

Semantic Theory

Lecture 5: Generalized Quantifiers

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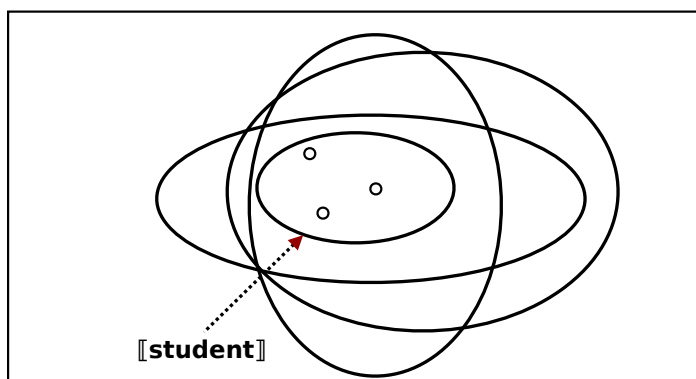
Generalized Quantifiers

- *Every student works*
 - $\forall x(\text{student}'(x) \rightarrow \text{work}'(x))$
 - *Every student* $\mapsto \lambda Q \forall x(\text{student}'(x) \rightarrow Q(x))$
 - $\llbracket \text{Every student} \rrbracket^M = \{ P \subseteq U_M \mid \llbracket \text{student} \rrbracket \subseteq P \}$
- **A generalized quantifier** is a set of properties
 - property = set of individuals
- A sentence of the form $[_S \text{ NP VP}]$ is true iff $\llbracket \text{VP} \rrbracket \in \llbracket \text{NP} \rrbracket$
 - $\llbracket \text{Every student works} \rrbracket = 1$ iff $\llbracket \text{work} \rrbracket \in \llbracket \text{every student} \rrbracket$

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$\llbracket \text{every student} \rrbracket$

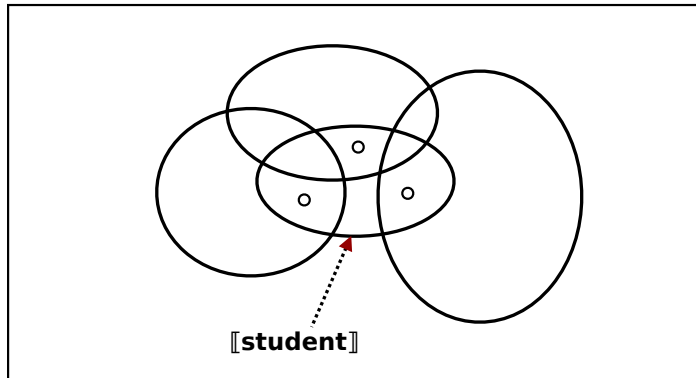
- $\llbracket \text{every student} \rrbracket$ denotes the set of properties that apply to every student (i.e., all supersets of $\llbracket \text{student} \rrbracket$)



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[[a student]]

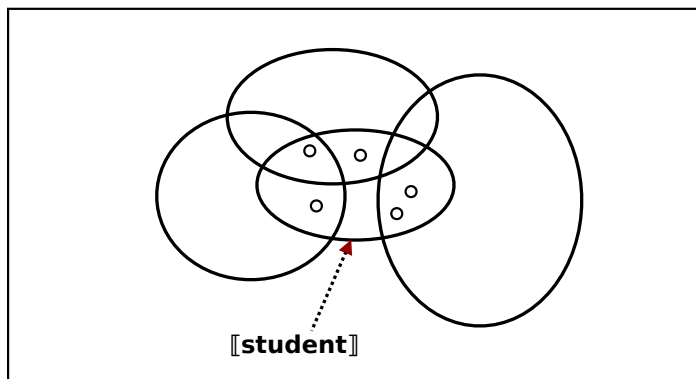
- [[a student]] denotes the set of properties that apply to at least one student.



