Semantic Theory

Lecture 5 – Generalized Quantifiers

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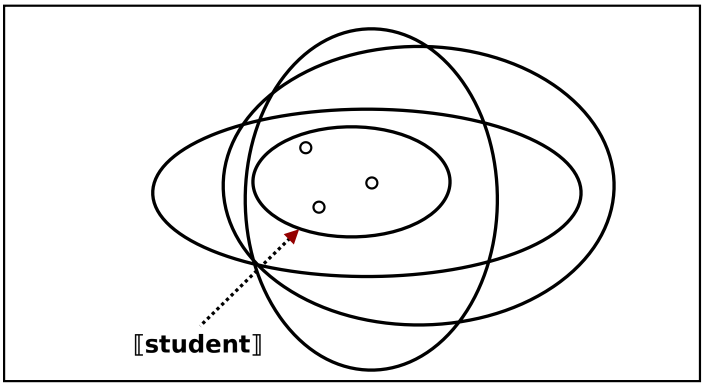
Generalized Quantifiers

- *Every student works*
  - $\forall x(\text{student}'(x) \rightarrow \text{work}'(x))$
  - *Every student* $\mapsto \lambda Q \forall x(\text{student}'(x) \rightarrow Q(x))$
  - $\llbracket \textit{Every student} \rrbracket^M = \{ P \subseteq U_M \mid \llbracket \textit{student} \rrbracket \subseteq P \}$
- **A generalized quantifier** is a set of properties
  - property = set of individuals
- A sentence of the form $[_S$ NP VP$]$ is true iff $\llbracket \text{VP} \rrbracket \in \llbracket \text{NP} \rrbracket$
  - $\llbracket \textit{Every student works} \rrbracket = 1$ iff $\llbracket \textit{work} \rrbracket \in \llbracket \textit{every student} \rrbracket$

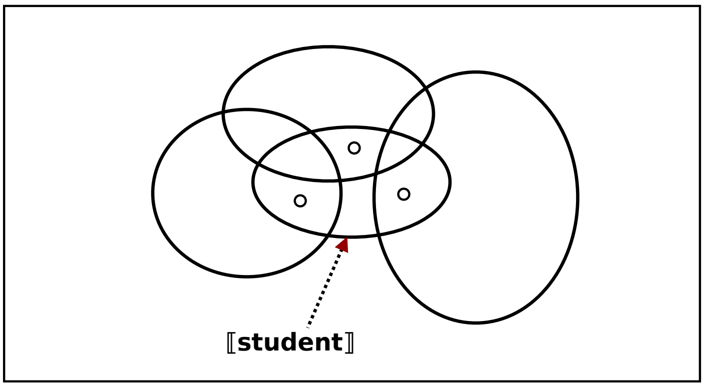
⟦*every student*⟧

- ⟦***every student***⟧ denotes the set of properties that apply to every student (i.e., all supersets of ⟦student⟧)



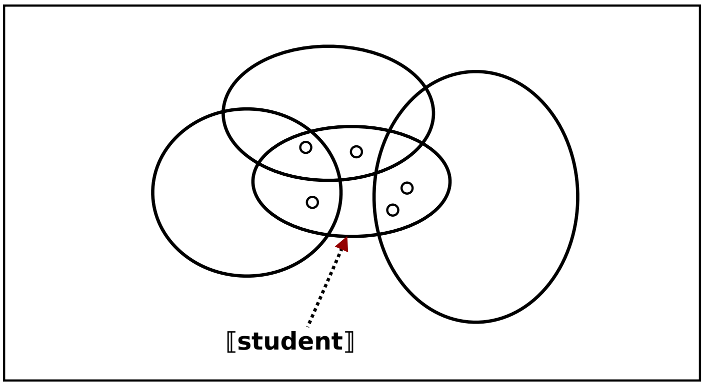
⟦*a student*⟧

- ⟦***a student***⟧ denotes the set of properties that apply to at least one student.



