

1 Formalising natural-language sentences

Give formulas of first-order predicate logic that represent the truth conditions of the following sentences as closely as possible. You can ignore tense and use a single predicate symbol to represent several words where appropriate.

- (a) It is raining.
- (b) John loves Mary.
- (c) All students are intelligent.
- (d) If Mary attends a semantics course, she is bored or she learns something interesting.
- (e) Nobody is loved by no one.
- (f) Dolphins are mammals that live in the sea.
- (g) Every student presents at least one exercise solution in class.

2 Status of logical formulas

For each of the following logical sentences, decide whether it is logically valid, unsatisfiable, or contingent. For valid and unsatisfiable sentences, explain why you believe it is valid or unsatisfiable. For contingent sentences, give a model and a countermodel.

- (a) $\forall x.P(x) \wedge \forall y.Q(y) \wedge (\exists z.\neg P(z) \vee \exists x.\neg Q(x))$
- (b) $(\forall x.\text{student}(x) \rightarrow \text{intelligent}(x)) \rightarrow (\exists y.\text{student}(y) \wedge \text{intelligent}(y))$
- (c) $(\forall x.P(x) \wedge \forall y.Q(y)) \rightarrow \forall x.(P(x) \wedge Q(x))$
- (d) $\forall x.(\text{student}(x) \rightarrow \text{person}(x))$

3 Logical entailment

For each of the following pairs of logical sentences, decide whether the first sentence logically entails the second. If entailment holds, explain why you believe it does. If entailment doesn't hold, give a countermodel.

- (a) $\forall x.P(x) \models \exists x.P(x)$
- (b) $\exists x.P(x) \wedge \exists x.Q(x) \models \exists x.(P(x) \wedge Q(x))$

4 Limits of first-order logic

Consider the following sentence:

This example shows that there are natural-language sentences whose truth conditions can't be adequately represented in first-order logic.

Try to represent the truth conditions of this sentence as a formula of first-order logic. As the sentence suggests, this is not really possible, so you may make changes to the sentence in order to make it representable. Make as few changes as possible, and explain why each change is necessary. You can use atomic predicate symbols to represent the meanings of “natural-language sentence”, “truth condition”, and “first-order logic”.

5 Lexical ambiguity

Explore the WordNet lexicon and find a word with a really high number of senses. You can access WordNet online at <http://wordnet.princeton.edu/cgi-bin/webwn>.