Semantic Theory
(Ontologies and) Description Logic

Manfred Pinkal
Stefan Thater

Universität des Saarlandes

2009-06-02

Ontologies

- An ontology is a “formal, explicit specification of a shared conceptualisation” (Gruber, 1993)
- An ontology is a formal representation of
  - a set of concepts within a domain
  - and the relationships between those concepts.
- Domain ontologies: concepts and relations within a specific domain.
- Upper ontologies: concepts and relations of foundational, domain-independant relevance.

Ontologies – Some Components

- Concepts
  - Person, Relative, Body, …
- Relations between concepts
  - Relative is-a Person, Body is-part-of Person, …
- Instances
  - Bill, Mary, …
- Properties of and relations between individuals
  - Bill is-a Person, Bill is-married-to Mary, …

Is WordNet an Ontology?

- WordNet hierarchically organizes nouns, verbs, adjectives, and adverbs into synsets refering to lexical concepts.

```
person, individual, someone, somebody, mortal, soul
   ⇒ organism, being
      ⇒ living thing, animate thing
         ⇒ whole, unit
            ⇒ object, physical object
               ⇒ physical entity
                  ⇒ entity
   ⇒ causal agent, cause, causal agency
      ⇒ physical entity
         ⇒ entity
```

Is WordNet an Ontology?

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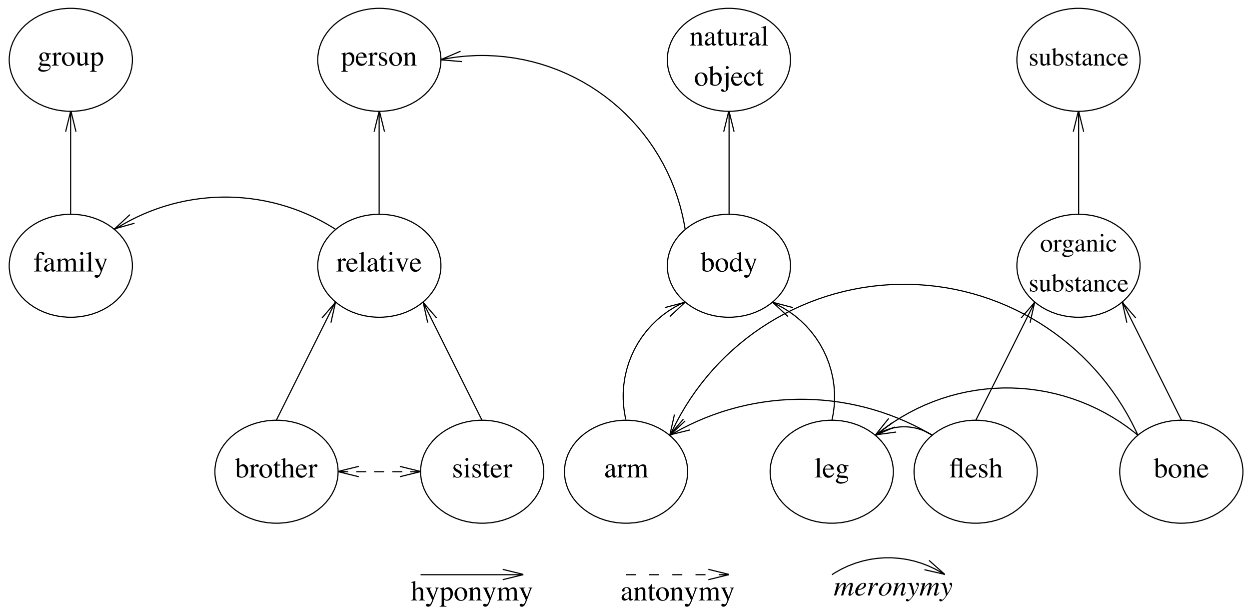
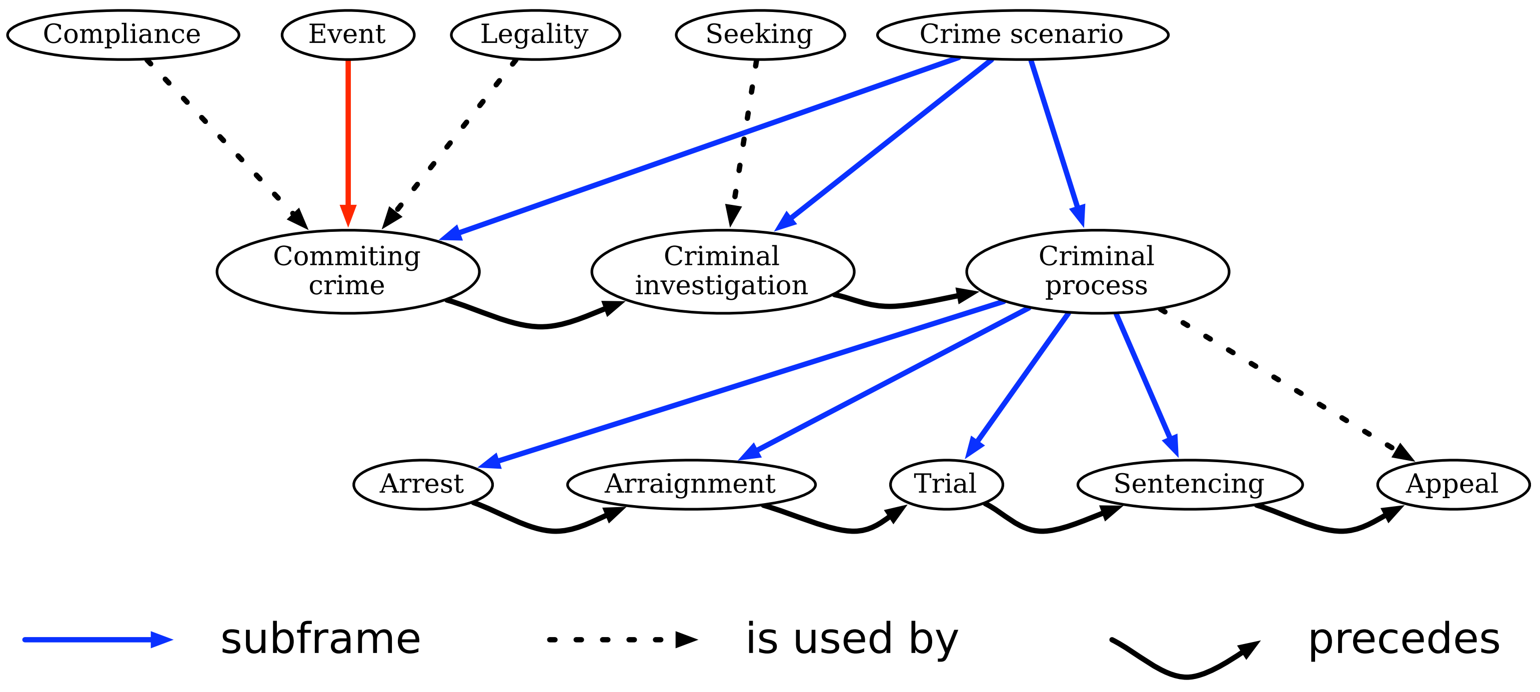