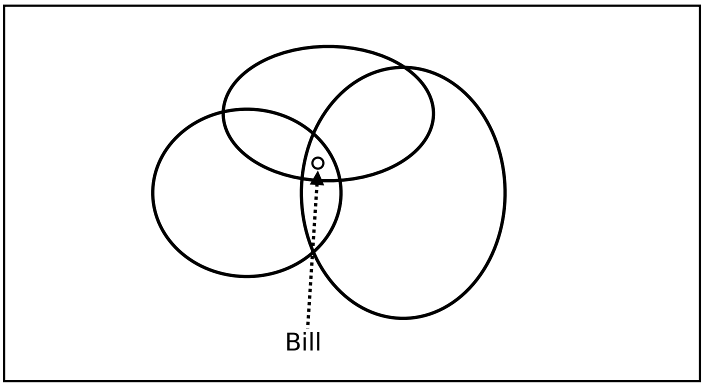
⟦*two students*⟧

- ⟦***two students***⟧ denotes the set of properties that apply to at least (exactly) two students.



⟦*Bill*⟧

- ⟦***Bill***⟧ denotes the set of properties that apply to Bill



Noun Phrase Interpretations

$\llbracket \textit{all N} \rrbracket^M = \{ P \subseteq U_M \mid \llbracket N \rrbracket \cap P = \llbracket N \rrbracket \}$

$\llbracket \textit{a N} \rrbracket^M = \{ P \subseteq U_M \mid \llbracket N \rrbracket \cap P \neq \emptyset \}$

$\llbracket \textit{not all N} \rrbracket^M =$

$\llbracket \textit{no N} \rrbracket^M =$

$\llbracket \textit{exactly n N} \rrbracket^M =$

$\llbracket \textit{at most n N} \rrbracket^M =$

$\llbracket \textit{at least n N} \rrbracket^M =$

Generalized Quantifier Theory

- What formal properties do quantifiers have?
- What natural subclasses can be distinguished?
- Which subclasses actually represent meanings of natural language noun phrases?

Negative Polarity Items

(1) a. *John **need**n't go there*
    b. **John **need** go there*

(2) a. *Nobody saw **any**thing*
    b. **Somebody saw **any**thing*

(3) a. *No student has **ever** been in Saarbrücken*
    b. **Some student has **ever** been in Saarbrücken*

Negative polarity items (any, ever, ...)
⇒ items that can occur only in "negative contexts"

Question: What licenses negative polarity items?

There-Sentences

(1) *There is someone in the garden*

(2) *There is no one in the garden*

(3) *There are two unicorns in the garden*

(4) **There is/are everyone in the garden*

(5) **There is John in the garden*

(6) **There are the two unicorns in the garden*

Question: which noun phrases can appear in “there” sentences (and why)?

Note: as exclamations (4) - (6) are correct

Coordination

(1) *No man and few women walked*

(2) *None of the girls and at most three boys walked*

(3) **A man and few women walked*

(4) **John and no woman saw Jane*

Question: which noun phrases can be coordinated?

Inference Patterns

(1) *All men* ***walked rapidly*** ⊨ *All men* ***walked***

(2) *No man* ***walked*** ⊨ *No man* ***walked rapidly***

(3) *A girl* ***smoked a cigar*** ⊨ *A girl* ***smoked***

(4) *Few girls* ***smoked*** ⊨ *Few girls* ***smoked a cigar***

Upward Monotonicity

- *All men walked rapidly* ⊨ *All men walked*
  - Note: ⟦walked rapidly⟧ ⊆ ⟦walked⟧
- *A girl smoked a cigar* ⊨ *A girl smoked*
  - Note: ⟦smoked a cigar⟧ ⊆ ⟦smoked⟧
- **Observation:**
  A sentence [$_S$ NP VP] remains true if the denotation of the verb phrase is made "larger"

Upward Monotonicity

- A quantifier Q is **upward monotonic** in M = ⟨U, V⟩ iff Q is closed under supersets:
  - for all X, Y ⊆ U: if X ∈ Q and X ⊆ Y, then Y ∈ Q
- A noun phrase is upward monotonic if it denotes an upward monotonic quantifier.

