

FLST: Semantics II

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Semantics: The Logical Paradigm

- ❑ Validation of semantic representations via truth-conditional interpretation
- ❑ Semantically controlled inference through entailment and deduction
- ❑ A rigid model of compositionality

Deduction: A Question Answering Example



- Question: *Which element is Thallium said to look like?*
- Support passage: *Thallium is a metallic element that resembles lead.*
- Answer: *Lead*

Watson Again

- We show that "lead" is a correct answer by deriving the representation of the question instantiated with "lead" (the "conclusion" or "hypothesis") from the representation of the answer passage (the "premiss").

- Given:
 - $\text{metallic}(\text{thallium}) \wedge \text{element}(\text{thallium}) \wedge \text{resemble}(\text{thallium}, \text{lead})$

- Wanted:
 - $\text{element}(\text{lead}) \wedge \text{look_like}(\text{thallium}, \text{lead})$

More Ingredients for the Derivation

- We need some more deduction rules. These are justified by corresponding entailment relations: truth preserving transition from premises to conclusion (please, check!).
 - $A \wedge B \vdash A, A \wedge B \vdash B$ (Conjunction Elimination)
 - $A, B \vdash A \wedge B$ (Conjunction Introduction)
 - $A, A \rightarrow B \vdash B$ (Modus Ponens)
 - $A \leftrightarrow B \vdash A \rightarrow B, A \leftrightarrow B \vdash B \rightarrow A$ (Equivalence Elimination)
 - $\forall x A \vdash A[b/x]$ (Universal Instantiation)

- We need some extra bits of knowledge (axioms, taken e.g. from a lexical-semantic knowledge base):
 - `element(lead)`
 - $\forall x \forall y (\text{resemble}(x,y) \leftrightarrow \text{look_like}(x,y))$

Example Derivation

- | | | |
|-----|--|---------------------------------|
| (1) | $\text{metallic}(\text{th}) \wedge \text{element}(\text{th}) \wedge \text{resemble}(\text{th}, \text{lead})$ | Premise |
| (2) | $\text{resemble}(\text{th}, \text{lead})$ | 2x Conjunction Elim (1) |
| (3) | $\forall x \forall y (\text{resemble}(x, y) \leftrightarrow \text{look_like}(x, y))$ | Axiom |
| (4) | $\forall y (\text{resemble}(\text{th}, y) \leftrightarrow \text{look_like}(\text{th}, y))$ | Univ. Instantiation th/x, (3) |
| (5) | $\text{resemble}(\text{th}, \text{lead}) \leftrightarrow \text{look_like}(\text{th}, \text{lead})$ | Univ. Instantiation lead/y, (4) |
| (6) | $\text{resemble}(\text{th}, \text{lead}) \rightarrow \text{look_like}(\text{th}, \text{lead})$ | Equivalence Elim, (5) |
| (7) | $\text{look_like}(\text{th}, \text{lead})$ | Modus Ponens (2), (6) |
| (8) | $\text{element}(\text{lead})$ | Axiom |
| (9) | $\text{element}(\text{lead}) \wedge \text{look_like}(\text{th}, \text{lead})$ | Conjunction Intro (7), (8) |

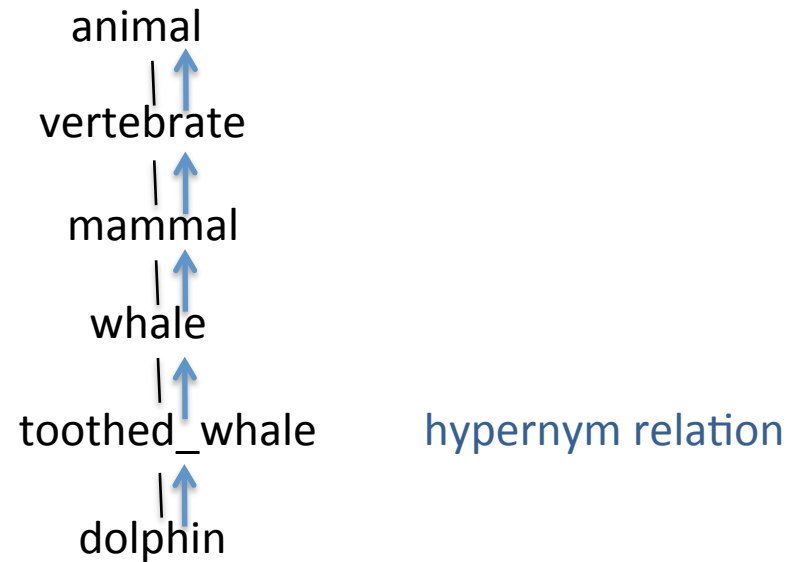
Word Meaning in the Logical Paradigm

- ❑ Atomic predicates represent word senses, but are not very informative in themselves.

