Semantic Theory

Lexical Semantics III

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Lattices and Semi-lattices

- A partial order is a structure <A, ≤> with reflexive, transitive, and antisymmetric ≤ .
- Let <A, ≤> be a partial order:

  The join of a and b ∈ A: a ⊔ b is the lowest upper bound for a and b.

  The meet of a and b ∈ A: a ⊓ b is the highest lower bound for a and b.
- A lattice is a partial order <A, ≤> which is closed under meet and join (also written <A, ⊓, ⊔ >).

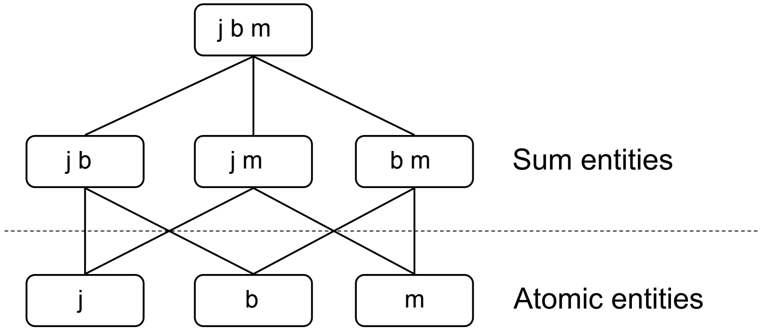
Lattices and Semi-lattices

- A lattice may or may not have one maximal and minimal element. If it has such elements, they are named **1** and **0**, respectively, and the lattice is called bounded.
- An a ∈ A is an atom, if a ≠ **0** and there is no b ≠ **0** in A such that b<a.
- A lattice <A, ≤> is atomic, if for every a ≠ **0 there is an atom b ≤ a.**
- A join semi-lattice is a partial order <A, ≤> which is closed under join (also written <A, ⊔>).

Model structure for plural terms

- A model structure is a pair M = ⟨⟨U, ≤⟩, V⟩, where
  - <U, ≤>/<U, ⊔> is an atomic join semi-lattice with universe U and individual part relation ≤.
  - V is a value assignment function.
- A⊆U is the set of atoms in <U ,≤>.
- U\A is the set of non-atomic elements, i.e., the sum objects in U.

Structured Universe – Example



Logic for plural terms

- Like standard FOL. We add a summation operator ⊕, a one-place predicate At for "atom" and a two-place relation ◁ for "(proper) individual part".
- Application examples:
  - j ⊕ b, translating "John and Bill"
  - j ⊕ b ◁ the_committee: "John and Bill are individual parts / members of the committee"

Interpretation

Like standard FOL interpretation

- with additional clauses for ⊕ and ◁ :

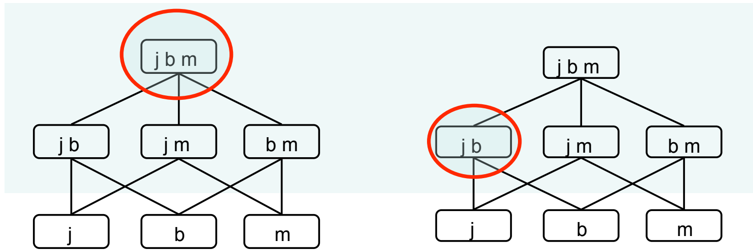
  $[\![a \oplus b]\!]^{M,g} = [\![a]\!]^{M,g} \sqcup [\![b]\!]^{M,g}$

  $[\![a \lhd b]\!]^{M,g} = 1$ iff $[\![a]\!]^{M,g} < [\![b]\!]^{M,g}$

  $[\![At(a)]\!]^{M,g} = 1$ iff $[\![a]\!]^{M,g} \in A$
- with standard individual constants (proper names, definite descriptions) denoting atoms ($V_M(a) \in A$),
- group constants (plural and group NPs) denoting sums ($V_M(a) \in U\backslash A$)
- predicates satisfying specific constraints with respect to their collectivity status

