1 Type theory: Lexicon

(a) Proper names:
John $\Rightarrow \lambda F.F(j^*)$

(b) Determiners:
every $\Rightarrow \lambda F \lambda G \forall x.(F(x) \rightarrow G(x))$
a $\Rightarrow \lambda F \lambda G \exists x.(F(x) \wedge G(x))$
no $\Rightarrow \lambda F \lambda G \neg\exists x.(F(x) \wedge G(x))$

(c) Most content words are simply analysed as constants (note: transitive verbs get type $\langle\langle\langle e,t\rangle,t\rangle,\langle e,t\rangle\rangle$). But sometimes, the semantics of a word can be represented more precisely by a complex term, e.g.

- edible $\Rightarrow \lambda P \lambda x.P(x) \wedge \Diamond\exists y.\mathsf{eat}^*(x)(y)$
- unmarried $\Rightarrow \lambda P \lambda x.P(x) \wedge \neg\exists y.\mathsf{is\_married\_to}'(y)(x)$

2 Modal Logic

Terms:

$[\![x]\!]^{M,g,w,t} = g(x)$	if x is a variable
$[\![a]\!]^{M,g,w,t} = V_M(a)$	if a is a constant.

Formulas:

$[\![R(t_1,\ldots,t_n)]\!]^{M,g,w,t} = 1$	iff $\langle[\![t_1]\!]^{M,g,w,t},\ldots,[\![t_n]\!]^{M,g,w,t}\rangle \in V_M(R)(w,t)$
$[\![t_1 = t_2]\!]^{M,g,w,t} = 1$	iff $[\![t_1]\!]^{M,g,w,t} = [\![t_2]\!]^{M,g,w,t}$
$[\![\neg\varphi]\!]^{M,g,w,t} = 1$	iff $[\![\varphi]\!]^{M,g,w,t} = 0$
$[\![\varphi \wedge \psi]\!]^{M,g,w,t} = 1$	iff $[\![\varphi]\!]^{M,g,w,t} = 1$ and $[\![\psi]\!]^{M,g,w,t} = 1$
$[\![\varphi \vee \psi]\!]^{M,g,w,t} = 1$	iff $[\![\varphi]\!]^{M,g,w,t} = 1$ or $[\![\psi]\!]^{M,g,w,t} = 1$
$[\![\exists x\varphi]\!]^{M,g,w,t} = 1$	iff there is $a \in U_M$, $[\![\varphi]\!]^{M,g[x/a],w,t} = 1$
$[\![\forall x\varphi]\!]^{M,g,w,t} = 1$	iff for all $a \in U_M$, $[\![\varphi]\!]^{M,g[x/a],w,t} = 1$
$[\![\Box\varphi]\!]^{M,g,w,t} = 1$	iff for all $w' \in W$, $[\![\varphi]\!]^{M,g,w',t} = 1$
$[\![\Diamond\varphi]\!]^{M,g,w,t} = 1$	iff there is $w' \in W$, $[\![\varphi]\!]^{M,g,w',t} = 1$
$[\![\mathbf{F}\varphi]\!]^{M,g,w,t} = 1$	iff there is $t' > t$, $[\![\varphi]\!]^{M,g,w,t'} = 1$
$[\![\mathbf{G}\varphi]\!]^{M,g,w,t} = 1$	iff for all $t' > t$, $[\![\varphi]\!]^{M,g,w,t'} = 1$
$[\![\mathbf{P}\varphi]\!]^{M,g,w,t} = 1$	iff there is $t' < t$, $[\![\varphi]\!]^{M,g,w,t'} = 1$
$[\![\mathbf{H}\varphi]\!]^{M,g,w,t} = 1$	iff for all $t' < t$, $[\![\varphi]\!]^{M,g,w,t'} = 1$

3 Nested Cooper Storage

Transitive verbs are analysed as constants of type $\langle\langle\langle e,t\rangle,t\rangle,\langle e,t\rangle\rangle$.