- ❑ Axioms express word-semantic information:
 - ❑ semantic relations between different words:
 $\forall x \forall y (\text{look_like}(x, y) \leftrightarrow \text{resemble}(x, y))$
 - ❑ semantic properties of words:
 $\forall x \forall y (\text{resemble}(x, y) \rightarrow \text{resemble}(y, x))$

- ❑ Where can we get these axioms from???
- Axioms can be read off lexical-semantic taxonomies like WordNet

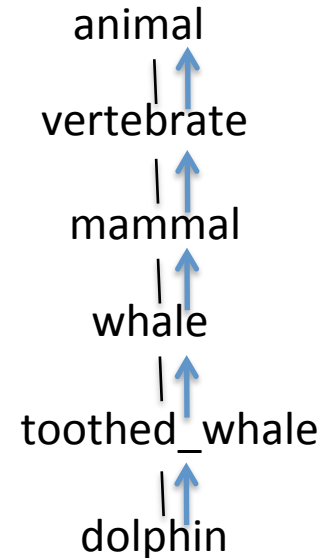
WordNet Meaning Relations



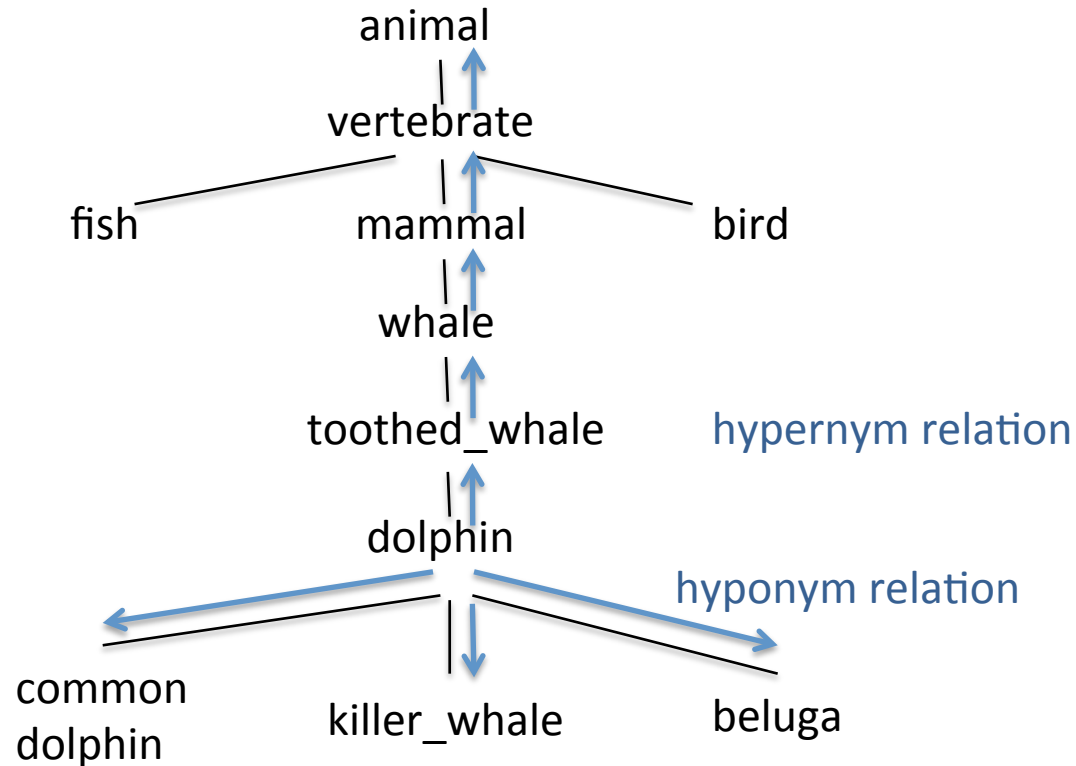
Axioms Expressing Semantic Relations

- B hypernym of A $\Rightarrow \forall x(A(x) \rightarrow B(x))$
 - $\forall x(\text{dolphin}(x) \rightarrow \text{toothed_whale}(x))$
 - $\forall x(\text{toothed_whale}(x) \rightarrow \text{whale}(x))$
 - $\forall x(\text{whale}(x) \rightarrow \text{mammal}(x))$
