Semantic Theory
Lecture 5: Generalized Quantifiers

Manfred Pinkal & Stefan Thater
FR 4.7 Allgemeine Linguistik (Computerlinguistik)
Universität des Saarlandes

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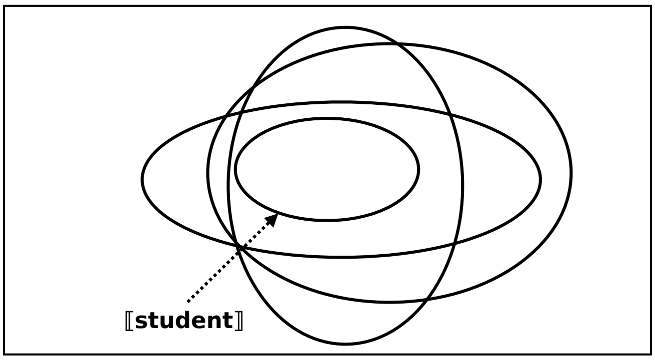
Generalized Quantifiers

- *Every student works*
  - ∀x(student'(x) → work'(x))
  - *Every student* ↦ λQ∀x(student'(x) → Q(x))
  - ⟦*Every student*⟧ = { P ⊆ U_M | ⟦*student*⟧ ⊆ P }
- **A generalized quantifier** is a set of properties
  - property = set of individuals
- A sentence of the form [$_S$ NP VP] is true iff ⟦VP⟧ ∈ ⟦NP⟧
  - ⟦*Every student works*⟧ = 1 iff ⟦*work*⟧ ∈ ⟦*every student*⟧

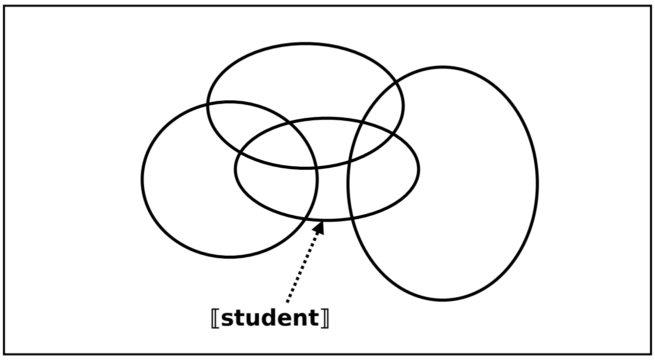
⟦*every student*⟧

- ⟦***every student***⟧ denotes the set of properties that apply to every student (i.e., all supersets of ⟦student⟧)



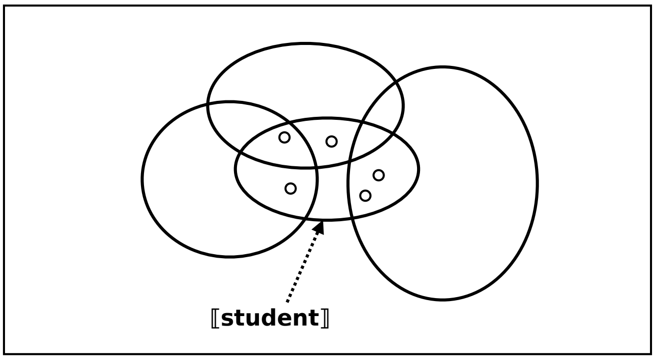
⟦*a student*⟧

- ⟦***a student***⟧ denotes the set of properties that apply to at least one student.



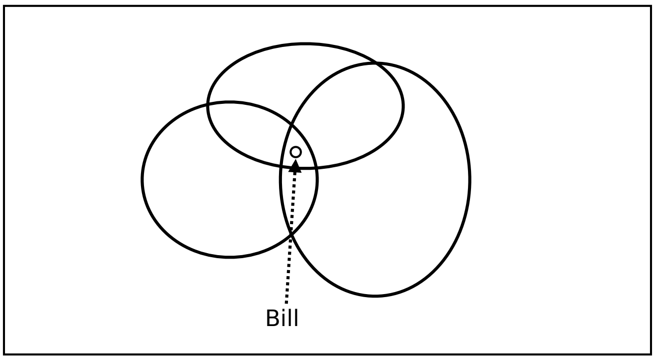
⟦*two students*⟧

- ⟦***two students***⟧ denotes the set of properties that apply to at least (exactly) two students.



