Phrases

Natural language contains a wide variety of NPs, serving as quantifiers

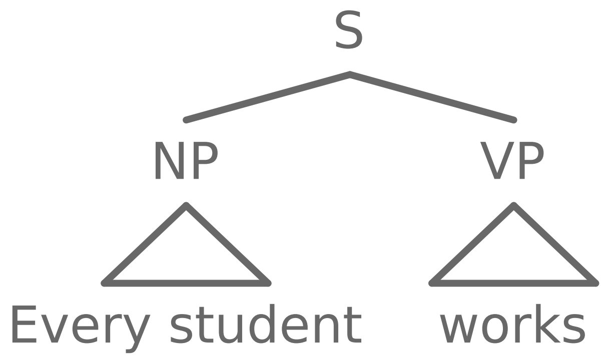
- all students, no woman, not every man, everything, nothing, three books, the ten professors, John, John and Mary, only John, firemen, at least five horses, most girls, all but ten marbles, less than half of the audience, John's, some student's, no student except Mary, more male than female cats, usually, never, each other.

Aristotle: “Quantifiers are second-order relations between sets”

Frege: “All quantifiers can be defined in terms of ∀ (and ∃)”

NP interpretation

"Every student"



- $\mapsto \lambda P \forall x(\text{student}'(x) \rightarrow P(x))$
- Type: $\langle\langle e, t\rangle, t\rangle$
- Interpretation: "Every student" denotes the *set of properties* that apply to every student (property = set of individuals).
- $[\![\text{Every student}]\!]^M = \{ P \subseteq U_M \mid \text{every student has property } P \}$
  $= \{ P \subseteq U_M \mid [\![\text{student}]\!] \subseteq P\}$
- $[\![\text{Every student works}]\!]^M = 1$ iff $[\![\text{work}]\!]^M \in [\![\text{every student}]\!]^M$

Generalized Quantifiers

Generalized quantifiers sets of subsets of M (i.e., sets of properties)

every student $\mapsto \lambda P \forall x(\text{student}'(x) \rightarrow P(x))$

- $[\![\text{every student}]\!]^M = \{ P \subseteq U_M \mid [\![\text{student}]\!] \subseteq P\}$

"the set of properties P such that all students are P"

a student $\mapsto \lambda P \exists x(\text{student}'(x) \wedge P(x))$

- $[\![\text{a student}]\!]^M = \{ P \subseteq U_M \mid [\![\text{student}]\!] \cap P \neq \varnothing \}$

"the set of properties P such that at least one student is P"

