Exercises are due on: Tuesday, May 5th, 10 AM (before class)

Semantic Theory 2015: Exercise sheet 1

Exercise 1

Translate the following sentences into first-order predicate logic. You can freely introduce predicates, but try to retain as much of the structure as possible. Also provide the key to the translation.

a. Geoffrey is young and mean, but still a king.

b. Every Lannister pays his debt.

c. If one family rules the throne, all other families will fight for it.

d. A dire wolf is not a pet.

e. If someone is a Stark, (s)he is brave.

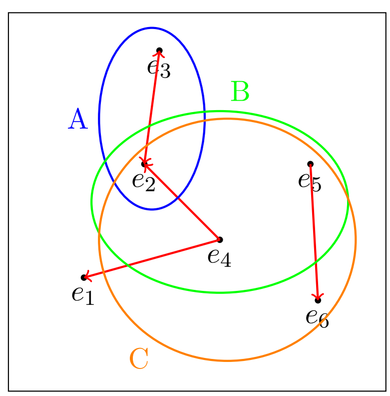
f. Fire-breathing dragons only obey Khaleesi.

g. Although Jaime lost a hand, he wins every fight unless he loses his other hand.

Exercise 2

Consider the following model $M_1 = \langle U_1, V_1 \rangle$, with $U_1 = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. The interpretation function V_1 is defined as follows:

M1:



- $V_1(j) = e_1$
- $V_1(m) = e_4$
- $V_1(b) = e_6$
- $V_1(A) = \{e_2, e_3\}$
- $V_1(B) = \{e_2, e_4, e_5\}$
- $V_1(C) = \{e_2, e_4, e_5, e_6\}$
- $V_1(R) = \{\langle e_2, e_3\rangle, \langle e_3, e_2\rangle\langle e_4, e_1\rangle, \langle e_4, e_2\rangle, \langle e_5, e_6\rangle\}$

Let the assignment function g_1 be defined as follows:
$g_1(x_1) = e_4$, $g_1(x_2) = e_2$, $g_1(x_3) = e_3$ and for all $n \geq 4$: $g_1(x_n) = e_5$.

2.1 Evaluate the following formulas in model M_1, with respect to assignment function g_1, showing the crucial steps.

a. $[\![R(x_1, x_2) \wedge R(x_4, b)]\!]^{M_1,g_1} = 1$

b. $[\![\exists x_2(B(x_2) \wedge R(x_2, j))]\!]^{M_1,g_1} = 1$

c. $[\![\forall x_1 \exists x_4(R(x_4, x_1) \vee R(x_1, x_4))]\!]^{M_1,g_1} = 1$

d. $[\![\forall x_1(B(x_1) \rightarrow (A(x_1) \vee \neg\exists x_3(R(x_3, x_1))))]\!]^{M_1,g_1} = 1$

2.2 Provide the full definition of a model M_2 and assignment function g_2 that satisfy the following formulas (NB: c_1 and c_2 are constants):

- $R(x_1, x_2)$
- $\forall x_1(A(x_1) \vee \exists x_2(R(x_1, x_2)))$
- $\neg\exists x_1(R(x_1, c_1))$
- $\exists x_3(A(x_3) \wedge \neg\exists x_2(A(x_2) \wedge R(x_2, x_3)))$
- $\forall x_2(B(x_2) \rightarrow (A(x_2) \vee R(x_2, c_2)))$

2.3 (Bonus) Can you think of a sensible (or: funny) interpretation for the predicates A, B and R, and the constants c_1 and c_2 in your model of the previous exercise? Given this interpretation, what is the translation of the formulas given in exercise 2.2?