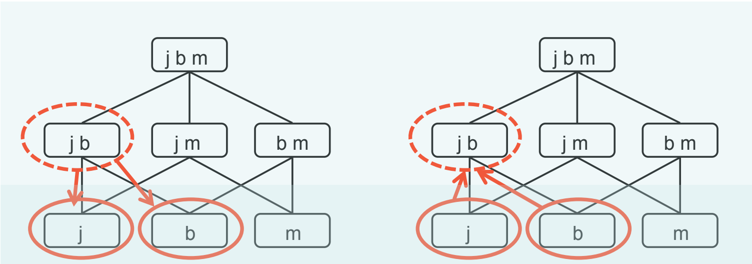
Collective predicates

- Collective predicates F (like *meet, collaborate*):
  $V_M(F) \subseteq U\backslash A$



Distributive predicates

- Distributive predicates F (like *work, tall, student*):
  $V_M(F) \subseteq U$, such that the following axiom is satisfied:
  $\forall x(F(x) \leftrightarrow \forall y(At(y) \wedge y \triangleleft x \rightarrow F(y)))$



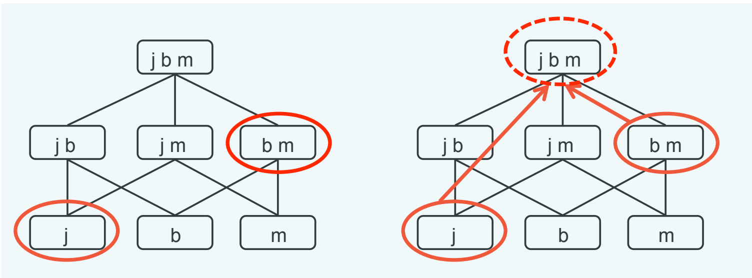
$\rightarrow$: Distributivity $\leftarrow$: Closure under summation

Distributive predicates

- The denotation of distributive predicates is uniquely determined by their atomic members.
- If a predicate applies to a set $M \subseteq A$, then the full denotation of the predicate is the join semi-lattice generated by M.

Interpretation of predicates

- Mixed predicates F (e.g., *carrying a piano, solving the exercise*):
  $V_M(F) \subseteq U$



Non-distributive, but closed under summation

Mass Nouns

- *Examples: water, gold, wood, money, soup, ...*
- Mass nouns don't have individual entities. They are "divisive": Every amount of water can be subdivided into proper parts which are also amounts of water.
- The basic property of mass nouns is a basic problem for FOL-based semantics: There are no atomic individuals as basic material of model structures.
- The close similarity between plurals and mass nouns suggests a solution to this problem.

Plural nouns and mass nouns

- Plural nouns and mass nouns
  - are both cumulative:
    If you add students to students, the resulting group are students.
    If you add water to water, the resulting entity is an amount of water.
  - come both with cardinalities: 5 students, 5 l of water.
  - share grammatical properties; e.g., indefinite plural and mass term NPs don't take an article.

Model structure for mass nouns

- We add the sort of "material entities" M, which forms a join semi-lattice <M,≤> with the material part relation ≤ – similar to the semi-lattice for plurals, except the facts that <M, ≤> is non-atomic and dense.
  - An ordering relation ≤ is dense, if the following holds:
    $\forall x \forall y\ (x<y \rightarrow \exists z (x<z \wedge z<y)$
    E.g., the (strict) order of real numbers is dense, the order of natural numbers is not.

