Semantic Theory
Lecture 14: Dynamic Predicate Logic

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Contributions of DRT

(1) *A student works. She is successful.*

(2) *If a student works, she is successful.*

Sentence Meaning (Recap)

- **Truth-conditional interpretation:** The meaning of a declarative sentence is given by its truth conditions
- **Compositionality:** The meaning of a complex expression is a function of the meanings of its parts and of the syntactic rules by which they are combined

Accessibility & Truth-conditions

(1) *There is a book that John doesn't own.*
He wants to buy it.

(2) *John does not own every book.*
** He wants to buy it.*

(3) *One of the ten balls is not in the bag.*
It must be under the sofa.

(4) *Nine of the ten balls are in the bag.*
** It must be under the sofa.*

DRT & Compositionality

- **DRT is non-compositional** on truth conditions: The different discourse-semantic status of the text pairs is not predictable through the (identical) truth conditions of its component sentences.
- Since structural information which cannot be reduced to truth conditions is required to compute the semantic value of texts, DRT is called **a representational theory of meaning.**

Dynamic Predicate Logic (DPL)

- **Dynamic Predicate Logic** is a dynamic theory of meaning (just like DRT) but admits (in contrast to DRT) a compositional interpretation
- **The DRT approach:**
  - alternative representations ("boxes")
  - interpretation not fully compositional
- **The DPL approach:**
  - conventional representations (predicate logic)
  - compositional (but more complex) interpretation

Dynamic Predicate Logic (DPL)

- The DPL approach is closely related to the denotational approach to the semantics of programming languages.
- A program p denotes a set of pairs of start and end configurations:
  - $\langle f,g \rangle \in [\![p]\!]^M$ iff g is an end configuration that can be reached from the start configuration f by running p.
- Semantics of complex programs can be determined compositionally. For instance:
  - $\langle f,g \rangle \in [\![p_1; p_2]\!]^M$ iff there is an intermediate configuration h that can be reached from f by running p_1 ($\langle f, h \rangle \in [\![p_1]\!]^M$) and from which g can be reached by running p_2 ($\langle h, g \rangle \in [\![p_2]\!]^M$)

Dynamic Predicate Logic (DPL)

- Logical formulas are programs. Configurations are variable assignments.
- A formula denotes a set of pairs of start and end configurations (input and output assignments).
  - $[\![\varphi]\!]^M = \{ \langle g, h \rangle \mid \dots \}$
- Certain formulas and connectives are instructions for changing the assignments.
  - for instance: "∃x" modifies the value of x by overwriting it with an arbitrary individual from the universe.
- Other formulas are tests: "F(x)" checks whether the value of x in the current assignment has the property F.

Syntax of DPL

- The syntax of dynamic predicate logic (DPL) is the syntax of first-order predicate logic.
- Translation of natural languages expressions into DPL:
  - "a" and "every" into the respective quantifier
  - pronouns into (possibly free) variables.
- Example:
  - Somebody works. She is successful.
  - $\exists x\ \mathrm{work}(x) \wedge \mathrm{successful}(x)$
  - **Note:** "x" in the right conjunct not in the scope of "∃x"

Semantics of DPL

- Ordinary first order model structures
  - Universe + interpretation function
- **Interpretation of terms** as in predicate logic
  - $[\![c]\!]^{M,g} = V_M(c)$ (constants)
  - $[\![x]\!]^{M,g} = g(x)$ (variables)
- **Interpretation of formulae:** The semantic value $[\![\varphi]\!]$ of a formula φ is a binary relation between variable assignments:
  - $[\![\varphi]\!]^M = \{ \langle g, h \rangle \mid \ldots \}$

Semantics of DPL

- **Atomic Formulae:**
  - $[\![R(t_1, \ldots, t_n)]\!]^M = \{ \langle g, h \rangle \mid g=h \wedge \langle [\![t_1]\!]^{M,g}, \ldots, [\![t_n]\!]^{M,g} \rangle \in V_M(R)\}$
- **Existential quantification:**
  - $[\![\exists x\varphi]\!]^M = \{ \langle g, h \rangle \mid \exists k\text{: } k[x]g \wedge \langle k, h \rangle \in [\![\varphi]\!]^M \}$
- Read "**h[x]g**" as "assignments h and g differ at most in the value for x"

Example (∃)