(a) Storage:

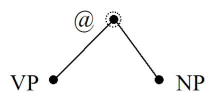
$$\frac{B \;\Rightarrow\; \langle\gamma,\Gamma\rangle \qquad B \text{ is an NP node}}{B \;\Rightarrow\; \langle\lambda P.P(x_i),\{\langle\gamma,\Gamma\rangle_i\}\rangle \qquad i\in\mathbf{N} \text{ is a new index}}$$

(b) Retrieval:

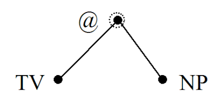
$$\frac{A \;\Rightarrow\; \langle\alpha,\Delta\cup\{\langle\gamma,\Gamma\rangle_i\}\rangle \qquad A \text{ is any sentence node}}{A \;\Rightarrow\; \langle\gamma(\lambda x_i.\alpha),\Delta\cup\Gamma\rangle}$$

4 Dominance graphs: Semantics construction

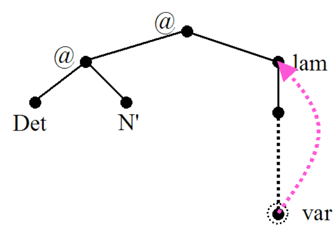
S → NP VP:



VP → TV NP:

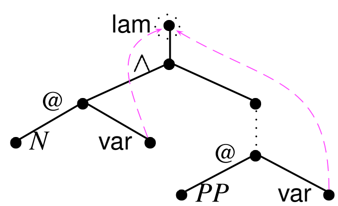


NP → Det N':



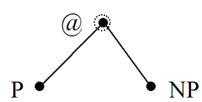
VP → IV:

N' → N PP:



NP → PN:

PP → P NP:



N' → N:

5 Dominance graphs: Solving

The three rules of the dominance graph solver:

(a) *Choice:* If a node u has two dominance parents v and w, generate two new dominance graphs containing the edges (v,w) and (w,v), and continue the search for solved forms for both new graphs.

(b) *Parent Normalisation:* If (u,v) is a dominance edge, and v has a father w over a tree edge, replace (u,v) by (u,w).

(c) *Redundancy Elimination:* If $e=(u,v)$ is an edge and there is a path from u to v that doesn't use e, delete e from the dominance graph.

6 DRT: Syntax and Semantics

A discourse representation structure (DRS) K is a pair $\langle U_K, C_K \rangle$ where

- U_K is a set of discourse referents
- C_K is a set of conditions.

Conditions:

$R(u_1, \ldots, u_n)$	R is an n-place relation, $u_i \in U_K$
$u = v$	$u, v \in U_K$
$u = a$	$u \in U_K$, a a proper name
$K_1 \Rightarrow K_2$	K_1 and K_2 DRSs
$K_1 \vee K_2$	K_1 and K_2 DRSs
$\neg K_1$	K_1 is a DRS

7 DRT: Embedding, verifying embedding

Let U_D be a set of discourse referents, $K = \langle U_K, C_K \rangle$ a DRS with $U_K \subseteq U_D$, $M = \langle U_M, V_M \rangle$ a model structure of first-order predicate logic that is suitable for K. An *embedding* of U_D into M is a (partial) function that assigns individuals from U_M to discourse referents.

An embedding f *verifies* the DRS K in M ($f \models_M K$) iff

(a) $U_K \subseteq \text{Dom}(f)$ and

(b) f verifies each condition $\alpha \in C_K$.

f verifies a condition α in M ($f \models_M \alpha$) in the following cases:

$f \models_M R(u_1, \ldots, u_n)$	iff $\langle f(u_1), \ldots, f(u_n) \rangle \in V_M(R)$
$f \models_M u = v$	iff $f(u) = f(v)$
$f \models_M u = a$	iff $f(u) = V_M(a)$
$f \models_M K_1 \Rightarrow K_2$	iff for all $g \supseteq_{U_{K_1}} f$ such that $g \models_M K_1$, there is $h \supseteq_{U_{K_2}} g$ such that $h \models_M K_2$
$f \models_M \neg K_1$	iff there is no $g \supseteq_{U_{K_1}} f$ such that $g \models_M K_1$
$f \models_M K_1 \vee K_2$	iff there is a $g_1 \supseteq_{U_{K_1}} f$ such that $g_1 \models_M K_1$, or there is a $g_2 \supseteq_{U_{K_2}} f$ such that $g_2 \models_M K_2$.

8 Presuppositions (van der Sandt)

A proto-DRS is a triple $\langle U_K, C_K, A_K \rangle$, where

- U_K is a set of discourse referents
- C_K is a set of conditions
- A_K is a set of "anaphoric" (alpha-) DRSs.

9 Resolution of α-DRSs

Let K and K' be proto-DRSs such that K' is a sub-DRS of K. Let $\gamma = \alpha x K_s$ be an alpha-free alpha-DRS in K', and let K_t be a sub-DRS of K that is accessible for γ.