 - $\forall x(\text{mammal}(x) \rightarrow \text{vertebrate}(x))$
 - $\forall x(\text{vertebrate}(x) \rightarrow \text{animal}(x))$

hypernym relation



WordNet Meaning Relations



Axioms Expressing Semantic Relations

□ B hypernym of A $\Rightarrow \forall x(A(x) \rightarrow B(x))$

$\forall x(\text{dolphin}(x) \rightarrow \text{toothed_whale}(x))$

$\forall x(\text{toothed_whale}(x) \rightarrow \text{whale}(x))$

$\forall x(\text{whale}(x) \rightarrow \text{mammal}(x))$

$\forall x(\text{mammal}(x) \rightarrow \text{vertebrate}(x))$

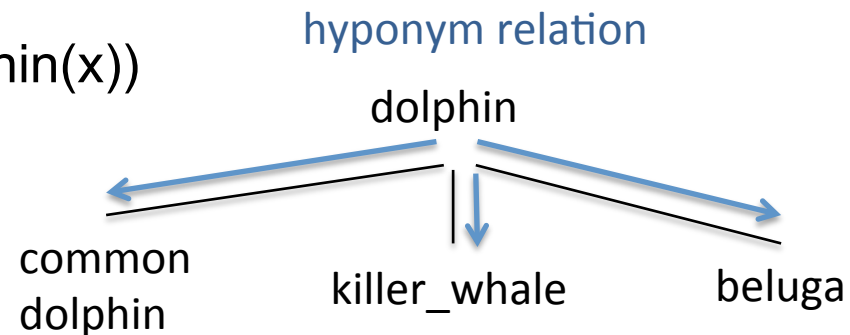
$\forall x(\text{vertebrate}(x) \rightarrow \text{animal}(x))$

□ B hyponym of A $\Rightarrow \forall x(B(x) \rightarrow A(x))$

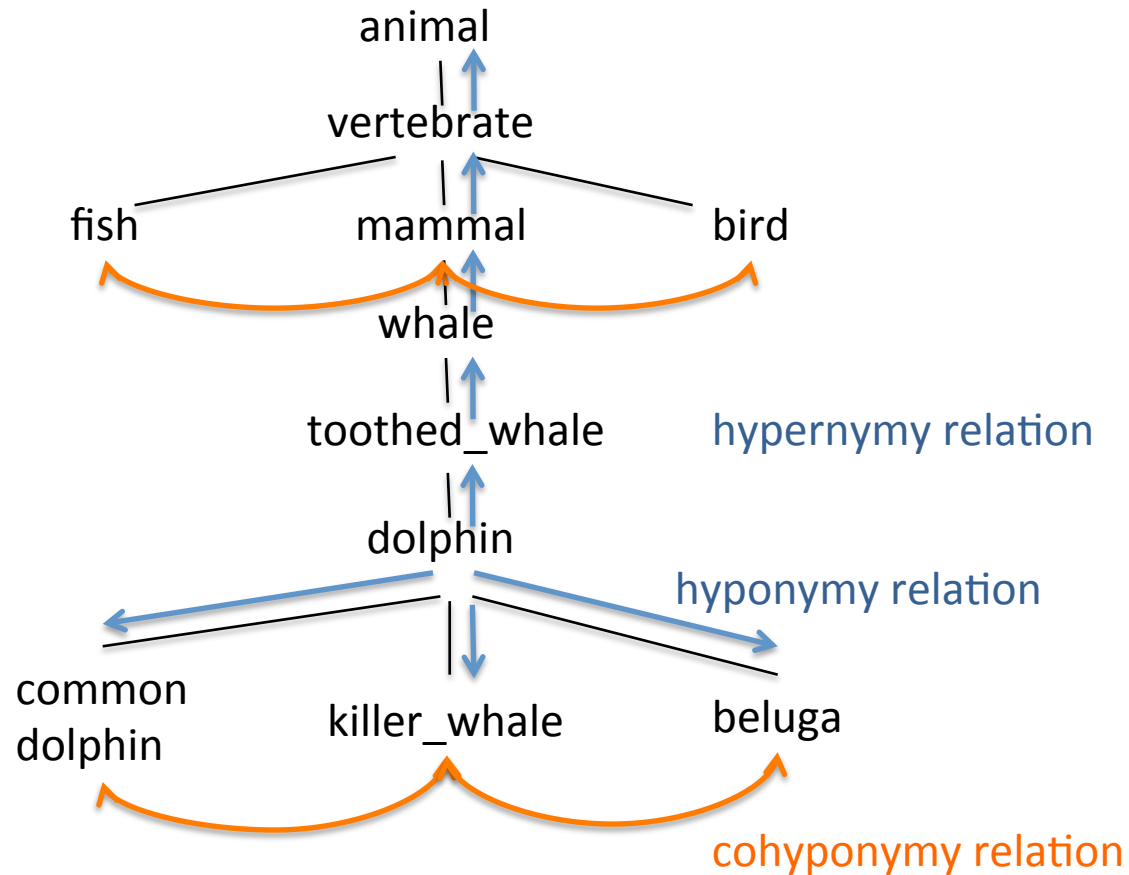
$\forall x(\text{common_dolphin}(x) \rightarrow \text{dolphin}(x))$