⟦*Bill*⟧

- ⟦***Bill***⟧ denotes the set of properties that apply to Bill



Noun Phrase Interpretations

$[\![\textit{all N}]\!]^M = \{ P \subseteq U_M \mid [\![N]\!] \cap P = [\![N]\!] \}$

$[\![\textit{a N}]\!]^M = \{ P \subseteq U_M \mid [\![N]\!] \cap P \neq \emptyset \}$

$[\![\textit{not all N}]\!]^M = \{ P \subseteq U_M \mid [\![N]\!] \cap P \neq [\![N]\!] \}$

$[\![\textit{no N}]\!]^M = \{ P \subseteq U_M \mid [\![N]\!] \cap P = \emptyset \}$

$[\![\textit{exatcly n N}]\!]^M = \{ P \subseteq U_M \mid \text{card}([\![N]\!] \cap P) = n \}$

$[\![\textit{at most n N}]\!]^M = \{ P \subseteq U_M \mid \text{card}([\![N]\!] \cap P) \leq n \}$

$[\![\textit{at least n N}]\!]^M = \{ P \subseteq U_M \mid \text{card}([\![N]\!] \cap P) \geq n \}$

Generalized Quantifier Theory

- What formal properties do quantifiers have?
- What natural subclasses can be distinguished?
- Which subclasses actually represent meanings of natural language noun phrases?

Negative Polarity Items

(1) a. *John needn't go there*
b. **John need go there*

(2) a. *Nobody saw anything*
b. **Somebody saw anything*

(3) a. *No student has ever been in Saarbrücken*
b. **Some student has ever been in Saarbrücken*

- **Negative polarity items** (any, ever, ...)
⇒ items that can occur only in "negative contexts"
- **Question:** What licenses negative polarity items?

There-Sentences

(1) *There is someone in the garden*

(2) *There is no one in the garden*

(3) *There are two unicorns in the garden*

(4) **There is/are everyone in the garden*

(5) **There is John in the garden*

(6) **There are the two unicorns in the garden*

- **Question:** which noun phrases can appear in “there” sentences (and why)?

Coordination

(1) *No man and few women walked*

(2) *None of the girls and at most three boys walked*

(3) **A man and few women walked*

(4) **John and no woman saw Jane*

- **Question:** which noun phrases can be coordinated?

Inference Patterns

(1) *All men* ***walked rapidly*** $\models$ *All men* ***walked***

(2) *No man* ***walked*** $\models$ *No man* ***walked rapidly***

(3) *A girl* ***smoked a cigar*** $\models$ *A girl* ***smoked***

(4) *Few girls* ***smoked*** $\models$ *Few girls* ***smoked a cigar***

Upward Monotonicity

- *All men walked rapidly* ⊨ *All men walked*
  - ⟦walked rapidly⟧ ⊆ ⟦walked⟧
- *A girl smoked a cigar* ⊨ *A girl smoked*
  - ⟦smoked a cigar⟧ ⊆ ⟦smoked⟧
- **Observation:**
  A sentence $[_S$ NP VP] remains true if the denotation of the verb phrase is made “larger”

Upward Monotonicity

- A quantifier Q is **upward monotonic** in M = ⟨U, V⟩ iff Q is closed under supersets:
  - for all X, Y ⊆ U: if X ∈ Q and X ⊆ Y, then Y ∈ Q
- A noun phrase is upward monotonic if it denotes an upward monotonic quantifier.

Upward Monotonicity Tests

- **If ⟦VP_1⟧ ⊆ ⟦VP_2⟧, then NP VP_1 ⊨ NP VP_2**
  - *All men walked rapidly* ⊨ *All men walked*
  - *No man walked rapidly* ⊭ *No man walked*
  - ⟦walked rapidly⟧ ⊆ ⟦walked⟧
- **NP VP_1 and VP_2 ⊨ NP VP_1 and NP VP_2**
  - *All men smoked and drank* ⊨
    All men smoked and all men drank
  - *No man smoked and drank* ⊭
    No man smoked and no man drank
- ⟦VP_1 and VP_2⟧ = ⟦VP_1⟧ ∩ ⟦VP_2⟧

Upward Monotonicity

- The set of upward monotonic quantifiers is closed under conjunction and disjunction:
  - the intersection (union) of two upward monotonic quantifiers is an upward monotonic quantifier.
- *All boys and a girl walked rapidly* ⊨ *All boys and a girl walked*
- $[\![NP_1 \text{ and } NP_2]\!] = [\![NP_1]\!] \cap [\![NP_2]\!]$,

