1 Formalising Natural Language Sentences

Translate the following sentences into formulas of modal predicate logic, representing the truth-conditions of the sentences as closely as possible. To keep things simple, you might treat "pass the exam" and "works hard" as one-place predicate symbols. Please ignore tense.

1. It is possible that every student will pass the exam, but it is not necessary.
2. Its impossible that Bill won't pass the exam if he works hard.
3. If anybody can be smarter than everybody else, then everybody can be the smartest.
4. The president is necessarily a democrat.

2 Modal Propositional Logic

Consider the following model structure:

- $W = \{w_1, w_2, w_3, w_4\}$
- $\mathrm{R} = \{(w_1, w_2), (w_2, w_3), (w_3, w_1), (w_3, w_4), (w_4, w_2)\}$
- p is true at w_1, w_3 (and false at all other worlds), q is true at w_1, w_2.

(a) Compute the truth-conditions of (i) $\Box q$ and (ii) $\Box\neg(p \rightarrow \Diamond q)$, and decide at which worlds the two formulae are true.

(b) Decide whether (i) $\Diamond\Box p \wedge \Diamond\Diamond\Box p$ and (ii) $\Box p \rightarrow \neg p$ are valid in the model, i.e. true at all points.

3 Entaiment

Translate the following two sentences into modal predicate logic, and show that (the translation of) the first sentence entails the second one, i.e., that (b) is true in all worlds in which (a) is true.

(a) Bill possibly has a degree in LST

(b) Bill possibly has a degree.

To be turned in by Friday, 2009-05-22