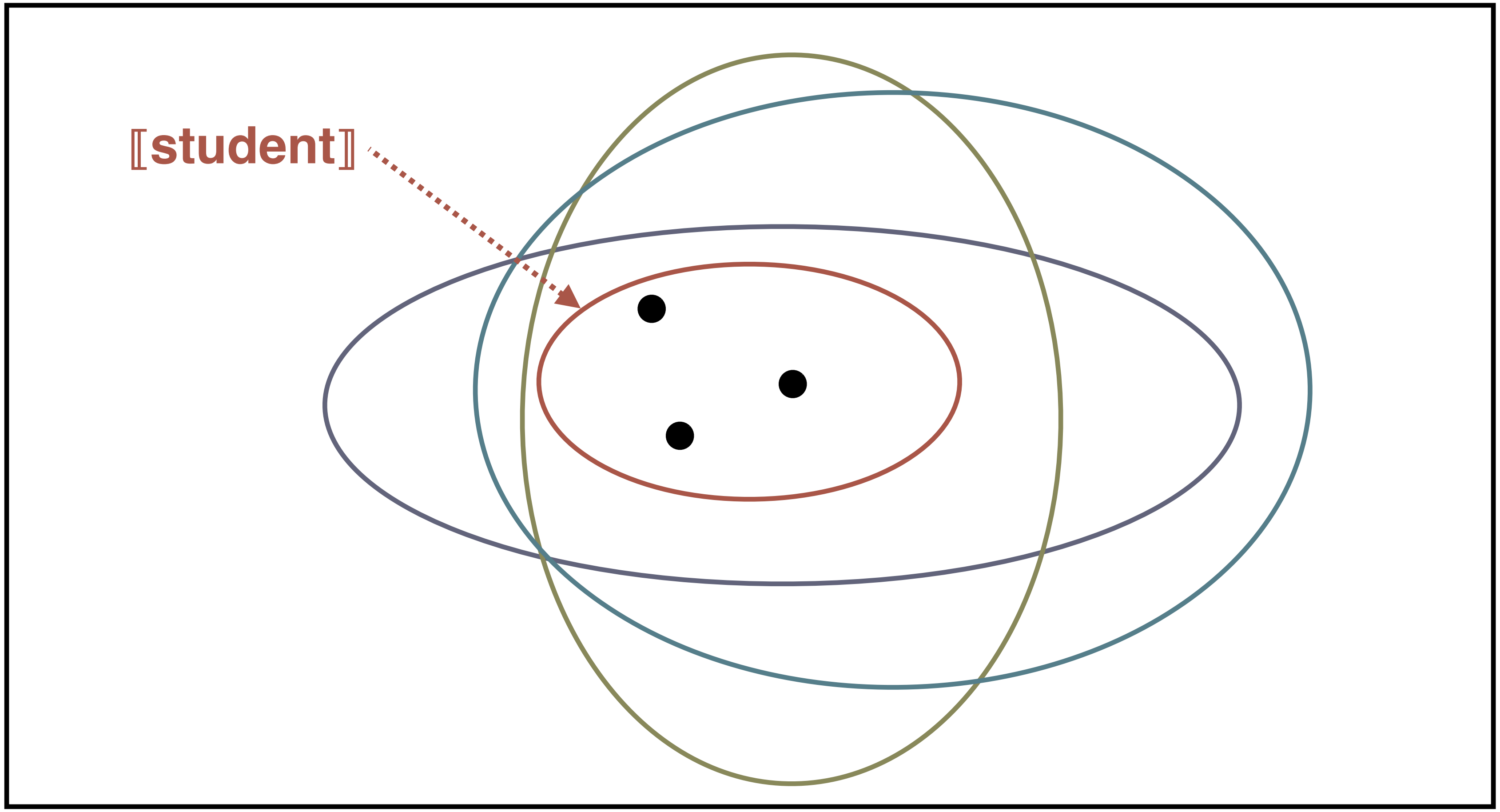
Bill $\mapsto \lambda P.P(b^*)$

- $[\![\text{Bill}]\!]^M = \{ P \subseteq U_M \mid b^* \in P\}$

"the set of properties P, such that Bill is P"

