Semantic Theory
Week 3– Some more type theory

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Exercises are due on: Tuesday, May 5th, 10 AM (before class)

Semantic Theory 2015: Exercise sheet 1

Exercise 1

Translate the following sentences into first-order predicate logic. You can freely introduce predicates, but try to retain as much of the structure as possible. Also provide the key to the translation.

a. Geoffrey is young and mean, but still a king.

b. Every Lannister pays his debt.

c. If one family rules the throne, all other families will fight for it.

d. A dire wolf is not a pet.

e. If someone is a Stark, (s)he is brave.

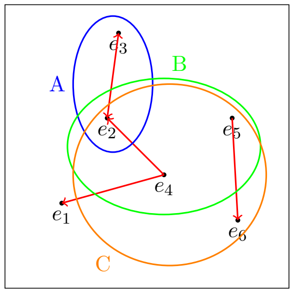
f. Fire-breathing dragons only obey Khaleesi.

g. Although Jaime lost a hand, he wins every fight unless he loses his other hand.

Exercise 2

Consider the following model $M_1 = \langle U_1, V_1 \rangle$, with $U_1 = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. The interpretation function V_1 is defined as follows:

M1:



- $V_1(j) = e_1$
- $V_1(m) = e_4$
- $V_1(b) = e_6$
- $V_1(A) = \{e_2, e_3\}$
- $V_1(B) = \{e_2, e_4, e_5\}$
- $V_1(C) = \{e_2, e_4, e_5, e_6\}$
- $V_1(R) = \{\langle e_2, e_3\rangle, \langle e_3, e_2\rangle \langle e_4, e_1\rangle, \langle e_4, e_2\rangle, \langle e_5, e_6\rangle\}$

Let the assignment function g_1 be defined as follows:
$g_1(x_1) = e_4$, $g_1(x_2) = e_2$, $g_1(x_3) = e_3$ and for all $n \geq 4$: $g_1(x_n) = e_5$.

2.1 Evaluate the following formulas in model M_1, with respect to assignment function g_1, showing the crucial steps.

a. $[\![R(x_1, x_2) \wedge R(x_4, b)]\!]^{M_1, g_1} = 1$

b. $[\![\exists x_2(B(x_2) \wedge R(x_2, j))]\!]^{M_1, g_1} = 1$

c. $[\![\forall x_1 \exists x_4(R(x_4, x_1) \vee R(x_1, x_4))]\!]^{M_1, g_1} = 1$

d. $[\![\forall x_1(B(x_1) \rightarrow (A(x_1) \vee \neg\exists x_3(R(x_3, x_1))))]\!]^{M_1, g_1} = 1$

2.2 Provide the full definition of a model M_2 and assignment function g_2 that satisfy the following formulas (NB: c_1 and c_2 are constants):

- $R(x_1, x_2)$
- $\forall x_1(A(x_1) \vee \exists x_2(R(x_1, x_2)))$
- $\neg\exists x_1(R(x_1, c_1))$
- $\exists x_3(A(x_3) \wedge \neg\exists x_2(A(x_2) \wedge R(x_2, x_3)))$
- $\forall x_2(B(x_2) \rightarrow (A(x_2) \vee R(x_2, c_2)))$

2.3 (Bonus) Can you think of a sensible (or: funny) interpretation for the predicates A, B and R, and the constants c_1 and c_2 in your model of the previous exercise? Given this interpretation, what is the translation of the formulas given in exercise 2.2?

Type Theory: Recap

- Basic Types: **e**, **t**
- Complex Types: ⟨**σ, τ**⟩, for any two types **σ** and **τ**
- Function application: ⟨**α, β**⟩(**α**) ↦ **β**
- Characteristic functions: functions with a *range* of {0,1}
- First-order one-place predicates (Px) are *functions* of type ⟨**e, t**⟩
  (i.e., characteristic functions describing the set of entities that have the property P)

Type Theory — Syntax

For every type τ, the set of well-formed expressions WE_τ is defined as follows:

(i) $CON_\tau \subseteq WE_\tau$ and $VAR_\tau \subseteq WE_\tau$;

(ii) If $\alpha \in WE_{\langle\sigma, \tau\rangle}$, and $\beta \in WE_\sigma$, then $\alpha(\beta) \in WE_\tau$;
(function application)

(iii) If A, B are in WE_t, then $\neg A$, $(A \wedge B)$, $(A \vee B)$, $(A \rightarrow B)$, $(A \leftrightarrow B)$ are in WE_t;

(iv) If A is in WE_t and x is a variable of arbitrary type, then $\forall xA$ and $\exists xA$ are in WE_t;

(v) If α, β are well-formed expressions of the same type, then $\alpha = \beta \in WE_t$;

Type inferencing: examples

1. Yoda_e is <u>faster than</u> Palpatine_e.
2. Yoda_e is <u>much</u> faster than Palpatine_e.
3. $\text{Wookiee}_{\langle e,t \rangle}$ is a <u>hairier</u> species than $\text{Ewok}_{\langle e,t \rangle}$.
4. $[\text{Han Solo}]_e$ fights <u>because</u> $[\text{the Dark Side}]_e$ is rising.
5. Anakin_e <u>believes</u> he will be a Yedi.
6. Obi-Wan_e <u>told</u> $[\text{Qui-Gon Jinn}]_e$ that he will take $[\text{the Yedi-exam}]_e$.
7. Obi-Wan_e <u>expects</u> to pass.
8. Yoda_e <u>encouraged</u> Obi-Wan_e to take the exam.