- *Somebody works.*
  - ∃x work(x)
- **Interpretation:**
  - $\langle g, h \rangle \in [\![\exists x\, \text{work}(x)]\!]^M$
  - iff there is a k such that k[x]g and $\langle k, h \rangle \in [\![\text{work}(x)]\!]^M$
  - iff there is a k such that k[x]g and $\langle k, h \rangle \in \{ \langle k', h' \rangle \mid k' = h' \text{ and } [\![x]\!]^{M,h'} \in V_M(\text{work}) \}$
  - iff there is a k such that k[x]g and k = h and $[\![x]\!]^{M,h} \in V_M(\text{work})$
  - iff h[x]g and $[\![x]\!]^{M,h} \in V_M(\text{work})$

Example (∃)

- *Somebody works.*
  - $\exists x\, \text{work}(x)$
- **Interpretation:**
  - $\langle g, h\rangle \in [\![\exists x\, \text{work}(x)]\!]^M$ iff $h[x]g$ and $[\![x]\!]^{M,h} \in V_M(\text{work})$
- Alternative notation:
  - $[\![\exists x\, \text{work}(x)]\!]^M = \{ \langle g, h\rangle \mid h[x]g \text{ and } [\![x]\!]^{M,h} \in V_M(\text{work}) \}$

Conjunctions

- *Somebody works. She is successful*
  - $\exists x\, \text{work}(x) \wedge \text{successful}(x)$

Conjunctions

- Atomic Formulae:
  - $[\![R(t_1, \ldots, t_n)]\!]^M = \{ \langle g, h\rangle \mid g=h \wedge \langle [\![t_1]\!]^{M,g}, \ldots, [\![t_n]\!]^{M,g}\rangle \in V_M(R)\}$
- Existential quantification:
  - $[\![\exists x\varphi]\!]^M = \{ \langle g, h\rangle \mid \exists k\colon k[x]g \wedge \langle k, h\rangle \in [\![\varphi]\!]^M \}$
- **Conjunctions:**
  - $[\![\varphi \wedge \psi]\!]^M = \{ \langle g, h\rangle \mid \exists k\colon \langle g, k\rangle \in [\![\varphi]\!]^M \wedge \langle k, h\rangle \in [\![\psi]\!]^M \}$

Conjunctions

- *Somebody works. She is successful*
  - $\exists x\, work(x) \wedge successful(x)$
- **Interpretation:**
  - $\langle g, h \rangle \in [\![\exists x\, work(x) \wedge successful(x)]\!]^M$
  - iff there is a k such that
    - $\langle g, k \rangle \in [\![\exists x\, work(x)]\!]^M$ and $\langle k, h \rangle \in [\![successful(x)]\!]^M$
  - iff there is a k such that
    - $k[x]g$ and $k(x) \in V_M(work)$ and $k = h$ and $k(x) \in V_M(successful)$

Static & dynamic operators

- Existential quantifiers are dynamic operators:
  - the output assignment of a formula $\exists x\psi$ can be different than the input assignment
- Conjunctions are also interpretet dynamically
  - the left-hand subformula can change the input assignment for the right-hand subformula
- Atomic formulae are interpreted statically:
  - input and output assignment must be identical

Conditionals

- *If somebody works, she is successful*
  - $\exists x\, work(x) \rightarrow successful(x)$

Conditionals

- Atomic Formulae:
  - $[\![R(t_1, \ldots, t_n)]\!]^M = \{ \langle g, h \rangle \mid g = h \wedge \langle [\![t_1]\!]^{M,g}, \ldots, [\![t_n]\!]^{M,g} \rangle \in V_M(R)\}$
- Conjunctions
  - $[\![\varphi \wedge \psi]\!]^M = \{ \langle g, h \rangle \mid \exists k: \langle g, k \rangle \in [\![\varphi]\!]^M \wedge \langle k, h \rangle \in [\![\psi]\!]^M \}$
- Existential quantification:
  - $[\![\exists x \varphi]\!]^M = \{ \langle g, h \rangle \mid \exists k: k[x]g \wedge \langle k, h \rangle \in [\![\varphi]\!]^M \}$
- **Conditionals:**
  - $[\![\varphi \rightarrow \psi]\!]^M = \{ \langle g, h \rangle \mid g = h \wedge \forall k: \langle g, k \rangle \in [\![\varphi]\!]^M \Rightarrow \exists j: \langle k, j \rangle \in [\![\psi]\!]^M\}$