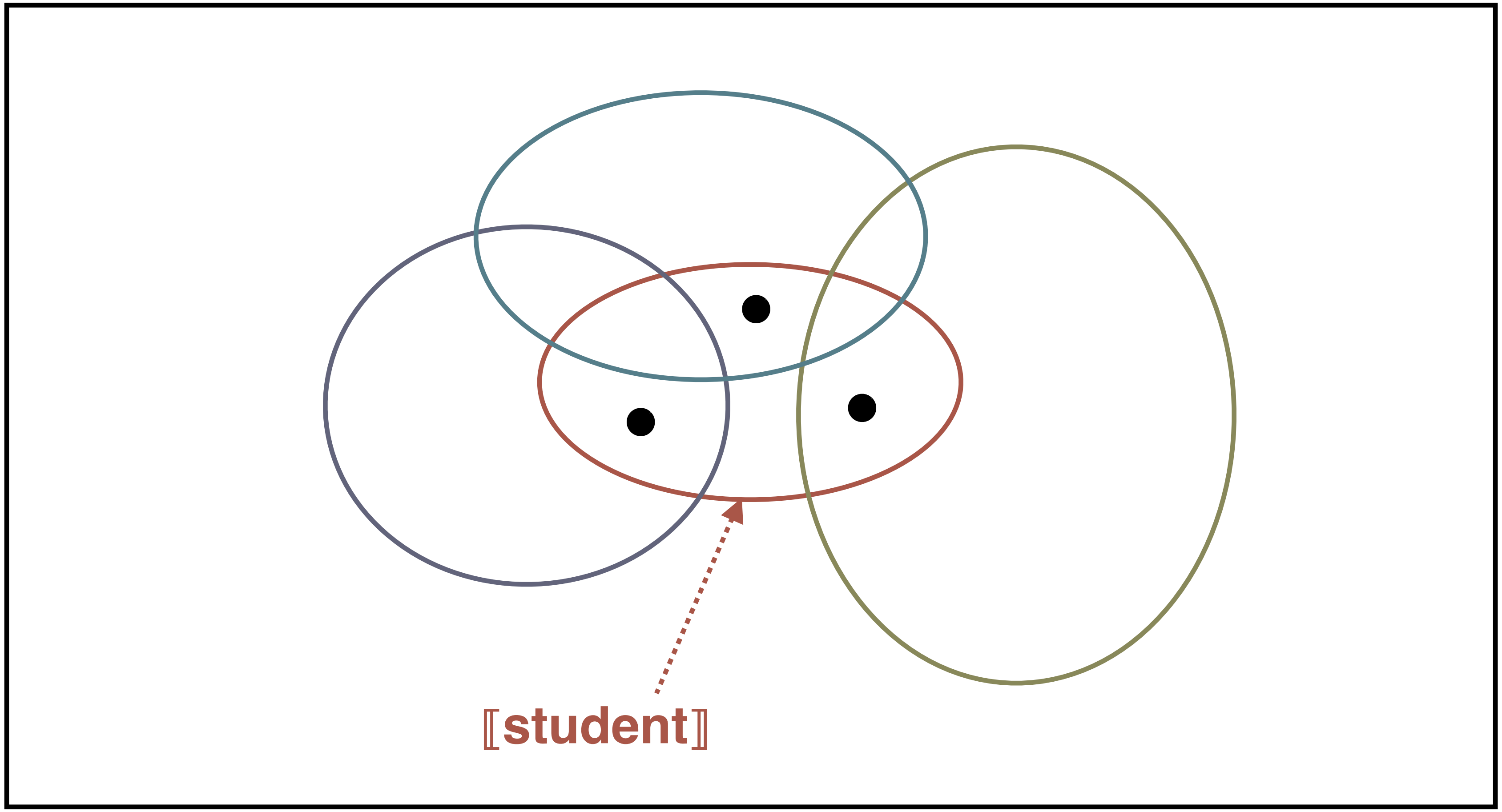
⟦*every student*⟧

- “every student” denotes the set of properties that apply to every student (i.e., all supersets of ⟦student⟧)