(Miller, 1993)

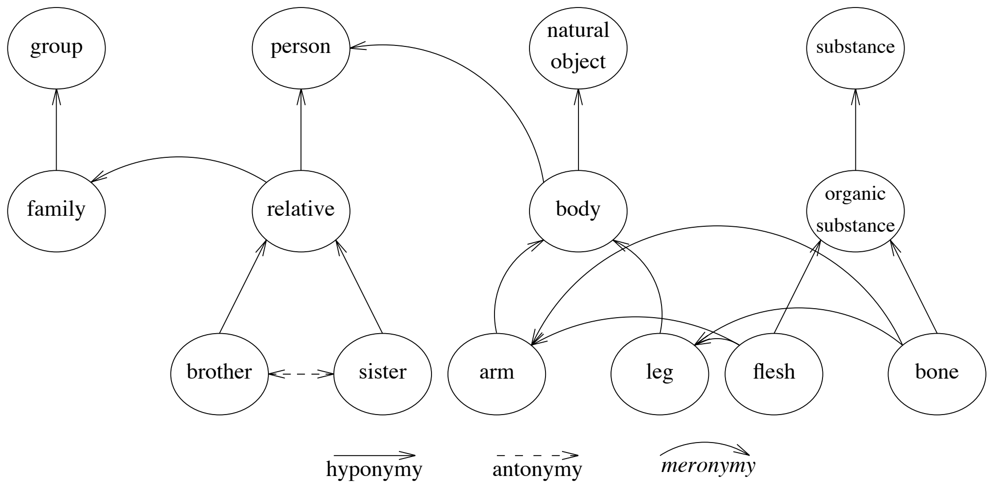
FrameNet



Compliance
Event
Legality
Seeking
Crime scenario
Commiting crime
Criminal investigation
Criminal process
Arrest
Arraignment
Trial
Sentencing
Appeal
subframe
is used by
precedes

