„Semantic Theory“ SS 05 Exercise 2 (21/04/2005)

http://www.coli.uni-saarland.de/courses/semantics-05/

Note: This exercise contains a bonus question, indicated by a star (*). The (difficult) bonus question does not count towards the total number of points you can get for this exercise sheet, so you can get more than the total number of points this week.

1 Terms and types

Which of the following terms are well-formed terms of type theory? For those that are well-formed, determine the types. In both cases, justify your answer, i.e. explain why it is correct. Assume that the constant a has type e, f has type $\langle e,e\rangle$, P has type $\langle e,t\rangle$, and C has type $\langle\langle e,e\rangle,t\rangle$.

(a) $P(a)$

(b) $C(a)$

(c) $C(f(a))$

(d) $C(\lambda x.f(f(x)))$

(e) $\lambda x.C(f)$

(f) $C(\lambda x\lambda y.P(x))$

2 Semantic representations in type theory

Find formulas of λ-free type theory that represent the semantics of the following sentences. You can represent all noun phrases (not all nouns) by constants of type e.

(a) John gives Mary the book.

(b) Peter owns a red car.

(c) The president rarely sleeps.

(d) Presumably, the president lives in a very nice house.

(e) Mary eats a sandwich.

(f) Mary eats only a sandwich.

3 Denotations

Compute the denotations of the following terms (without β-reducing them first):

(a) $(\lambda F\lambda G\neg\exists x.F(x)\wedge G(x))(\mathsf{student}')(\mathsf{work})'$
"No student works."

(b) $(\lambda F.F(\mathbf{j}^*)\vee F(\mathbf{p}^*))(\mathsf{work}')$
"Either John or Peter works."

Then β-reduce the term (b) into a formula of first-order predicate logic and compute the denotation of this formula. Compare the two denotations.

4 And

1. What type would you have to assign the semantic representation of "and" in each of the following sentences so the representation for the whole sentence gets type t?
   (a) John works and Mary reads a book.
   (b) John works and reads a book.
   (c) John reads a book and three articles.
   (d) John works quickly and thoroughly.
2. Represent the semantics of "and" in each sentence as a λ-term.

5 Untyped β-reduction

Untyped λ-calculus is a system of λ-terms in which the terms and variables are not assigned types; any combination of λ-abstractions and applications is well-formed. The operation of β-reduction can be defined on untyped λ-terms exactly as for typed λ-terms.

β-reduce the following untyped λ-term as far as possible:

$$(\lambda x.x(x))(\lambda x.x(x))$$

What is your observation? Can you assign a type to x such that the term becomes a well-formed expression of type theory? Do you believe something similar is possible in type theory? Explain your intuition.

6 * Defining Logical Connectives

It is possible to extend type theory with a family of identity symbols $=_\tau$ of type $\langle\tau, \langle\tau, t\rangle\rangle$. For each type τ, the interpretation of $=_\tau$ is fixed to be the identity relation $\{(a, a) \mid a \in D_\tau\}$.

Using only identity, the λ-operator, and functional application, it is possible to define all other connectives and quantifiers of type theory. As an auxiliary notion we give the definition of the tautology $\top$:

$$\top \ := \ \lambda P.P =_{\langle t,t\rangle} \lambda P.P,$$

where P is a variable of type t. Give the definitions of negation, conjunction, and the universal quantifier.

To be turned in by 28/04/2005, 11:15 am