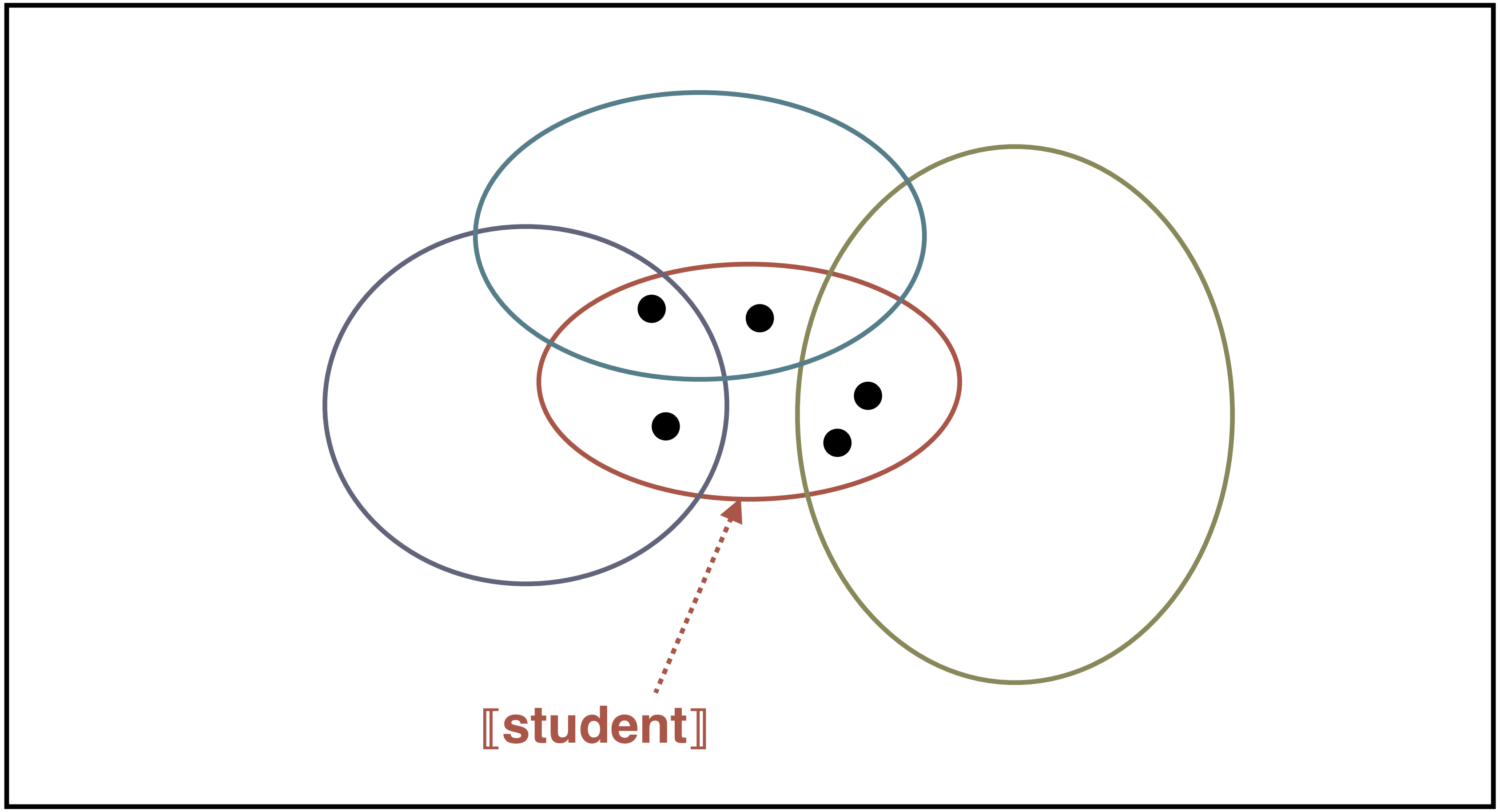
⟦*a student*⟧

- “a student” denotes the set of properties that apply to at least one student.