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[[two students]]

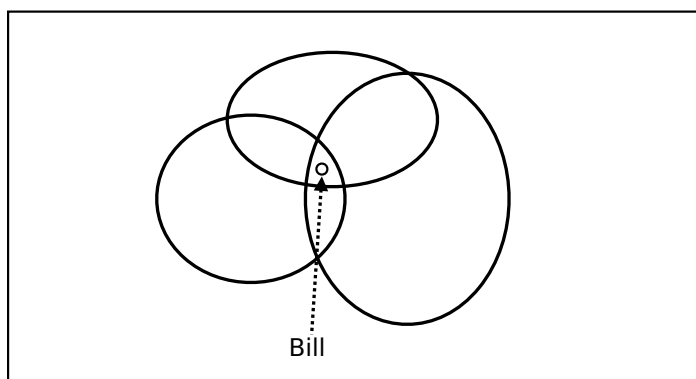
- [[two students]] denotes the set of properties that apply to at least (exactly) two students.



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[[Bill]]

- [[Bill]] denotes the set of properties that apply to Bill



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Noun Phrase Interpretations

$$\llbracket \text{all } N \rrbracket^M = \{ P \subseteq U_M \mid \llbracket N \rrbracket \cap P = \llbracket N \rrbracket \}$$

$$\llbracket \text{a } N \rrbracket^M = \{ P \subseteq U_M \mid \llbracket N \rrbracket \cap P \neq \emptyset \}$$

$$\llbracket \text{not all } N \rrbracket^M = \{ P \subseteq U_M \mid \llbracket N \rrbracket \cap P \neq \llbracket N \rrbracket \}$$

$$\llbracket \text{no } N \rrbracket^M = \{ P \subseteq U_M \mid \llbracket N \rrbracket \cap P = \emptyset \}$$

$$\llbracket \text{exactly } n \text{ } N \rrbracket^M = \{ P \subseteq U_M \mid \text{card}(\llbracket N \rrbracket \cap P) = n \}$$

$$\llbracket \text{at most } n \text{ } N \rrbracket^M = \{ P \subseteq U_M \mid \text{card}(\llbracket N \rrbracket \cap P) \leq n \}$$

$$\llbracket \text{at least } n \text{ } N \rrbracket^M = \{ P \subseteq U_M \mid \text{card}(\llbracket N \rrbracket \cap P) \geq n \}$$

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Generalized Quantifier Theory

- What formal properties do quantifiers have?
- What natural subclasses can be distinguished?
- Which subclasses actually represent meanings of natural language noun phrases?

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Negative Polarity Items

- (1) a. *John needn't go there*
b. **John need go there*
- (2) a. *Nobody saw anything*
b. **Somebody saw anything*
- (3) a. *No student has ever been in Saarbrücken*
b. **Some student has ever been in Saarbrücken*

- **Negative polarity items** (any, ever, ...)
⇒ items that can occur only in “negative contexts”
- **Question:** What licenses negative polarity items?

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There-Sentences

- (1) *There is someone in the garden*
- (2) *There is no one in the garden*
- (3) *There are two unicorns in the garden*
- (4) **There is/are everyone in the garden*
- (5) **There is John in the garden*
- (6) **There are the two unicorns in the garden*

- **Question:** which noun phrases can appear in “there” sentences (and why)?

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Coordination

- (1) *No man and few women walked*
- (2) *None of the girls and at most three boys walked*
- (3) **A man and few women walked*
- (4) **John and no woman saw Jane*

- **Question:** which noun phrases can be coordinated?

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Inference Patterns

- (1) *All men **walked rapidly** ⇨ All men **walked***
- (2) *No man **walked** ⇨ No man **walked rapidly***
- (3) *A girl **smoked a cigar** ⇨ A girl **smoked***
- (4) *Few girls **smoked** ⇨ Few girls **smoked a cigar***

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Upward Monotonicity

- *All men walked rapidly* \models *All men walked*
 - Note: $\llbracket \text{walked rapidly} \rrbracket \subseteq \llbracket \text{walked} \rrbracket$
- *A girl smoked a cigar* \models *A girl smoked*
 - Note: $\llbracket \text{smoked a cigar} \rrbracket \subseteq \llbracket \text{smoked} \rrbracket$
- **Observation:**
A sentence $[{}_s \text{ NP VP}]$ remains true if the denotation of the verb phrase is made “larger”

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Upward Monotonicity

- A quantifier Q is **upward monotonic** in $M = \langle U, V \rangle$ iff Q is closed under supersets:
 - for all $X, Y \subseteq U$: if $X \in Q$ and $X \subseteq Y$, then $Y \in Q$
- A noun phrase is upward monotonic if it denotes an upward monotonic quantifier.