WordNet in First-order Logic

- ∀x(family(x) → group(x))
- ∀x(relative(x) → person(x))
- ∀x(person(x) → ∃y(substance_m(y, x) ∧ body(y))
- ∀x(body(x) → ∃y(part_m(y, x) ∧ leg(y))
- …



… and in Description Logic

- Family ⊑ Group
  - ∀x(family(x) → group(x))
- Relative ⊑ Person
  - ∀x(relative(x) → person(x))
- Person ⊑ ∃substance_m.Body
  - ∀x(person(x) → ∃y(substance_m(y, x) ∧ body(y))
- Body ⊑ ∃part_m.Leg
  - ∀x(body(x) → ∃y(part_m(y, x) ∧ leg(y))

Description Logic

- Description logics are fragments of first order logic tailored to represent terminological information
- Domains are described in terms of concepts, roles (relations) and individuals.
- Properties:
  - most description logics are decidable
  - closely related to propositional (multi-) modal logic.
  - sound and complete decision procedures
  - implemented systems: FaCT, Racer, …

Description Logic – Basics

- Concepts (unary predicates)
  - Person, Father, Mother, ...
- Roles (binary predicates)
  - hasChild, isMarriedTo, ...
- Individual names (constants)
  - john, mary, ...
- Subsumption (relations between concepts)
  - Father $\sqsubseteq$ Person
- Operators (for forming concepts and roles)
  - $\sqcap$ (conjunction) , $\sqcup$ (disjunction), $\neg$ (negation)
  - $\forall$ (universal qualifier), $\exists$ (existent qualifier)

Description Logic – Simple example

- Concept
- Role
- Individual
- TBox – terminological knowledge
- ABox – assertional knowledge about individuals

relative $\sqsubseteq$ person person $\sqsubseteq$ $\exists$substance_m.body
person(john)

Description Logic – Concepts

- Vocabulary:
  - atomic concepts A
  - atomic roles R
- Concepts C, D:
  - A (atomic concept)
  - $\top$ (universal concept)
  - $\bot$ (bottom concept)
  - $\neg C$ (negation)
  - $C \sqcap D$ (intersection)
  - $C \sqcup D$ (union)
  - $\forall R.C$ (value restriction)
  - $\exists R.C$ (existential quantification)

Model-theoretic interpretation

- Model structures: M = (D, I), where D is a domain of individuals, and I an interpretation function mapping
  - atomic concepts to subsets of D
  - roles to binary relations over D
  - individual names to elements of D
- Interpretation function tells us how to interpret atomic concepts, properties and individuals.
- The semantics of concept forming operators is given by extending the interpretation function (next slide).
- An interpretation that satisifies all axioms in an ontology (TBox and ABox) is also called a model of the ontology.

Interpretation of concepts

$$
\begin{aligned}
I(\top) &= D \\
I(\bot) &= \emptyset \\
I(\neg C) &= D \setminus I(C) \\
I(C \sqcap D) &= I(C) \cap I(D) \\
I(C \sqcup D) &= I(C) \cup I(D) \\
I(\forall R.C) &= \{\ a \in D \mid \forall b.\ (a,b) \in I(R) \rightarrow b \in I(C)\ \} \\
I(\exists R.C) &= \{\ a \in D \mid \exists b.\ (a,b) \in I(R) \land b \in I(C)\ \}
\end{aligned}
$$

Axioms & Terminologies

- A terminolgy (“TBox”) is a set of terminological axioms:
  - Inclusions: $C_1 \sqsubseteq C_2$
  - Equalities: $C_1 \equiv C_2$
- Definitions: If the left-hand side of an equality is an atomic concept, the axiom is called a definition.
  - $Mother \equiv Woman \sqcap \exists hasChild.Person$
- An interpretation I satisfies
  - $C_1 \sqsubseteq C_2$ iff $I(C_1) \subseteq I(C_2)$
  - $C_1 \equiv C_2$ iff $I(C_1) = I(C_2)$

An example

$$\text{Woman} \equiv \text{Person} \sqcap \text{Female}$$

$$\text{Man} \equiv \text{Person} \sqcap \neg\text{Woman}$$

$$\text{Mother} \equiv \text{Woman} \sqcap \exists\text{hasChild.Person}$$

$$\text{Father} \equiv \text{Man} \sqcap \exists\text{hasChild.Person}$$

$$\text{Parent} \equiv \text{Mother} \sqcup \text{Father}$$

$$\text{Grandmother} \equiv \text{Mother} \sqcap \exists\text{hasChild.Parent}$$

(Baader & Nutt, 2003)