Downward Monotonicity

(1) *No man walked* ⊨ *No man walked rapidly*

(2) *Not every woman was asleep* ⊨ *Not every woman was dreaming*

(3) *Less than half of the girls smoked* ⊨ *Less than half of the girls smoked cigars*

(4) *Few boys were playing* ⊨ *Few boys were playing out on the street*

Downward Monotonicity

- A quantifier Q is **downward monotonic** in $M = \langle U, V \rangle$ iff Q is closed under inclusion:
  - for all $X, Y \subseteq U$: if $X \in Q$ and $X \supseteq Y$, then $Y \in Q$
- A noun phrase is downward monotonic if it denotes a downward monotonic quantifier.

Downward Monotonicity Tests

- **If ⟦VP_1⟧ ⊇ ⟦VP_2⟧, then NP VP_1 ⊨ NP VP_2**
  - *All men walked* ⊭ *All men walked rapidly*
  - *No man walked* ⊨ *No man walked rapidly*
  - ⟦walked⟧ ⊇ ⟦walked rapidly⟧
- **NP VP_1 or VP_2 ⊨ NP VP_1 and NP VP_2**
  - *Neither girl was drinking or smoking* ⊨
    Neither girl was drinking and neither girl was smoking.
  - *All boys sing or dance* ⊭
    All boys sing and all boys dance.
- ⟦VP_1 or VP_2⟧ = ⟦VP_1⟧ ∪ ⟦VP_2⟧

Negative Polarity Items

(1) a. *John needn't go there*
b. **John need go there*

(2) a. *Nobody saw anything*
b. **Somebody saw anything*

(3) a. *No student has ever been in Saarbrücken*
b. **Some student has ever been in Saarbrücken*

- ⇒ negative polarity items are licensed only in downward monotonic contexts.

Coordination

(1) *No man and few women walked*

(2) *None of the girls and at most three boys walked*

(3) **A man and few women walked*

(4) **John and no woman saw Jane*

- Non-comparative noun phrases can be coordinated iff they have the same direction of monotonicity.

Coordination

(1) **A man and few women walked*

(2) *A man but few woman walked*

(3) **John and no woman saw Jane*

(4) *John but no woman saw Jane*

- Coordination with the connective “but” requires noun phrases of different direction of monotonicity.

Language Universals

- **Monotonicity Constraint** (Barwise & Cooper 1981)
  The simple noun phrases of any natural language express monotone quantifiers or conjunctions of monotone quantifiers.
- **Simple noun phrase:** Proper names or noun phrases of the form $[_{NP}$ DET N$]$

Negation of Quantifiers

- **External negation:** $\neg Q = \{ P \subseteq U_M \mid P \notin Q \}$

  $\neg[\![\textit{all N}]\!] = \{ P \subseteq U_M \mid P \notin [\![\text{all N}]\!] \}$
  $= \{ P \subseteq U_M \mid [\![N]\!] \cap P \neq [\![N]\!] \}$

  $\neg[\![\textit{all N}]\!] = [\![\textit{not all N}]\!]$

- **Internal negation:** $Q\neg = \{ P \subseteq U_M \mid (U_M - P) \in Q \}$

  $[\![\textit{all N}]\!]\neg = \{ P \subseteq U_M \mid (U_M - P) \in Q \}$
  $= \{ P \subseteq U_M \mid [\![N]\!] \cap (U_M - P) \neq \emptyset \}$
  $= \{ P \subseteq U_M \mid [\![N]\!] \cap P = \emptyset \}$

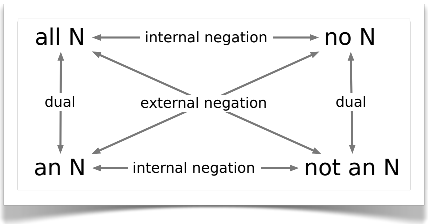
  $[\![\textit{all N}]\!]\neg = [\![\textit{no N}]\!]$

Negation of Quantifiers

- If Q is an upward monotonic quantifier, then both ¬Q and Q¬ are downward monotonic.
- If Q is an downward monotonic quantifier, then both ¬Q and Q¬ are upward monotonic.
- Examples:
  - *All N – Not all N*
  - *At least n N – At most n N*

Duals

- **The dual Q* of a quantifier Q in M**
  - $Q^* = \neg Q \neg = \{ P \subseteq U_M \mid (U_M - P) \in \neg Q \}$
    $= \{ P \subseteq U_M \mid (U_M - P) \notin Q \}$.
- If Q is upward monotonic, then Q* is upward monotonic.
- If Q is downward monotonic, then Q* is downward monotonic.