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Upward Monotonicity Tests

- If $\llbracket \text{VP}_1 \rrbracket \subseteq \llbracket \text{VP}_2 \rrbracket$, then $\text{NP VP}_1 \models \text{NP VP}_2$
 - *All men walked rapidly* \models *All men walked*
 - *No man walked rapidly* $\not\models$ *No man walked*
 - Note: $\llbracket \text{walked rapidly} \rrbracket \subseteq \llbracket \text{walked} \rrbracket$
- NP VP_1 and $\text{VP}_2 \models \text{NP VP}_1$ and NP VP_2
 - *All men smoked and drank* \models
All men smoked and all men drank
 - *No man smoked and drank* $\not\models$
No man smoked and no man drank
 - Note: $\llbracket \text{VP}_1 \text{ and } \text{VP}_2 \rrbracket = \llbracket \text{VP}_1 \rrbracket \cap \llbracket \text{VP}_2 \rrbracket$

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Upward Monotonicity

- The set of upward monotonic quantifiers is closed under conjunction and disjunction:
 - the intersection (union) of two upward monotonic quantifiers is an upward monotonic quantifier.
- *All boys and a girl walked rapidly* \models
All boys and a girl walked
- Note:
 - $\llbracket \text{NP}_1 \text{ and } \text{NP}_2 \rrbracket = \llbracket \text{NP}_1 \rrbracket \cap \llbracket \text{NP}_2 \rrbracket$
 - $\llbracket \text{NP}_1 \text{ or } \text{NP}_2 \rrbracket = \llbracket \text{NP}_1 \rrbracket \cup \llbracket \text{NP}_2 \rrbracket$

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Downward Monotonicity

- (1) *No man walked* \models
No man walked rapidly
- (2) *Not every woman was asleep* \models
Not every woman was dreaming
- (3) *Less than half of the girls smoked* \models
Less than half of the girls smoked cigars
- (4) *Few boys were playing* \models
Few boys were playing out on the street

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Downward Monotonicity

- A quantifier Q is **downward monotonic** in $M = \langle U, V \rangle$ iff Q is closed under inclusion:
 - for all $X, Y \subseteq U$: if $X \in Q$ and $X \supseteq Y$, then $Y \in Q$
- A noun phrase is downward monotonic if it denotes a downward monotonic quantifier.

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Downward Monotonicity Tests

- If $[[VP_1]] \supseteq [[VP_2]]$, then $NP VP_1 \models NP VP_2$
 - *All men walked* $\not\models$ *All men walked rapidly*
 - *No man walked* \models *No man walked rapidly*
 - Note: $[[walked]] \supseteq [[walked rapidly]]$
- $NP VP_1$ or $VP_2 \models NP VP_1$ and $NP VP_2$
 - *Neither girl was drinking or smoking* \models
Neither girl was drinking and neither girl was smoking.
 - *All boys sing or dance* $\not\models$
All boys sing and all boys dance.
 - Note: $[[VP_1 \text{ or } VP_2]] = [[VP_1]] \cup [[VP_2]]$

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Negative Polarity Items

- (1) a. *John needn't go there*
b. **John need go there*
 - (2) a. *Nobody saw anything*
b. **Somebody saw anything*
 - (3) a. *No student has ever been in Saarbrücken*
b. **Some student has ever been in Saarbrücken*
- \Rightarrow negative polarity items are licensed only in downward monotonic contexts.

