„Semantic Theory“ SS 06 Exercise 2 (02/05/2006)

http://www.coli.uni-saarland.de/courses/semantics-06/

Note: This exercise contains a bonus question, indicated by a star (*). The (difficult) bonus question does not count towards the total number of points you can get for this exercise sheet, so you can get more than the total number of points this week.

1 Terms and types

Which of the following terms are well-formed terms of type theory? For those that are well-formed, determine the types. In both cases, justify your answer, i.e. explain why it is correct. Assume that the constant a has type e, f has type $\langle e, e\rangle$, P has type $\langle e, t\rangle$, and C has type $\langle\langle e, e\rangle, t\rangle$.

(a) $P(a)$

(b) $C(a)$

(c) $C(f(a))$

(d) $C(\lambda x.f(f(x)))$

(e) $\lambda x.C(f)$

(f) $C(\lambda x\lambda y.P(x))$

2 Interpreting type theory

Compute the truth conditions of the following expressions. That is, derive statements of the form "$[\![\mathsf{sleep}(\mathsf{peter})]\!]^{M,g} = 1$ iff $V_M(\mathsf{peter}) \in V_M(\mathsf{sleep})$" for expressions of type t and statements of the form "$[\![\mathsf{sleep}]\!]^{M,g} = V_M(\mathsf{sleep})$" for (sub)expressions of other types. You may abbreviate intermediate results appropriately, as long as you show the most important interpretation steps. Don't β-reduce anything for now.

(a) $(\lambda F \lambda G \neg\exists x.F(x) \wedge G(x))(\mathsf{student})(\mathsf{work})$
"No student works."

(b) $(\lambda F.F(\mathbf{j}^*) \vee F(\mathbf{p}^*))(\mathsf{work})$
"Either John or Peter works."

Then β-reduce the term (b) into a formula of first-order predicate logic and compute its truth conditions as a FOL formula. Compare the two interpretations.

3 Semantics construction

Construct semantic representations for each of the following sentences using type theory the rules from the lecture. First give a representation for each word (counting word sequences that are connected by a hyphen, such as "not-all", as a single word). Then combine them compositionally into a representation for the whole sentence. Give the type of each term in your derivation. Use lambda abstraction as necessary, and β-reduce your results (including intermediate representations) as far as possible.

(a) Every student works hard.

(b) Not-all blond students sleep.

(c) Most intelligent people don't drink and drive.
(Use a constant most of type $\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$ to represent the determiner.)

(d) John says-that Peter is-a-criminal.
(* For extra credit, find an analysis in which "a criminal" is represented as in the lecture. You will have to do something clever to represent "is" in this case.)

(e) * John reads every book.
(You will have to analyse "reads" as an expression that doesn't have the type $\langle e,\langle e,t\rangle\rangle$.)

4 And

1. What type would you have to assign the semantic representation of "and" in each of the following sentences so the representation for the whole sentence gets type t?
   (a) John sleeps and Mary works.
   (b) John works hard and is-successful.
   (c) All students and some professors work hard.
   (d) John works quickly and thoroughly.
2. Represent the semantics of "and" in each sentence as a λ-term.

5 * Defining Logical Connectives

It is possible to define all connectives of type theory in terms of identity $s = t$, functional application $\alpha(\beta)$ and λ-abstraction $\lambda v \alpha$. That is, every well-formed expression of type theory can be translated into a well-formed expression with the same type and the same interpretation that only contains these three connectives.

Give definitions of negation, conjunction, and the universal quantifier. As an auxiliary notion (and an example), we give the definition of the tautology $\top$, i.e. the expression of type t whose interpretation is 1 regardless of the model, here:

$$\top \ := \ \lambda P.P =_{\langle t,t\rangle} \lambda P.P,$$

where P is a variable of type t.

To be turned in by 09/05/2006, 11:15 am