Upward Monotonicity Tests

- **If ⟦VP_1⟧ ⊆ ⟦VP_2⟧, then NP VP_1 ⊨ NP VP_2**
  - *All men walked rapidly* ⊨ *All men walked*
  - *No man walked rapidly* ⊭ *No man walked*
  - Note: ⟦walked rapidly⟧ ⊆ ⟦walked⟧
- **NP VP_1 and VP_2 ⊨ NP VP_1 and NP VP_2**
  - *All men smoked and drank* ⊨
    All men smoked and all men drank
  - *No man smoked and drank* ⊭
    No man smoked and no man drank
  - Note: ⟦VP_1 and VP_2⟧ = ⟦VP_1⟧ ∩ ⟦VP_2⟧

Upward Monotonicity

- The set of upward monotonic quantifiers is closed under conjunction and disjunction:
  - the intersection (union) of two upward monotonic quantifiers is an upward monotonic quantifier.
- *All boys and a girl walked rapidly* ⊨ *All boys and a girl walked*
- Note:
  - ⟦NP_1 and NP_2⟧ = ⟦NP_1⟧ ∩ ⟦NP_2⟧
  - ⟦NP_1 or NP_2⟧ = ⟦NP_1⟧ ∪ ⟦NP_2⟧

Downward Monotonicity

(1) *No man walked* ⊨
No man walked rapidly

(2) *Not every woman was asleep* ⊨
Not every woman was dreaming

(3) *Less than half of the girls smoked* ⊨
Less than half of the girls smoked cigars

(4) *Few boys were playing* ⊨
Few boys were playing out on the street

Downward Monotonicity

- A quantifier Q is **downward monotonic** in M = ⟨U, V⟩ iff Q is closed under inclusion:
  - for all $X, Y \subseteq U$: if $X \in Q$ and $X \supseteq Y$, then $Y \in Q$
- A noun phrase is downward monotonic if it denotes a downward monotonic quantifier.

Downward Monotonicity Tests

- **If ⟦VP$_1$⟧ ⊇ ⟦VP$_2$⟧, then NP VP$_1$ ⊨ NP VP$_2$**
  - *All men walked* ⊭ *All men walked rapidly*
  - *No man walked* ⊨ *No man walked rapidly*
  - Note: ⟦walked⟧ ⊇ ⟦walked rapidly⟧
- **NP VP$_1$ or VP$_2$ ⊨ NP VP$_1$ and NP VP$_2$**
  - *Neither girl was drinking or smoking* ⊨ *Neither girl was drinking and neither girl was smoking.*
  - *All boys sing or dance* ⊭ *All boys sing and all boys dance.*
  - Note: ⟦VP$_1$ or VP$_2$⟧ = ⟦VP$_1$⟧ ∪ ⟦VP$_2$⟧

Negative Polarity Items

(1) a. *John needn't go there*
b. **John need go there*

(2) a. *Nobody saw anything*
b. **Somebody saw anything*

(3) a. *No student has ever been in Saarbrücken*
b. **Some student has ever been in Saarbrücken*

⇒ negative polarity items are licensed only in downward monotonic contexts.

Coordination

(1) *No man and few women walked*

(2) *None of the girls and at most three boys walked*

(3) **A man and few women walked*

(4) **John and no woman saw Jane*

Non-comparative noun phrases can be coordinated iff they have the same direction of monotonicity.

Coordination

(1) **A man and few women walked*

(2) *A man but few women walked*

(3) **John and no woman saw Jane*

(4) *John but no woman saw Jane*

Coordination with the connective “but” requires noun phrases of different direction of monotonicity.

Language Universals

Monotonicity Constraint (Barwise & Cooper 1981)
The simple noun phrases of any natural language express monotone quantifiers or conjunctions of monotone quantifiers.

Simple noun phrase: Proper names or noun phrases of the form $[_{NP}\ \mathrm{DET}\ \mathrm{N}]$

Negation of Quantifiers

External negation: $\neg Q = \{ P \subseteq U_M \mid P \notin Q \}$

- $\neg\llbracket \text{all N} \rrbracket = \{ P \subseteq U_M \mid P \notin \llbracket \text{all N} \rrbracket \}$
  $= \{ P \subseteq U_M \mid \llbracket N \rrbracket \cap P \neq \llbracket N \rrbracket \}$
- $\neg\llbracket \text{all N} \rrbracket = \llbracket \text{not all N} \rrbracket$

Internal negation: $Q\neg = \{ P \subseteq U_M \mid (U_M - P) \in Q \}$

- $\llbracket \text{all N} \rrbracket\neg = \{ P \subseteq U_M \mid (U_M - P) \in Q \}$
  $= \{ P \subseteq U_M \mid \llbracket N \rrbracket \cap (U_M - P) \neq \emptyset \}$
  $= \{ P \subseteq U_M \mid \llbracket N \rrbracket \cap P = \emptyset \}$
- $\llbracket \text{all N} \rrbracket\neg = \llbracket \text{no N} \rrbracket$

Negation of Quantifiers

- If Q is an upward monotonic quantifier, then both ¬Q and Q¬ are downward monotonic.
- If Q is an downward monotonic quantifier, then both ¬Q and Q¬ are upward monotonic.