$\forall x(\text{killer_whale}(x) \rightarrow \text{dolphin}(x))$

$\forall x(\text{beluga}(x) \rightarrow \text{dolphin}(x))$



WordNet Meaning Relations



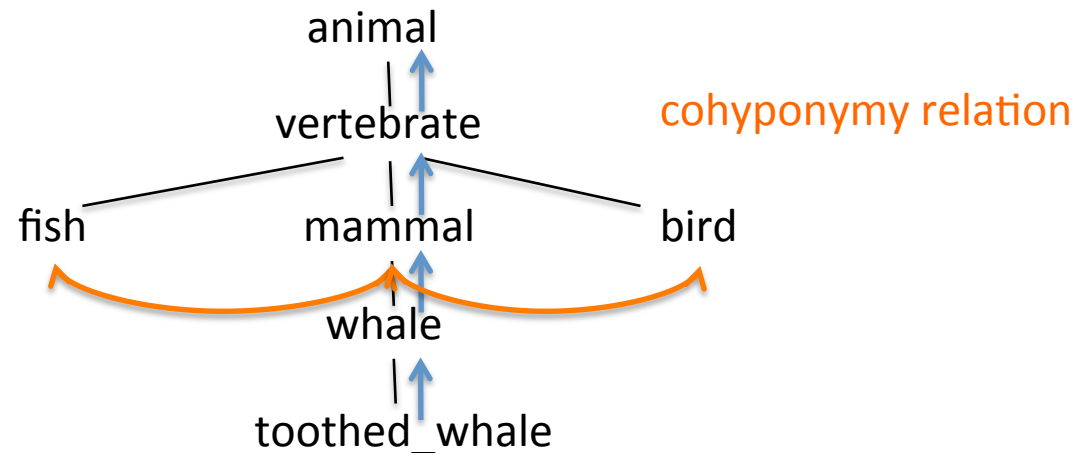
Axioms Expressing Semantic Relations

□ A and B cohyponyms $\Rightarrow \forall x(A(x) \rightarrow \neg B(x))$

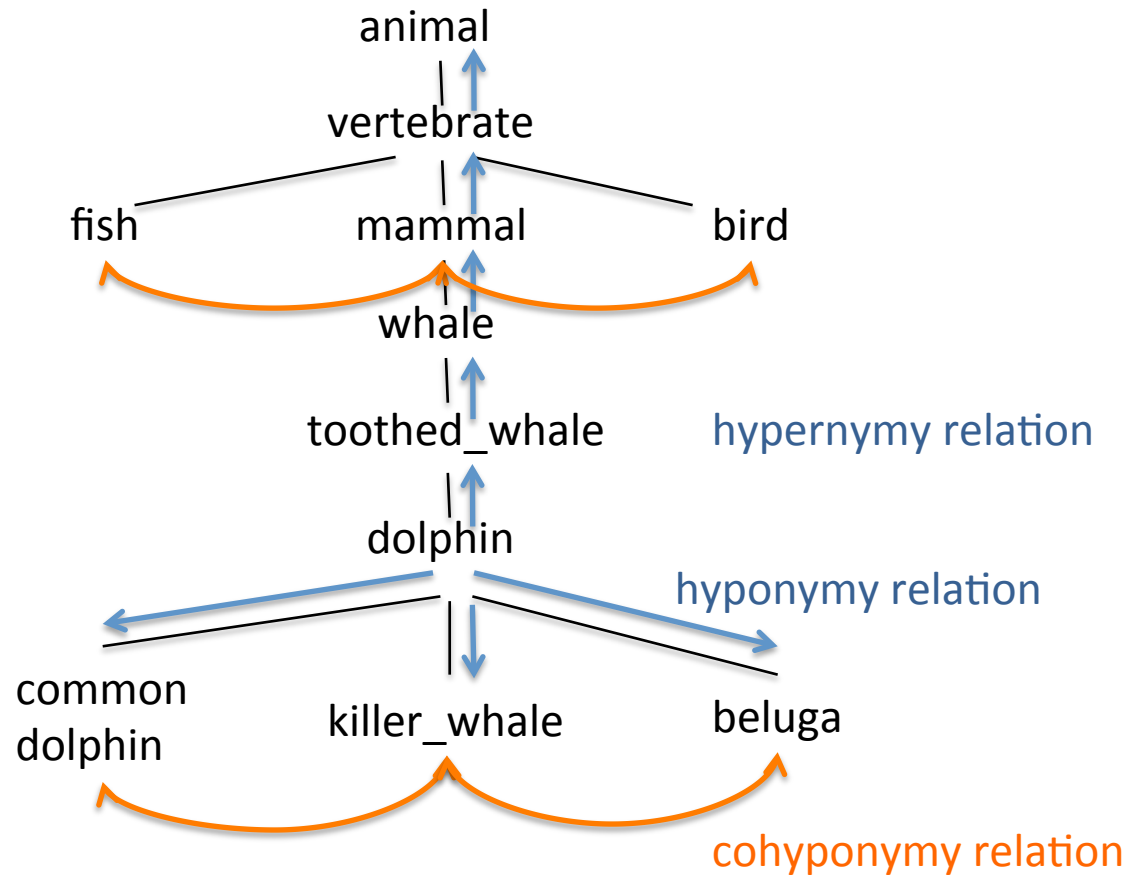
$\forall x(\text{mammal}(x) \rightarrow \neg \text{fish}(x))$