Type Theory — Interpretation

For every type **τ**, we can define a domain of interpretation **D_τ**

$D_e = U$; $D_t = \{0,1\}$; $D_{\langle e,t\rangle} = \{0,1\}^{D_e}$ (that is, the set of functions f: $D_e \rightarrow D_t$) ; etc.

Type theoretic expressions are interpreted relative to a model structure **M = ⟨U, V⟩**, and assignment function **g**, where:

- **U** is a non-empty domain of individuals
- **V** is an interpretation function, which assigns to every **α** $\in$ **CON_τ** an element of **D_τ** (where **τ** is an arbitrary type)
- **g** assigns to every typed variable **v** $\in$ **VAR_τ** an element of **D_τ**

Interpretation in Type Theory

Consider the following Model M:

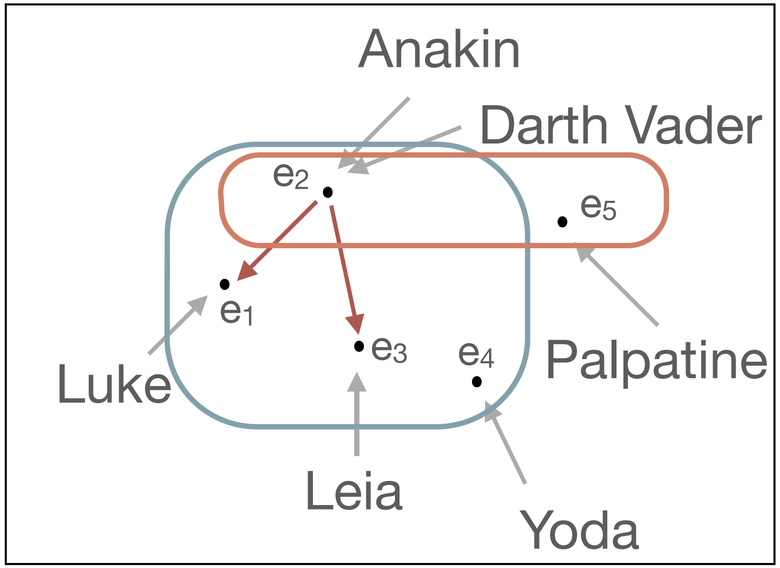
$D_e = U_M = \{e_1, e_2, e_3, e_4, e_5\}$

$V_M(\text{Anakin}_e) = V_M(\text{Darth Vader}_e) = e_2$

$V_M(\text{Yedi}_{\langle e,t\rangle}) = \begin{bmatrix} e_1 \rightarrow 1 \\ e_2 \rightarrow 1 \\ e_3 \rightarrow 1 \\ e_4 \rightarrow 1 \\ e_5 \rightarrow 0 \end{bmatrix}$ $V_M(\text{Dark\_Sider}_{\langle e,t\rangle}) = \begin{bmatrix} e_1 \rightarrow 0 \\ e_2 \rightarrow 1 \\ e_3 \rightarrow 0 \\ e_4 \rightarrow 0 \\ e_5 \rightarrow 1 \end{bmatrix}$

$$V_M(\text{Powerful}_{\langle\langle e,t\rangle\langle e,t\rangle\rangle}) = \begin{bmatrix} \begin{bmatrix} e_1 \rightarrow 1 \\ e_2 \rightarrow 1 \\ e_3 \rightarrow 1 \\ e_4 \rightarrow 1 \\ e_5 \rightarrow 0 \end{bmatrix} \rightarrow \begin{bmatrix} e_1 \rightarrow 0 \\ e_2 \rightarrow 1 \\ e_3 \rightarrow 0 \\ e_4 \rightarrow 1 \\ e_5 \rightarrow 0 \end{bmatrix} \\ \begin{bmatrix} e_1 \rightarrow 0 \\ e_2 \rightarrow 1 \\ e_3 \rightarrow 0 \\ e_4 \rightarrow 0 \\ e_5 \rightarrow 1 \end{bmatrix} \rightarrow \begin{bmatrix} e_1 \rightarrow 0 \\ e_2 \rightarrow 1 \\ e_3 \rightarrow 0 \\ e_4 \rightarrow 0 \\ e_5 \rightarrow 1 \end{bmatrix} \\ \ldots \end{bmatrix}$$

M:



NB: Powerful $X_{\langle e,t\rangle} \models X_{\langle e,t\rangle}$

Adjective classes & Meaning postulates

Some valid inferences in natural language:

- Bill is a poor piano player $\models$ Bill is a piano player
- Bill is a blond piano player $\models$ Bill is blond
- Bill is a former professor $\models$ Bill isn't a professor

These entailments do not hold in type theory. Why?

Meaning postulates: restrictions on models which constrain the possible meaning of certain words

Adjective classes & Meaning postulates (cont.)

Restrictive or Subsective adjectives ("poor")

- $[\![$ poor N $]\!] \subseteq [\![$ N $]\!]$
- Meaning postlate: $\forall G \forall x(poor(G)(x) \rightarrow G(x))$

Intersective adjectives ("blond")

- $[\![$ blond N $]\!] = [\![$ blond $]\!] \cap [\![$ N $]\!]$
- Meaning postlate: $\forall G \forall x(blond(G)(x) \rightarrow (blond^*(x) \wedge G(x))$
- NB: blond $\in WE_{\langle\langle e, t\rangle, \langle e, t\rangle\rangle} \neq$ blond* $\in WE_{\langle e, t\rangle}$

Privative adjectives ("former")

- $[\![$ former N $]\!] \cap [\![$ N $]\!] = \varnothing$
- Meaning postlate: $\forall G \forall x(former(G)(x) \rightarrow \neg G(x))$

Background reading material

- Gamut: Logic, Language, and Meaning Vol II
  — Chapter 4 (minus 4.3)