http://www.coli.uni-saarland.de/courses/semantics-05/

Note: Because May 5 is a public holiday, the course will not take place on that day, and we cannot collect solutions. Please turn in your answers to this exercise by 06/05/2005, 10:00 to the Sekretariat Pinkal (room 1.10).

1 Semantic representations for expressions

Represent the semantics of the following natural language expressions as terms of type theory. Use lambda abstraction and modal and tense operators as necessary.

- (a) *exactly one* (as in: "Exactly one student failed")
- (b) only (as in: "Only John failed")
- (c) *edible* (use the two-place predicate *eat'* to define it)
- (d) *unmarried* (use the two-place predicate *marry*', and ignore the possibility of divorces)

2 Semantic representations for sentences

Represent the semantics of the following sentences as formulas of (extensional) type theory. First give a representation for each word (counting word sequences that are connected by a hyphen, such as "believes-that", as a single word). Then combine them into a representation for the whole sentence, as in the lecture. Give the type of each term in your derivation. Use lambda abstraction and modal and tense operators as necessary, and β -reduce your result as far as possible.

- (a) John believes-that Peter saw a unicorn.
- (b) Every student who studies hard will pass-the-exam.
- (c) Every student in Saarbrücken is intelligent.
- (d) No student is (a) professor. (You can ignore the determiner "a". For extra credit, give an analysis where "a professor" is analysed as a term of type $\langle \langle e, t \rangle, t \rangle$.)
- (e) Not all blond students sleep.(What does "not" modify?)

3 Tense logic

- 1. A formula in tense logic is called *valid* iff it is true in all model structures at all time points. Prove that each of the following formulas is a valid formula of tense logic.
 - (a) $\mathbf{FF}A \to \mathbf{F}A$

- (b) $\mathbf{PP}A \rightarrow \mathbf{P}A$
- (c) $\mathbf{FP}A \to \mathbf{P}A \lor A \lor \mathbf{F}A$
- 2. For each of the three formulas, give a countermodel that shows that the reversed formula (e.g. $\mathbf{F}A \to \mathbf{F}\mathbf{F}A$) is not valid.
- 3. For each of the three formulas, specify how the "earlier than" relation has to be restricted to make the reversed formula valid. Do not impose any restrictions on the valuation V.

4 Predicate logic with tense and modality

Consider the following model structure for predicate logic with modal and tense operators. The order on the time points is $t_1 < t_2 < t_3$, and the universe is $U = \{a, b, c\}$.

$$\begin{array}{c|cccc} S & t_1 & t_2 & t_3 \\ \hline w_1 & \{a,b\} & \{a,c\} & \{b.c\} \\ w_2 & \{a,b,c\} & \{a\} & \{a,b\} \end{array}$$

Determine the truth values of the following formulas at w_1 and t_2 :

- (a) $\forall x.S(x)$
- (b) $\mathbf{F} \forall x.S(x)$
- (c) $\diamond \mathbf{P} \forall x.S(x)$
- (d) $\mathbf{G} \diamond \forall x.S(x)$

5 * Branching time

In the lecture, we required that the earlier-than relation must be a linear order. Now we want to change this to a nondeterministic model of *forward branching time*: For each time point t, the set of all time points that are earlier than t is linearly ordered, but the set of all time points that are later than t need only be partially ordered. Intuitively (but not formally!), the models of forward branching tense logic are trees.

- (a) Give a formal definition of the model structures for forward branching tense logic.
- (b) Assume that the tense operators **F**, **P**, etc. are defined as in standard tense logic, and find some formulas that are valid in standard tense logic, but not in forward branching tense logic. Are there formulas for which the converse is true? Why or why not?
- (c) Describe in your own words what the future operators \mathbf{F} and \mathbf{G} express in forward branching tense logic. You will notice that neither of the two operators is an adequate representation of the semantics of future in natural language. Define an operator \mathbf{E} (i.e., specify its interpretation) for NL future tense.