Model structure for mass nouns

- To distinguish the individual part and the material part relation, we write $\leq_i$ for the former, and $\leq_m$ for the latter.
- Also, we distinguish individual and material join: $\sqcup_i$ and $\sqcup_m$, respectively.
- We arrive at the following model structure concept:
  $M = \langle\langle U, \leq_i\rangle, \langle M, \leq_m\rangle, V\rangle$, where
  - $\langle U, \leq_i\rangle$ is an atomic join semi-lattice
  - $\langle M, \leq_m\rangle$ is a dense non-atomic join semi-lattice

Model structure for mass nouns

- There is a narrow relation between the two sub-domains of (singular and plural) individuals and material entities: Each individual consists of a portion of matter.
- We take this relation into account by introducing a "materialisation" function *h* into the model structure: a homomorphism that maps (atomic and pluralic) individuals to the matter they consist of.
- $M = \langle\langle U, \leq_i\rangle, \langle M, \leq_m\rangle, h, V\rangle$
- Because h is a homomorphism, the following hold:
  $a \leq_i b$ iff $h(a) \leq_m h(b)$
  $h(a \sqcup_i b) = h(a) \sqcup_m h(b)$

FOL Language for mass nouns

- In the logical language, we provide logical constants for
  - an individual summation and a material summation operation: $\oplus_i$, $\oplus_m$
  - an individual part and a material part relation: $\lhd_i$, and $\lhd_m$.
  - a materialisation function *m*
- Interpretation rules:

  $[\![a \oplus_i b]\!]^{M,g} = [\![a]\!]^{M,g} \sqcup_i [\![b]\!]^{M,g}$

  $[\![a \oplus_m b]\!]^{M,g} = [\![a]\!]^{M,g} \sqcup_m [\![b]\!]^{M,g}$

  $[\![a \lhd_i b]\!]^{M,g} = 1$ iff $[\![a]\!]^{M,g} <_i [\![b]\!]^{M,g}$

  $[\![a \lhd_m b]\!]^{M,g} = 1$ iff $[\![a]\!]^{M,g} <_m [\![b]\!]^{M,g}$

  $[\![m(a)]\!]^{M,g} = h([\![a]\!]^{M,g})$

Examples

We use two sorts of variables:

x, y, z, ... for (atomic and sum) individuals

X, Y, Z, ... for portions of matter

We add a function expression m for the matter function:

The ring is made of gold

→ $\exists y(\text{ring}(y) \wedge \text{gold}(m(y)))$

The ring contains gold

→ $\exists y \exists X (\text{ring}(y) \wedge X \lhd_m m(y) \wedge \text{gold}(X))$

Part of the ring is gold

Events and Temporal Precedence

- A model structure with events and temporal precedence is defined as

  $M = (U, E, <, e_0, V)$,

  with $U \cap E = \emptyset$,

  $< \subseteq E \times E$ an asymmetric relation (temporal precedence)

  $e_0 \in E$ the utterance event

  V an interpretation function like in standard FOL, with

  $D_e = U \cup E$
- Overlapping events:

  $e \circ e'$ iff neither $e < e'$ nor $e' < e$

Structured events

- A model structure with events and temporal precedence is defined as

  $M = (U, <E, \leq>, <, e_0, V)$,

  with $U \cap E = \emptyset$,

  $< \subseteq E \times E$ an asymmetric relation (temporal precedence)

  $<E, \leq>$ an atomic join semi-lattice

  $e_0 \in E$ the utterance event

  V an interpretation function like in standard FOL, with

  $D_e = U \cup E$
- Axioms needed to specify the relation between $\leq$ and $<$, e.g.:

  If $e \leq e'$ or $e' \leq e$, then $e \circ e'$

  If $e_1 < e_2$, $e_1 \leq e_3$, and $e_2 \leq e_4$, then $e_3 < e_4$

Events and processes

- A model structure with events and temporal precedence is defined as

  M = (U, <E_i , $\leq_i$>, <E_m , $\leq_m$>, ⋖, e_0, h, V),

  with U, E_i, E_m pairwise disjoint

  <E_i , $\leq_i$> an atomic join semi-lattice

  <E_m , $\leq_m$> a dense non-atomic join semi-lattice

  ⋖ an asymmetric relation defined on $E_i \cup E_m$

  $e_0 \in E_i$ the utterance event

  h: $E_i \rightarrow E_m$ a homomorphism

  V an interpretation function like in standard FOL, with

  $D_e = U \cup E_i \cup E$