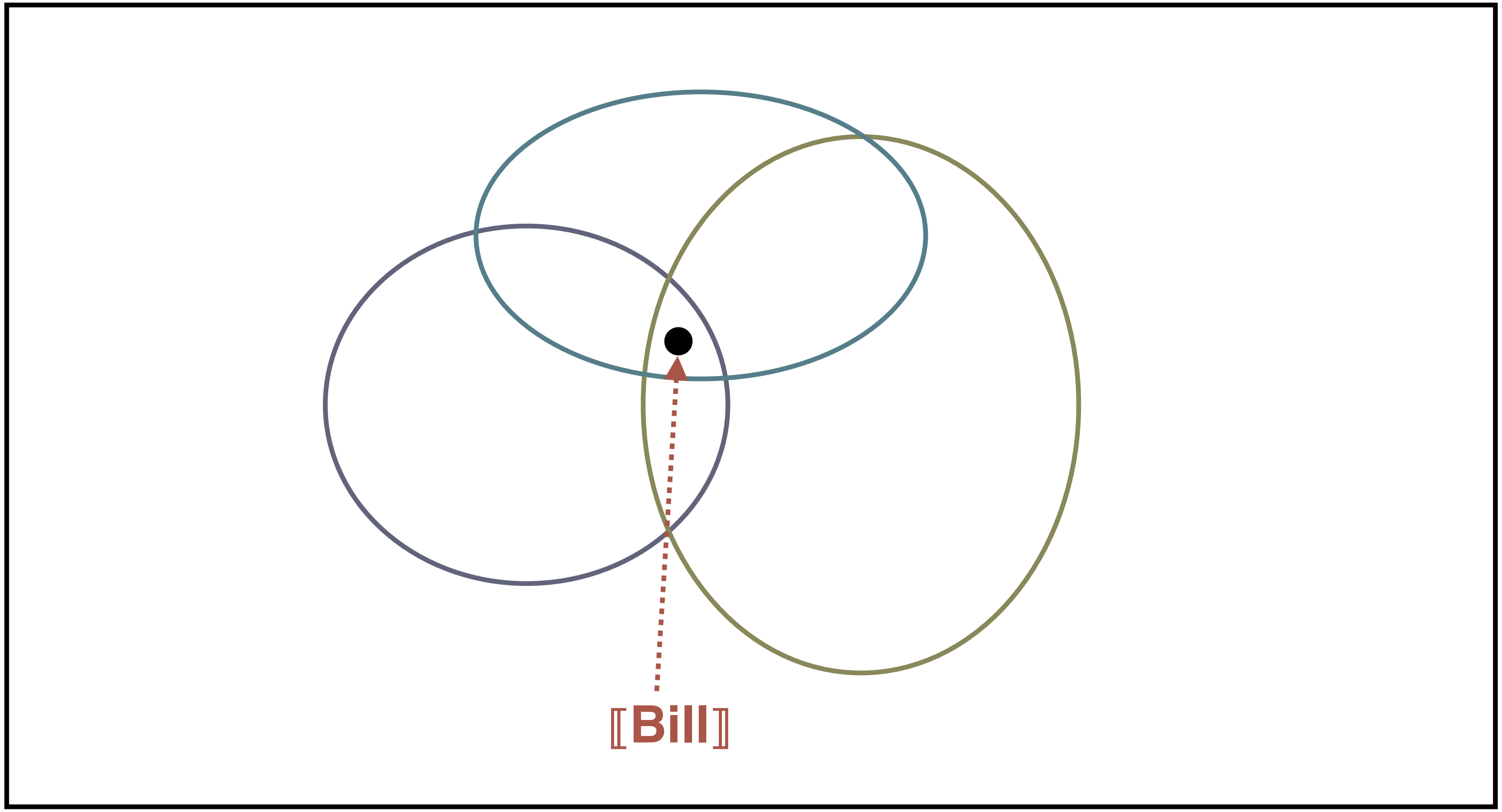
⟦*two students*⟧

- "two students" denotes the set of properties that apply to at least (exactly) two students.



⟦*Bill*⟧

- “Bill” denotes the set of properties that apply to Bill



Noun Phrase Interpretations

$[\![\text{all N}]\!]^M = \{ P \subseteq U_M \mid [\![N]\!] \cap P = [\![N]\!] \}$

$[\![\text{a(n) N}]\!]^M = \{ P \subseteq U_M \mid [\![N]\!] \cap P \neq \varnothing \}$

$[\![\text{bill}]\!]^M = \{ P \subseteq U_M \mid b^* \in P\}$

$[\![\text{not all N}]\!]^M = \{ P \subseteq U_M \mid [\![N]\!] \cap P \neq [\![N]\!] \}$

$[\![\text{no N}]\!]^M = \{ P \subseteq U_M \mid [\![N]\!] \cap P = \varnothing \}$

$[\![\text{exactly n N}]\!]^M = \{ P \subseteq U_M \mid \text{card}([\![N]\!] \cap P) = n \}$

$[\![\text{at most n N}]\!]^M = \{ P \subseteq U_M \mid \text{card}([\![N]\!] \cap P) \leq n \}$

$[\![\text{at least n N}]\!]^M = \{ P \subseteq U_M \mid \text{card}([\![N]\!] \cap P) \geq n \}$

Generalized Quantifier Theory

I. How do generalized quantifiers differ in terms of their formal properties?

II. What universal regularities govern the meaning of terms?

III. Which subclasses actually represent meanings of natural language noun phrases?

Observation 1: Inference Patterns

(1) *All men walked rapidly* $\models$ *All men walked*

(2) *A girl smoked a cigar* $\models$ *A girl smoked*

(3) *No man walked* $\models$ *No man walked rapidly*

(4) *Few girls smoked* $\models$ *Few girls smoked a cigar*

Q: How to explain the different inference patterns for quantifiers?