Conditionals

- *If somebody works, she is successful*
  - $\exists x\, \text{work}(x) \rightarrow \text{successful}(x)$
- **Interpretation:**
  - $\langle g, h \rangle \in [\![\exists x\, \text{work}(x) \rightarrow \text{successful}(x)]\!]^M$
  - iff g = h and for all k:
    - if $\langle h, k \rangle \in [\![\exists x\, \text{work}(x)]\!]^M$,
    - then there is a j such that $\langle k, j \rangle \in [\![\text{successful}(x)]\!]^M$
  - iff g = h and for all k:
    - if k[x]h and $k(x) \in V_M(\text{work})$,
    - then there is a j such that k = j and $j(x) \in V_M(\text{successful})$

Conditionals

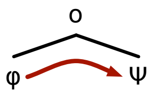
- *If somebody works, she is successful*
  - $\exists x\, \text{work}(x) \rightarrow \text{successful}(x)$
- **Interpretation:**
  - $\langle g, h \rangle \in [\![\exists x\, \text{work}(x) \rightarrow \text{successful}(x)]\!]^M$
  - iff g = h and for all k:
    - if k[x]h and $k(x) \in V_M(\text{work})$, then $k(x) \in V_M(\text{successful})$

Universals, Disjunction, Negation

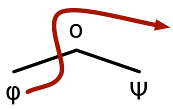
- **Negation**
  - $\llbracket \neg\varphi \rrbracket^M = \{ \langle g, h\rangle \mid g = h \wedge \neg\exists k: \langle g, k\rangle \in \llbracket\varphi\rrbracket^M \}$
- **Disjunction**
  - $\llbracket \varphi \vee \psi \rrbracket^M = \{ \langle g, h\rangle \mid g=h \wedge \exists k: \langle g, k\rangle \in \llbracket\varphi\rrbracket^M \vee \langle g, k\rangle \in \llbracket\psi\rrbracket^M\}$
- **Universal quantification**
  - $\llbracket \forall x\varphi \rrbracket^M = \{ \langle g, h\rangle \mid g = h \wedge \forall k: k[x]g \Rightarrow \exists m: \langle k, m\rangle \in \llbracket\varphi\rrbracket^M \}$

Static & dynamic operators

- A connective o is **internally dynamic** iff the left-hand subformula can change the input assignment for the right- hand subformula



- A connective o is **externally dynamic** iff the output assignment of a formula with main connective o can be different than the input assignment



- Formulas whose main connective is externally static are called **tests**: From $\langle g, h\rangle \in \llbracket\varphi\rrbracket^M$ follows that $g = h$

Connectives: Overview

Connective	**externally**	**internally**
¬	static	–
∧	dynamic	dynamic
∨	static	static
→	static	dynamic
∃	dynamic	dynamic
∀	static	dynamic

Sematics of DPL

- **Interpretation of formulas** with respect to a model structure M and variable assignment g.

$[\![R(t_1, ..., t_n)]\!]^M = \{\langle g, h\rangle \mid g = h \wedge \langle [\![t_1]\!]^{M,g}, ..., [\![t_n]\!]^{M,g}\rangle \in V_M(R)\}$

$[\![t_1 = t_2]\!]^M = \{\langle g, h\rangle \mid g = h \wedge [\![t_1]\!]^{M,g} = [\![t_2]\!]^{M,g}\}$

$[\![\neg\varphi]\!]^M = \{\langle g, h\rangle \mid g = h \wedge \neg\exists k: \langle g, k\rangle \in [\![\varphi]\!]^M\}$

$[\![\varphi \wedge \psi]\!]^M = \{\langle g, h\rangle \mid \exists k: \langle g, k\rangle \in [\![\varphi]\!]^M \wedge \langle k, h\rangle \in [\![\psi]\!]^M\}$

$[\![\varphi \vee \psi]\!]^M = \{\langle g, h\rangle \mid g = h \wedge \exists k: \langle g, k\rangle \in [\![\varphi]\!]^M \vee \langle g, k\rangle \in [\![\psi]\!]^M\}$

$[\![\varphi \rightarrow \psi]\!]^M = \{\langle g, h\rangle \mid g = h \wedge \forall k: \langle g, k\rangle \in [\![\varphi]\!]^M \Rightarrow \exists j: \langle k, j\rangle \in [\![\psi]\!]^M\}$