Acyclic TBoxes

- TBox T is acyclic if there are no definitorial cycles
- Expansion of acyclic TBox T: replace defined concept by their definition.

$$\begin{aligned}
\text{Woman} &\equiv \text{Person} \sqcap \text{Female} \\
\text{Man} &\equiv \text{Person} \sqcap \neg(\text{Person} \sqcap \text{Female}) \\
\text{Mother} &\equiv \text{Person} \sqcap \text{Female} \sqcap \exists \text{hasChild.Person} \\
\text{Father} &\equiv \text{Person} \sqcap \neg(\text{Person} \sqcap \text{Female}) \sqcap \exists \text{hasChild.Person} \\
&\ldots
\end{aligned}$$

Further constructors

- Number restrictions:
  - HappyFather ≡ Father ⊓ ≥3hasChild ⊓ ∀hasChild.Person
  - $I(\geq n\ R) = \{\ x \in D \mid card(\{y \mid (x, y) \in I(R)\}) \geq n\ \}$
  - $I(\leq n\ R) = \{\ x \in D \mid card(\{y \mid (x, y) \in I(R)\}) \leq n\ \}$
- Inverse roles: $hasChild^{-1}$
  - $I(R^{-1}) = \{\ (x, y) \mid (y, x) \in I(R)\ \}$
- Transitive roles: hasChild*
  - $I(R^*) = I(R)^*$
- etc.

World descriptions (“ABox”)

- The ABox contains assertions about individuals.
- Concept assertion: C(a)
  - Mother(mary)
- Role assertions: R(a, b)
  - hasChild(mary, tom)

Talking about Dolphins

Dolphins are mammals, not fish. They are warm blooded like man, and give birth to one baby called a calf [*at a time*]. They are [*highly*] sociable animals, living in pods which are [*fairly*] fluid, [*with dolphins from other pods interacting with each other from time to time*].

Atomic concepts and roles: Dolphin, Mammal, Fish, WarmBlooded, Man, Baby, Calf, Sociable, Animal, Pod, Fluid, giveBirthTo, liveIn

Basic inference problems

- Let T be a TBox
- C is satisfiable w.r.t. T
  - iff there exists some model I of T such that $I(C) \neq \emptyset$;
- C is subsumed by D w.r.t. T
  - iff for every model I of T, $I(C) \subseteq I(D)$
- C and D are equivallent w.r.t. T
  - iff for every model I of T, $I(C) = I(D)$
- C and D are disjoint w.r.t. T
  - iff for every model I of T, $I(C) \cap I(D) = \emptyset$

Basic inference problems

- Many inference tasks can be reduced to subsumption reasoning:
  - C is unsatisfiable $\Leftrightarrow$ C $\sqsubseteq \bot$
  - C and D are equivalent $\Leftrightarrow$ C $\sqsubseteq$ D and D $\sqsubseteq$ C
  - C and D are disjoint $\Leftrightarrow$ C $\sqcap$ D $\sqsubseteq \bot$
- Subsumption can be reduced to satisfiability:
  - C $\sqsubseteq$ D $\Leftrightarrow$ C $\sqcap \neg$D is unsatisfiable

Relation to FOL

- Description logic expressions can be translated into formulas of first-order predicate logic (with two variables).
- Translation of concepts:

$$t_x(A) = A(x)$$

$$t_x(C_1 \sqcap C_2) = t_x(C_1) \wedge t_x(C_1)$$

$$t_x(C_1 \sqcup C_2) = t_x(C_1) \vee t_x(C_1)$$

$$t_x(\exists R.C) = \exists y.R(x, y) \wedge t_y(C)$$

$$t_x(\forall R.C) = \forall y.R(x, y) \rightarrow t_y(C)$$

Relation to FOL

- Description logic expressions can be translated into formulas of first-order predicate logic (with two variables).
- Translation of axioms

  $t(C \sqsubseteq D) = \forall x. t_x(C) \rightarrow t_x(D)$

- A concept C is satisfiable wrt. some TBox T iff the first-oder formula $t_x(C) \wedge t(T)$ is satisfiable.
- A concept C is subsumed by a concept D wrt. T iff the formula $t_x(C) \wedge \neg t_x(D) \wedge t(T)$ is unsatisfiable.