Observation 2: Negative Polarity Items

NPIs (*any, ever, …*) can occur only in “negative contexts”

(1) a. *John needn’t go there.*

b. **John need go there.*

(2) a. *Nobody saw anything.*

b. **Somebody saw anything.*

(3) a. *No student has ever been in Saarbrücken.*

b. **Some student has ever been in Saarbrücken.*

Q: What licenses negative polarity items?

Observation 3: Coordination

(1) *No man and few women walked.*

(2) *None of the girls and at most three boys walked.*

(3) **A man and few women walked.*

(4) **John and no woman saw Jane.*

Q: which noun phrases can be coordinated?

Subsets and Supersets

(1) *All men walked rapidly* ⊨ *All men walked*

- Note: ⟦walked rapidly⟧ ⊆ ⟦walked⟧

(2) *A girl smoked a cigar* ⊨ *A girl smoked*

- Note: ⟦smoked a cigar⟧ ⊆ ⟦smoked⟧

Intuitively: For the given quantifiers, sentence [$_S$ NP VP] remains true if the denotation of the VP is made "larger"

Upward Monotonicity

A quantifier Q is upward monotonic (or: *monotone increasing*) in $M = \langle U, V \rangle$ iff Q is "closed under supersets", i.e.:

- for all $X, Y \subseteq U$: if $X \in Q$ and $X \subseteq Y$, then $Y \in Q$

- A noun phrase is upward monotonic if it denotes an upward monotonic quantifier.

Upward Monotonicity Tests

If $[\![VP_1]\!] \subseteq [\![VP_2]\!]$, then NP $VP_1 \vDash$ NP VP_2

- $[\![$walked rapidly$]\!] \subseteq [\![$walked$]\!]$
- *All men walked rapidly* ⊨ *All men walked* ☺
- *No man walked rapidly* ⊭ *No man walked* ☹

NP VP_1 and $VP_2 \vDash$ NP VP_1 and NP VP_2

- *All men smoked and drank* ⊨ *All men smoked and all men drank* ☺
- *No man smoked and drank* ⊭ *No man smoked and no man drank* ☹
- Note: $[\![VP_1 \text{ and } VP_2]\!] = [\![VP_1]\!] \cap [\![VP_2]\!]$

Upward Monotonicity and logical operators

Upward monotonic quantifiers are *closed under* conjunction and disjunction:

- *All boys and a girl walked rapidly* ⊨ *All boys and a girl walked*
- *John or a student arrived late* ⊨ *John or a student arrived*
- Note: $[\![NP_1 \text{ and } NP_2]\!] = [\![NP_1]\!] \cap [\![NP_2]\!]$
  $[\![NP_1 \text{ or } NP_2]\!] = [\![NP_1]\!] \cup [\![NP_2]\!]$

The intersection/union of two upward monotonic quantifiers is an upward monotonic quantifier.

Downward Monotonicity

(3) *No man walked* ⊨ *No man walked rapidly* ⟦*walked*⟧ ⊇ ⟦*walked rapidly*⟧

(4) *Few girls smoked* ⊨ *Few girls smoked a cigar.* ⟦*smoked*⟧ ⊇ ⟦*smoked a cigar*⟧

A quantifier Q is downward monotonic (or: *monotone decreasing*) in $M = \langle U, V \rangle$ iff Q is closed under inclusion:

- for all $X, Y \subseteq U$: if $X \in Q$ and $X \supseteq Y$, then $Y \in Q$

- A noun phrase is downward monotonic if it denotes a downward monotonic quantifier.