$[\![\exists x\varphi]\!]^M = \{\langle g, h\rangle \mid \exists k: k[x]g \wedge \langle k, h\rangle \in [\![\varphi]\!]^M\}$

$[\![\forall x\varphi]\!]^M = \{\langle g, h\rangle \mid g = h \wedge \forall k: k[x]g \Rightarrow \exists m: \langle k, m\rangle \in [\![\varphi]\!]^M\}$

Truth & Validity

- A formula φ is **true in M with respect to** an input assignment **g** iff there is a h such that $\langle g, h\rangle \in [\![\varphi]\!]^M$
- A formula φ is **true in M** iff φ is true in M wrt. every input assignment g.
- A formula φ is **valid** iff φ is true in every model structure M.

Equivalence

- **Satisfaction set** of a formula φ in M:
  $\backslash\varphi\backslash_M = \{ g \mid \text{there is a h such that } \langle g, h\rangle \in [\![\varphi]\!]^M \}$
- **Static equivalence** (s-equivalence)
  $\varphi \Leftrightarrow_s \psi$ iff for all model structures M: $\backslash\varphi\backslash_M = \backslash\psi\backslash_M$
- **Equivalence** (dynamic / full equivalence):
  $\varphi \Leftrightarrow \psi$ iff for all model structures M: $[\![\varphi]\!]^M = [\![\psi]\!]^M$

Entailment

- **Static entailment:** $\varphi \vDash_s \psi$
  iff for all model structures M and assignments g: if φ is true in M for g, then ψ is true in M for g
- **Meaning Inclusion:** $\varphi \leq \psi$
  iff for all model structures M, $[\![\varphi]\!]^M \subseteq [\![\psi]\!]^M$
- **Dynamic entailment:** $\varphi \vDash \psi$
  iff for all model structures M and assignments g and h: if $\langle g, h\rangle \in [\![\varphi]\!]$, then there is an assignment k such that $\langle h, k\rangle \in [\![\psi]\!]$

Logical Properties

- **The following equivalences hold:**
  - $\exists xA \wedge B \Leftrightarrow \exists x(A \wedge B)$
  - $\exists xA \rightarrow B \Leftrightarrow \forall x(A \rightarrow B)$
  - $(A \wedge B) \wedge C \Leftrightarrow A \wedge (B \wedge C)$
  - $A \rightarrow (B \rightarrow C) \Leftrightarrow (A \wedge B) \rightarrow C$
  - $A \vee B \Leftrightarrow B \vee A$
- **The following equivalences do not hold:**
  - $A \wedge B \Leftrightarrow B \wedge A$
  - $A \Leftrightarrow A \wedge A$
  - $\neg\forall xA \Leftrightarrow \exists x\neg A$

Definability of Connectives

- ∨, → and ∀ can be defined from ¬, ∧ und ∃
- But not vice versa!
- **Equivalences**
  - $A \rightarrow B \Leftrightarrow \neg(A \wedge \neg B)$
  - $A \vee B \Leftrightarrow \neg(\neg A \wedge \neg B)$
  - $A \vee B \Leftrightarrow \neg A \rightarrow B$
  - $\forall x\, A \Leftrightarrow \neg\exists x \neg A$
- **Non-equivalences**
  - $A \wedge B \quad \Leftrightarrow \quad \neg(A \rightarrow \neg B)$
  - $A \wedge B \quad \Leftrightarrow_s \quad \neg(\neg A \vee \neg B)$
  - $A \rightarrow B \quad \Leftrightarrow_s \quad \neg A \vee B$
  - $\exists x\, A \quad \Leftrightarrow \quad \neg\forall x \neg A$

Summary

- We can give a compositional interpretation to a theory of dynamic semantics:
  - relation between variable assignments.
- DPL uses standard syntax of predicate logic, but the different interpretation makes for interesting logical properties
  - for instance, some equivalences break
- We can translate DRSs into DPL formulas and further into (static) predicate logic formulas (see exercise).
- DRT can be equipped directly with a DPL-style interpretation.

Literature

- Jeroen Groenendijk, Martin Stokhof. *Dynamic Predicate Logic.* Linguistics and Philosophy, Vol. 14(1), 1991.
- L.T.F. Gamut (1991): *Logic, Language and Meaning, Vol II*, University of Chicago Press. Chapter 7.4.5