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Coordination

- (1) *No man and few women walked*
 - (2) *None of the girls and at most three boys walked*
 - (3) **A man and few women walked*
 - (4) **John and no woman saw Jane*
- Non-comparative noun phrases can be coordinated iff they have the same direction of monotonicity.

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Coordination

- (1) *A man and few women walked
- (2) A man but few woman walked
- (3) *John and no woman saw Jane
- (4) John but no woman saw Jane

- Coordination with the connective “but” requires noun phrases of different direction of monotonicity.

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Language Universals

- **Monotonicity Constraint** (Barwise & Cooper 1981)
The simple noun phrases of any natural language express monotone quantifiers or conjunctions of monotone quantifiers.
- **Simple noun phrase:** Proper names or noun phrases of the form $[_{NP} \text{ DET } N]$

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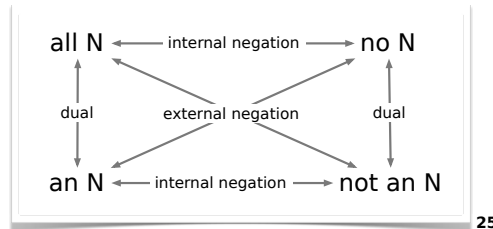
Negation of Quantifiers

- **External negation:** $\neg Q = \{ P \subseteq U_M \mid P \notin Q \}$
 $\neg \llbracket \text{all } N \rrbracket = \{ P \subseteq U_M \mid P \notin \llbracket \text{all } N \rrbracket \}$
 $= \{ P \subseteq U_M \mid \llbracket N \rrbracket \cap P \neq \llbracket N \rrbracket \}$
 $\neg \llbracket \text{all } N \rrbracket = \llbracket \text{not all } N \rrbracket$
- **Internal negation:** $Q\neg = \{ P \subseteq U_M \mid (U_M - P) \in Q \}$
 $\llbracket \text{all } N \rrbracket \neg = \{ P \subseteq U_M \mid (U_M - P) \in Q \}$
 $= \{ P \subseteq U_M \mid \llbracket N \rrbracket \cap (U_M - P) \neq \emptyset \}$
 $= \{ P \subseteq U_M \mid \llbracket N \rrbracket \cap P = \emptyset \}$
 $\llbracket \text{all } N \rrbracket \neg = \llbracket \text{no } N \rrbracket$

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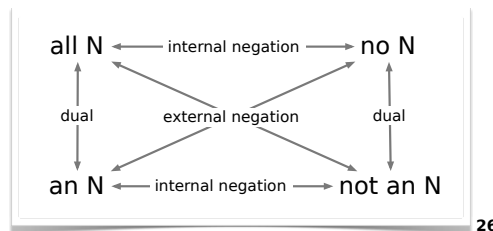
Negation of Quantifiers

- If Q is an upward monotonic quantifier, then both $\neg Q$ and $Q\neg$ are downward monotonic.
- If Q is a downward monotonic quantifier, then both $\neg Q$ and $Q\neg$ are upward monotonic.



Duals

- **The dual Q^* of a quantifier Q in M**
 - $Q^* = \neg Q\neg = \{ P \subseteq U_M \mid (U_M - P) \in \neg Q \}$
 $= \{ P \subseteq U_M \mid (U_M - P) \notin Q \}$.
- If Q is upward monotonic, then Q^* is upward monotonic.
- If Q is downward monotonic, then Q^* is downward monotonic.



Determiners

- *Every man walked* $\leftrightarrow \forall x(\text{man}'(x) \rightarrow \text{walk}'(x))$
 - *Every* $\Rightarrow \lambda P \lambda Q \forall x (P(x) \rightarrow Q(x))$
 - $\llbracket \text{Every} \rrbracket(A)(B) = 1$ iff $A \subseteq B$
- We can consider determiners as expressions that take a noun and a verb phrase to form a sentence.
- Semantically, the interpretation of a determiner can be seen as a relation between two sets.