Downward Monotonicity Tests

If ⟦VP1⟧ ⊇ ⟦VP2⟧, then NP VP1 ⊨ NP VP2

- ⟦walked⟧ ⊇ ⟦walked rapidly⟧
- *No man walked* ⊨ *No man walked rapidly* ☺
- *All men walked* ⊭ *All men walked rapidly* ☹

NP VP1 or VP2 ⊨ NP VP1 and NP VP2

- *Neither girl was drinking or smoking* ⊨
  Neither girl was drinking and neither girl was smoking. ☺
- *All boys sing or dance* ⊭ *All boys sing and all boys dance.* ☹
- Note: $⟦VP_1 \text{ or } VP_2⟧ = ⟦VP_1⟧ \cup ⟦VP_2⟧$ and $⟦VP_1 \text{ and } VP_2⟧ = ⟦VP_1⟧ \cap ⟦VP_2⟧$

Looking for Universals I: Monotonicity Constraint

> "The simple noun phrases of any natural language express monotone quantifiers or conjunctions of monotone quantifiers."
> (Barwise & Cooper 1981)

Simple noun phrase: Proper names or NPs of the form $[_{NP}$ DET N$]$

Monotone quantifiers: quantifiers that are either upward or downward monotonic

Back to
Observation 2: Negative Polarity Items

(1) a. *John needn't go there.*

b. **John need go there.*

(2) a. *Nobody saw anything.*

b. **Somebody saw anything.*

(3) a. *No student has ever been in Saarbrücken.*

b. **Some student has ever been in Saarbrücken.*

- NPIs are licensed only in downward monotonic contexts.

Back to
Observation 3: Coordination

(1) *No man and few women walked.*

(2) *None of the girls and at most three boys walked.*

(3) **A man and few women walked.*

(4) **John and no woman saw Jane.*

- (Non-comparative) NPs can be coordinated iff they have the same direction of monotonicity.

(3') *A man but few women walked.*

(4') *John but no woman saw Jane.*

- Coordination with the connective “but” requires NPs with a different direction of monotonicity.

Quantifier Negation

External negation

- $\neg Q = \{ P \subseteq U_M \mid P \notin Q \}$

$$\begin{aligned} \neg[\![\text{all } N]\!] &= \{ P \subseteq U_M \mid P \notin [\![\text{all } N]\!] \} \\ &= \{ P \subseteq U_M \mid [\![N]\!] \cap P \neq [\![N]\!] \} \\ &= [\![\text{not all } N]\!] \end{aligned}$$

Internal negation

- $Q\neg = \{ P \subseteq U_M \mid (U_M - P) \in Q \}$

$$\begin{aligned} [\![\text{all } N]\!]\neg &= \{ P \subseteq U_M \mid (U_M - P) \in [\![\text{all } N]\!] \} \\ &= \{ P \subseteq U_M \mid [\![N]\!] \cap (U_M - P) = [\![N]\!] \} \\ &= \{ P \subseteq U_M \mid [\![N]\!] \cap (U_M - P) \neq \varnothing \} \\ &= \{ P \subseteq U_M \mid [\![N]\!] \cap P = \varnothing \} \\ &= [\![\text{no } N]\!] \end{aligned}$$

- If Q is an *upward monotonic* quantifier, then both $\neg Q$ and $Q\neg$ are *downward monotonic*.

- If Q is an *downward monotonic* quantifier, then both $\neg Q$ and $Q\neg$ are *upward monotonic*.