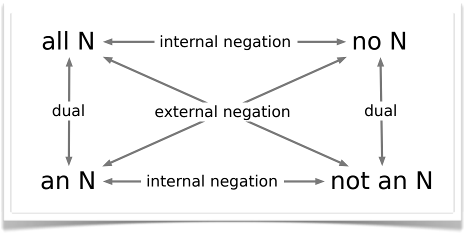
Duals

- **The dual Q* of a quantifier Q in M**

  $Q^* = \neg Q\neg \;= \{\, P \subseteq U_M \mid (U_M - P) \in \neg Q \,\}$

  $\qquad\qquad = \{\, P \subseteq U_M \mid (U_M - P) \notin Q \,\}$.

- If Q is upward monotonic, then Q* is upward monotonic.
- If Q is downward monotonic, then Q* is downward monotonic.



Determiners

- *Every man walked* ↦ $\forall x(\text{man}'(x) \rightarrow \text{walk}'(x))$
  - *Every* ⇒ $\lambda P \lambda Q \forall x(P(x) \rightarrow Q(x))$
  - $[\![\text{Every}]\!](A)(B) = 1$ iff $A \subseteq B$
- We can consider determiners as expressions that take a noun and a verb phrase to form a sentence.
- Semantically, the interpretation of a determiner can be seen as a relation between two sets.

Persistence

- **A determiner D is persistent** in M iff for all X, Y, Z:
  - if D(X, Z) and X ⊆ Y, then D(Y, Z)
- **Persistence test:**
  - If $[\![N_1]\!] \subseteq [\![N_2]\!]$, then DET N_1 VP ⊨ DET N_2 VP
  - *Some men walked* ⊨
    Some human beings walked
  - *At least four girls were smoking* ⊨
    At least four women were smoking.

Antipersistence

- **A determiner D is antipersistent** in M iff for all X,Y,Z:
  - if D(X, Z) and Y ⊆ X, then D(Y, Z)
- **Antipersistence test:**
  - If $[\![N_2]\!] \subseteq [\![N_1]\!]$, then DET N_1 VP ⊨ DET N_2 VP
  - *All children walked* ⊨
    All toddlers walked
  - *No woman was smoking* ⊨
    No girl was smoking
  - *At most three Englishmen agreed* ⊨
    At most three Londoners agreed.

Persistence and Monotonicity

- Persistence and monotonicity are closely related:
  - Persistence (antipersistence) is upward (downward) monotonicity of the first argument.
  - Upward (downward) monotonicity of noun phrases is upward (downward) monotonicity of the second argument of the determiner in the NP.
- Terminology:
  - left-monotonicity (↑mon and ↓mon)
  - right-monotonicity (mon↑ and mon↓)

Left and Right Monotonicity

↑mon↑	some, at least n, infinitely many
↓mon↑	all
↓mon↓	no, at most n, a finite number of
↑mon↓	not all

Conservativity

- **Conservativity:** for every A, B ⊆ U
  - Q(A, B) ⇔ Q(A, A ∩ B)
- **Test:** D N VP ⇔ D N are N that VP
  - *All students work ⇔*
    All students are students that work
  - *Some girls are dancing ⇔*
    Some girls are girls that are dancing
  - *Most teachers are motivated ⇔*
    Most teachers are teachers that are motivated

Lives on

- **A quantifier Q lives on** X iff for all Y,
  - Y ∈ Q iff X ∩ Y ∈ Q
- **Universal** (Barwise & Cooper 1981, cited from Gamut)
  In every natural language, simple determiners together with an N yield an NP which lives on ⟦N⟧
- **Apparent exception:** only
  - *Only men smoke cigars ⇎*
    Only men are men that smoke cigars
  - ⇒ “only” not a determiner?

Literature

- L.T.F. Gamut. Logic, Language, and Meaning. Vol 2. Chapter 7.
- Partee, ter Meulen, Wall. Mathematical Methods for Linguists. Chapter 14.
- Jon Barwise & Robin Cooper. Generalized Quantifiers. Linguistics and Philosophy. 1981.