Persistence

- **A determiner D is persistent** in M iff for all X, Y, Z:
 - if $D(X, Z)$ and $X \subseteq Y$, then $D(Y, Z)$
- **Persistence test:**
 - If $\llbracket N_1 \rrbracket \subseteq \llbracket N_2 \rrbracket$, then $\text{DET } N_1 \text{ VP} \models \text{DET } N_2 \text{ VP}$
 - *Some men walked* \models
Some human beings walked
 - *At least four girls were smoking* \models
At least four women were smoking.

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Antipersistence

- **A determiner D is antipersistent** in M iff for all X,Y,Z:
 - if $D(X, Z)$ and $Y \subseteq X$, then $D(Y, Z)$
- **Antipersistence test:**
 - If $\llbracket N_2 \rrbracket \subseteq \llbracket N_1 \rrbracket$, then $\text{DET } N_1 \text{ VP} \models \text{DET } N_2 \text{ VP}$
 - *All children walked* \models
All toddlers walked
 - *No woman was smoking* \models
No girl was smoking
 - *At most three Englishmen agreed* \models
At most three Londoners agreed.

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Persistence and Monotonicity

- Persistence and monotonicity are closely related:
 - Persistence (antipersistence) is upward (downward) monotonicity of the first argument.
 - Upward (downward) monotonicity of noun phrases is upward (downward) monotonicity of the second argument of the determiner in the NP.
- Terminology:
 - left-monotonicity ($\uparrow\text{mon}$ and $\downarrow\text{mon}$)
 - right-monotonicity ($\text{mon}\uparrow$ and $\text{mon}\downarrow$)

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Left and Right Monotonicity

$\uparrow\text{mon}\uparrow$ some, at least n, infinitely many

$\downarrow\text{mon}\uparrow$ all

$\downarrow\text{mon}\downarrow$ no, at most n, a finite number of

$\uparrow\text{mon}\downarrow$ not all

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Conservativity

- **Conservativity:** for every $A, B \subseteq U$
 - $Q(A, B) \Leftrightarrow Q(A, A \cap B)$
- **Test:** $D N VP \Leftrightarrow D N$ are N that VP
 - *All students work* \Leftrightarrow
All students are students that work
 - *Some girls are dancing* \Leftrightarrow
Some girls are girls that are dancing
 - *Most teachers are motivated* \Leftrightarrow
Most teachers are teachers that are motivated

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Lives on

- **A quantifier Q lives on X** iff for all Y,
 - $Y \in Q$ iff $X \cap Y \in Q$
- **Universal** (Barwise & Cooper 1981, cited from Gamut)
In every natural language, simple determiners together with an N yield an NP which lives on $\llbracket N \rrbracket$
- **Apparent exception:** only
 - *Only men smoke cigars* \nleftrightarrow
Only men are men that smoke cigars
 - \Rightarrow "only" not a determiner?

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Weak & Strong Determiners

- A determiner **D is positive strong** iff for every $X \subseteq U$:
 - $D(X, U)$ is true (whenever D is defined)
- A determiner **D is negative strong** iff for every $X \subseteq U$:
 - $D(X, U)$ is false (whenever D is defined)
- \Rightarrow sentences of the form [_S DET N exists] are either a tautology or a contradiction for strong determiners
 - e.g., every ist positive strong
- A determiner D that is neither positive nor negative strong is called **weak**.

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There-sentences

- A sentence [_S there is NP] is true iff $U_M \in \llbracket \text{NP} \rrbracket^M$
- $\llbracket \text{There is a unicorn in the garden} \rrbracket = 1$
 - iff $\llbracket \text{unicorn in the garden} \rrbracket \neq \emptyset$
 - iff $U_M \in \llbracket \text{a unicorn in the garden} \rrbracket$
- **There-sentences:**
only determiners are allowed to appear that don't make them tautological or contradictory
- \Rightarrow strong determiners are ruled out

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Literature

- L.T.F. Gamut. Logic, Language, and Meaning. Vol 2. Chapter 7.
- Partee, ter Meulen, Wall. Mathematical Methods for Linguists. Chapter 14.
- Jon Barwise & Robin Cooper. Generalized Quantifiers. Linguistics and Philosophy. 1981.

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