(a) $(g,h) \in [\![\neg\exists x\, perf(x)]\!]$ iff $g = h$ and $\neg\exists k : (g,k) \in [\![\exists x\, perf(x)]\!]$

$(g,k) \in [\![\exists x\, perf(x)]\!]$ iff $\exists l : l[x]g$ and $(l,k) \in [\![perf(x)]\!]$.

$(l,k) \in [\![perf(x)]\!]$ iff $l = k$ and $k(x) \in V(perf)$

iff $g = h$ and $\neg\exists k : \exists l : l[x]g$ and $l = k$ and $k(x) \in V(perf)$

iff $g = h$ and $\neg\exists k : k[x]g$ and $k(x) \in V(perf)$

$(g,h) \in [\![\exists x \neg perf(x)]\!]$ iff $\exists k : k[x]g$ and $(k,h) \in [\![\neg perf(x)]\!]$

$(k,h) \in [\![\neg perf(x)]\!]$ iff $k = h$ and $\neg\exists l : (k,l) \in [\![perf(x)]\!]$

$(k,l) \in [\![perf(x)]\!]$ iff $k = l$ and $k(x) \in V(perf)$.

iff $\exists k : k[x]g$ and $k = h$ and $\neg\exists l : k = l$ and $k(x) \in V(perf)$

iff $\exists k : k[x]g$ and $k = h$ and $k(x) \notin V(perf)$

iff $h[x]g$ and $h(x) \notin V(perf)$

$(g,h) \in [\![\neg perf(x)]\!]$

iff $g = h$ and $\neg\exists l : g = l$ and $g(x) \in V(perf)$

iff $g = h$ and $g(x) \notin V(perf)$

$$[\![\neg\exists x.perf(x)]\!] = \{(g,h) \mid g = h \text{ and } \neg\exists k : k[x]g \text{ and } k(x) \in V(perf)\}$$
$$[\![\exists x.\neg perf(x)]\!] = \{(g,h) \mid h[x]g \text{ and } h(x) \notin V(perf)\}$$
$$[\![\neg perf(x)]\!] = \{(g,h) \mid g = h \text{ and } g(x) \notin V(perf)\}$$

(b) (4) $\Leftrightarrow$ (5)? Formulae (4) and (5) cannot be fully/dynamically equivalent because (4) is a test, while (5) is externally dynamic.

(5) $\Leftrightarrow$ (6)? The same argument shows that (5) and (6) are not fully equivalent.

(4) $\Leftrightarrow$ (6)? Formulae (4) and (6) are not fully equivalent because the variable x occurs free in (6), while x occurs bound in (4). The denotation of (4) is either the set of all identical assignments, or the empty set. On the other hand, the denotation of (6) can be any set of pairs of assigments (g,g), depending on whether $g(x) \in V(perf)$.

Alternative answer: consider a model $M = (U,V)$ such that $U = \{a,b\}$ and $V(perf) = \{a\}$. Let h be an assignment such that $h(x) = b$. As one can see, $(h,h) \in [\![(6)]\!]$. But $(h,h) \notin [\![(4)]\!]$, hence (4) and (6) cannot be fully equivalent.

(c) (4) $\models_S$ (5)? In order to show that (4) statically entails (5), we have to show that for all models M, if (4) is true in M, then (5) is true in M. Now for (4) to be true in some model $M = (U,V)$, we must have that $V(perf) = \emptyset$, as otherwise $[\![(4)]\!] = \emptyset$. But then (5) is true in M as well: for each input assignment g, we can find an assignment h that differs from g at most in x, such that $h(x) \neq V(perf)$.

$(4) \models (5)$? Because (4) is a test, static and dynamic entailment coincide i.e., (4) dynamically entails (5).

$(5) \models_S (6)$? (5) does not statically entail (6). To see this, consider a model $M = (U, V)$ such that $U = \{a, b\}$ and $V(\mathit{perf}) = \{a\}$, and consider an assignment h such that $h(x) = b$. It is easy to see that $h \in \backslash(6)\backslash$, but $h \notin \backslash(5)\backslash$. That is, (6) is true in M for h, but (5) is not.

$(5) \models (6)$? (5) dynamically entails (6), because for all output assignments g of $[\![(5)]\!]$, we must have that $g(x) \notin V(\mathit{perf})$.

$(4) \models_S (6)$? (4) statically entails (6). As argued above, the denotation of (4) is either the set of all pairs of identical assignments, or the empty set.

If $V(\mathit{perf}) \neq \emptyset$, then $[\![(4)]\!] = \emptyset$ and there is nothing to show.

If $V(\mathit{perf}) = \emptyset$, then $[\![(4)]\!] = \{(g, h) \mid h = g\}$. Because all assignments g satisfy $g(x) \neq V(\mathit{perf})$, it follows that $[\![(5)]\!] = \{(g, h) \mid h = g\}$ as well. That is, if (4) is true in M for some assigment g, (6) is true in M for g as well.

$(4) \models (6)$? Because (4) is a test, (4) dynamically entails (6) as well.