(a) Accommodation: Remove γ from K', and extend K_t with U_{K_s} and C_{K_s}.

(b) Binding: Let further $y \in U_{K_t}$ be a discourse referent that is suitable for γ. Then remove γ from K', and extend K_t with U_{K_s} and C_{K_s} and the condition $x = y$.

10 DPL: Interpretation

Terms:

$$
\begin{array}{ll}
[\![x]\!]^{M,h} = h(x) & \text{if } x \text{ is a variable} \\
[\![a]\!]^{M,h} = V_M(a) & \text{if } a \text{ is a constant.}
\end{array}
$$

Formulas:

$$
\begin{array}{lcl}
[\![R(t_1,\ldots,t_n)]\!]^M & = & \{\langle g,h\rangle \mid h = g \text{ and } \langle [\![t_1]\!]^{M,h},\ldots,[\![t_n]\!]^{M,h}\rangle \in V_M(R)\} \\
[\![t_1 = t_2]\!]^M & = & \{\langle g,h\rangle \mid h = g \text{ and } [\![t_1]\!]^{M,h} = [\![t_2]\!]^{M,h}\} \\
[\![\neg\varphi]\!]^M & = & \{\langle g,h\rangle \mid h = g \text{ and ex. no } k \text{ s.t. } \langle g,k\rangle \in [\![\varphi]\!]^M\} \\
[\![\varphi \wedge \psi]\!]^M & = & \{\langle g,h\rangle \mid \text{ex. } k \text{ s.t. } \langle g,k\rangle \in [\![\varphi]\!]^M \text{ and } \langle k,h\rangle \in [\![\psi]\!]^M\} \\
[\![\varphi \vee \psi]\!]^M & = & \{\langle g,h\rangle \mid h = g \text{ and ex. } k \text{ s.t. } \langle g,k\rangle \in [\![\varphi]\!]^M \text{ or } \langle g,k\rangle \in [\![\psi]\!]^M\} \\
[\![\varphi \rightarrow \psi]\!]^M & = & \{\langle g,h\rangle \mid h = g \text{ and for all } k\text{: if } \langle g,k\rangle \in [\![\varphi]\!]^M\text{, then ex. } j \text{ s.t. } \langle k,j\rangle \in [\![\psi]\!]^M\} \\
[\![\exists x.\varphi]\!]^M & = & \{\langle g,h\rangle \mid \text{ex. } k[x]g \text{ s.t. } \langle k,h\rangle \in [\![\varphi]\!]^M\} \\
[\![\forall x.\varphi]\!]^M & = & \{\langle g,h\rangle \mid h = g \text{ and for each } k[x]g\text{, there is an } m \text{ s.t. } \langle k,m\rangle \in [\![\varphi]\!]^M\}
\end{array}
$$

11 DPL: Truth, equivalence, entailment

(a) Truth and validity:

- A formula φ is *true in M with respect to an input assignment g* iff there is a h s.t. $\langle g,h\rangle \in [\![\varphi]\!]^M$.
- A formula φ is *true in M* iff φ is true in M with respect to every input assignment.
- φ is *valid* iff it is true in every model structure.

(b) Notions of equivalence:

- *Satisfaction set*: $\backslash\varphi\backslash_M = \{g \mid \text{exists } h \text{ s.t. } \langle g,h\rangle \in [\![\varphi]\!]^M\}$
- *s-equivalence* (static equivalence): $\varphi \Leftrightarrow_S \psi$ iff for all M, $\backslash\varphi\backslash_M = \backslash\psi\backslash_M$
- *full equivalence* (dynamic equivalence): $\varphi \Leftrightarrow \psi$ iff for all M, $[\![\varphi]\!]^M = [\![\psi]\!]^M$

(c) Notions of entailment:

- *Static entailment*: $\varphi \models_S \psi$ iff for all M, g: If φ is true wrt. M and g, then ψ is true wrt. M and g.
- *Meaning inclusion*: $\varphi \leq \psi$ iff $[\![\varphi]\!]^M \subseteq [\![\psi]\!]^M$.
- *Dynamic entailment*: $\varphi \models \psi$ iff for all M, g, h: if $\langle g,h\rangle \in [\![\varphi]\!]^M$, then there exists k s.t. $\langle h,k\rangle \in [\![\psi]\!]^M$.