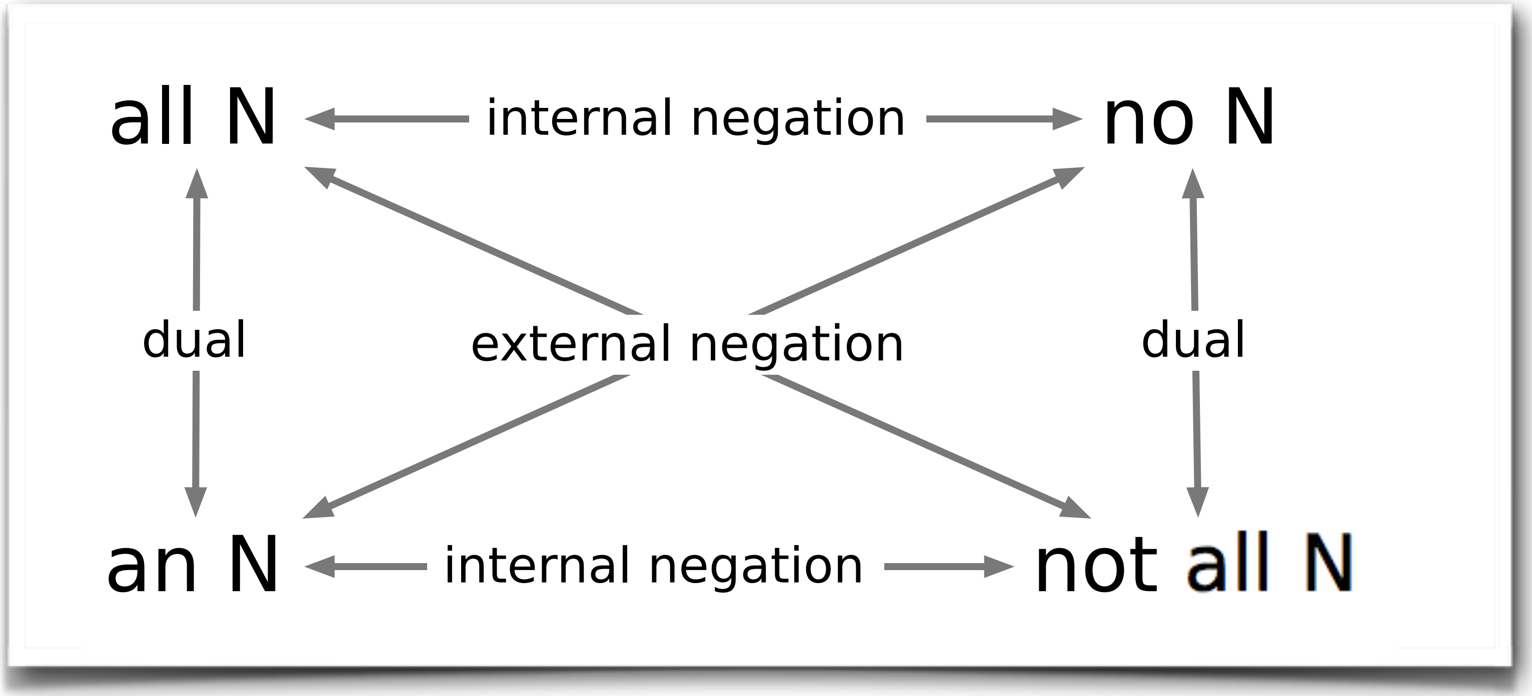
Duals

The dual Q* of a quantifier Q in M

$$Q^* = \neg Q \neg \quad = \{ P \subseteq U_M \mid (U_M - P) \in \neg Q \}$$
$$= \{ P \subseteq U_M \mid (U_M - P) \notin Q \}.$$

- If Q is *upward monotonic*, then Q* is *upward monotonic.*
- If Q is *downward monotonic*, then Q* is *downward monotonic*.

The “Square of Opposition”



From NPs to Determiners

Every man walked $\mapsto \forall x(\text{man'}(x) \rightarrow \text{walk'}(x))$

- *Every* $\Rightarrow \lambda P \lambda Q \forall x(P(x) \rightarrow Q(x))$
- $[\![\text{Every}]\!](A)(B) = 1$ iff $A \subseteq B$

Syntactically, determiners are expressions that take a noun and a verb phrase to form a sentence.

Semantically, the interpretation of a determiner can be seen as:

- a *function* from sets of entities to sets of properties: $\langle\langle e, t\rangle,\langle\langle e, t\rangle, t\rangle\rangle$
- a *relation* between two sets A and B, denoted by the NP and VP, respectively

Literature

- L.T.F. Gamut. Logic, Language, and Meaning. Vol 2. Chapter 7.
- Partee, ter Meulen, Wall. Mathematical Methods for Linguists. Chapter 14.
- Jon Barwise & Robin Cooper. Generalized Quantifiers. Linguistics and Philosophy. 1981.