Determiners

- *Every man walked* ↦ $\forall x(\text{man}'(x) \rightarrow \text{walk}'(x))$
  - *Every* ⇒ $\lambda P \lambda Q \forall x(P(x) \rightarrow Q(x))$
  - $[\![\text{Every}]\!](A)(B) = 1$ iff $A \subseteq B$
- We can consider determiners as expressions that take a noun and a verb phrase to form a sentence.
- Semantically, the interpretation of a determiner can be seen as a relation between two sets.

Persistence

- **A determiner D is persistent** in M iff for all X, Y, Z:
  - if D(X, Z) and X ⊆ Y, then D(Y, Z)
- **Persistence test:**
  - If $[\![N_1]\!] \subseteq [\![N_2]\!]$, then DET N_1 VP ⊨ DET N_2 VP
  - *Some men walked* ⊨
    Some human beings walked
  - *At least four girls were smoking* ⊨
    At least four women were smoking.

Antipersistence

- **A determiner D is antipersistent** in M iff for all X,Y,Z:
  - if D(X, Y) and Y ⊆ X, then D(Y, Z)
- **Antipersistence test:**
  - If $[\![N_2]\!] \subseteq [\![N_1]\!]$, then DET N_1 VP ⊨ DET N_2 VP
  - *All children walked* ⊨
    All toddlers walked
  - *No woman was smoking* ⊨
    No girl was smoking
  - *At most three Englishmen agreed* ⊨
    At most three Londoners agreed.

Persistence and Monotonicity

- Persistence and monotonicity are closely related:
  - Persistence (antipersistence) is upward (downward) monotonicity of the first argument.
  - Upward (downward) monotonicity of noun phrases is upward (downward) monotonicity of the second argument of the determiner in the NP.
- Terminology:
  - left-monotonicity (↑mon and ↓mon)
  - right-monotonicity (mon↑ and mon↓)

Left and Right Monotonicity

↑mon↑	some, at least n, infinitely many
↓mon↑	all
↓mon↓	no, at most n, a finite number of
↑mon↓	not all

Conservativity

- **Conservativity:** for every A, B ⊆ U
  - Q(A, B) ⇔ Q(A, A ∩ B)
- **Test:** D N VP ⇔ D N are N that VP
  - *All students work ⇔*
    All students are students that work
  - *Some girls are dancing ⇔*
    Some girls are girls that are dancing
  - *Most teachers are motivated ⇔*
    Most teachers are teachers that are motivated

Lives on

- **A quantifier Q lives on** X iff for all Y,
  - Y ∈ Q iff X ∩ Y ∈ Q
- **Universal** (Barwise & Cooper 1981, cited from Gamut)
  In every natural language, simple determiners together with an N yield an NP which lives on ⟦N⟧
- **Apparent exception:** only
  - *Only men smoke cigars ⇎*
    Only men are men that smoke cigars
  - ⇒ “only” not a determiner?

Weak & Strong Determiners

- A determiner **D is postive strong** iff for every $X \subseteq U$:
  - $D(X, U)$ is true (whenever D is defined)
- A determiner **D is negative strong** iff for every $X \subseteq U$:
  - $D(X, U)$ is false (whenever D is defined)
- ⇒ sentences of the form $[_S$ DET N exists] are either a tautology or a contradiction for strong determiners
  - e.g., every ist positive strong
- A determiner D that is neither positive nor negative strong is called **weak**.

There-sentences

- A sentence $[_S$ there is NP] is true iff $U_M \in [\![NP]\!]^M$
- $[\![$There is a unicorn in the garden$]\!] = 1$
  - iff $[\![$unicorn in the garden$]\!] \neq \emptyset$
  - iff $U_M \in [\![$a unicorn in the garden$]\!]$
- **There-sentences:**
  only determiners are allowed to appear that don't make them tautological or contradictory
- ⇒ strong determiners are ruled out

Literature

- L.T.F. Gamut. Logic, Language, and Meaning. Vol 2. Chapter 7.
- Partee, ter Meulen, Wall. Mathematical Methods for Linguists. Chapter 14.
- Jon Barwise & Robin Cooper. Generalized Quantifiers. Linguistics and Philosophy. 1981.