$\forall x(\text{fish}(x) \rightarrow \neg \text{bird}(x))$

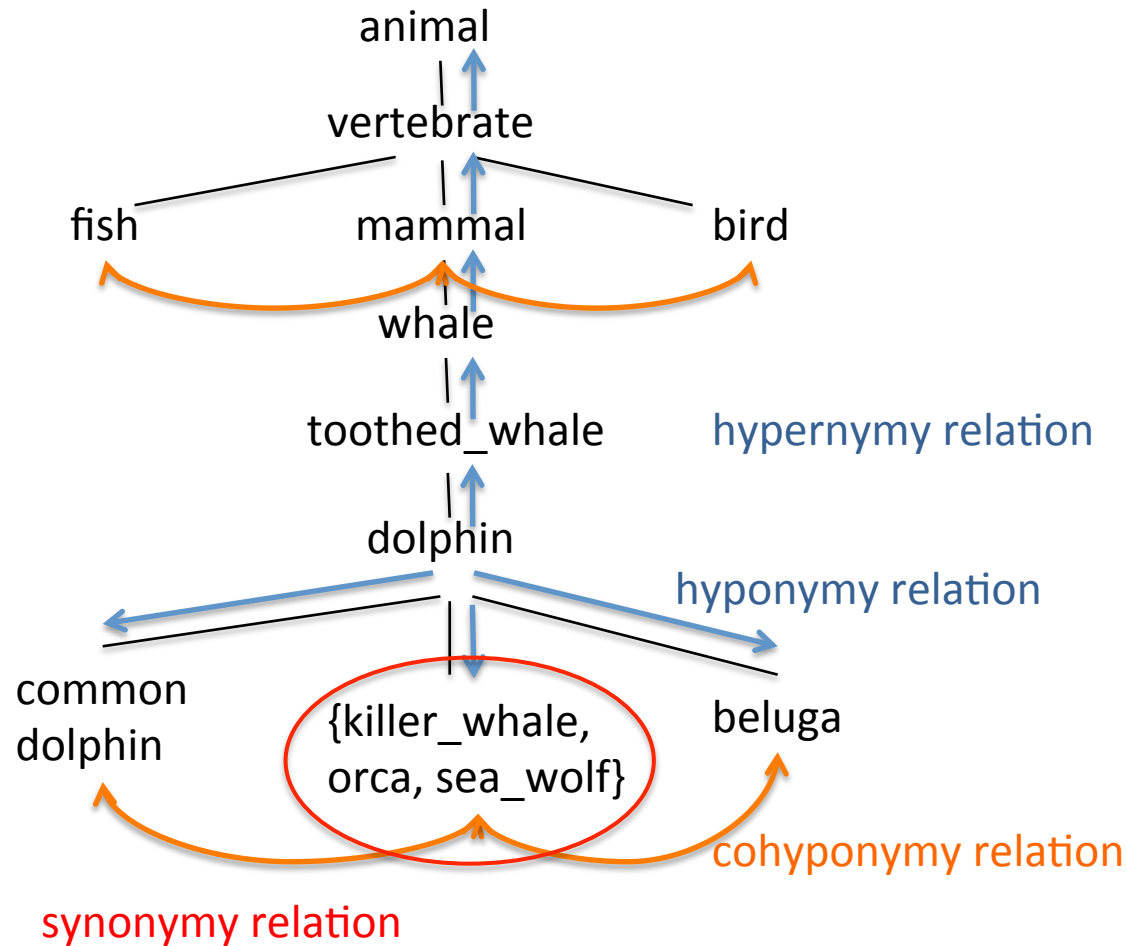
$\forall x(\text{bird}(x) \rightarrow \neg \text{mammal}(x))$



WordNet Meaning Relations



WordNet Meaning Relations



Axioms Expressing Semantic Relations

□ A and B cohyponyms $\Rightarrow \forall x(A(x) \rightarrow \neg B(x))$

$\forall x(\text{mammal}(x) \rightarrow \neg \text{fish}(x))$

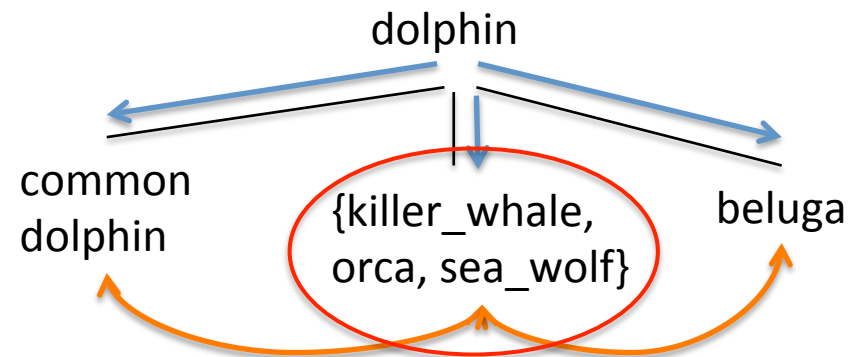
$\forall x(\text{fish}(x) \rightarrow \neg \text{bird}(x))$

$\forall x(\text{bird}(x) \rightarrow \neg \text{mammal}(x))$

□ A and B synonyms $\Rightarrow \forall x(A(x) \leftrightarrow B(x))$

$\forall x(\text{killer_whale}(x) \leftrightarrow \text{orca}(x))$

$\forall x(\text{killer_whale}(x) \leftrightarrow \text{sea_wolf}(x))$



synonymy relation

Semantics: The Logical Paradigm

- ❑ Validation of semantic representations via truth-conditional interpretation
- ❑ Semantically controlled inference through entailment and deduction
- ❑ A rigid model of compositionality

Semantic Composition

Principle of Compositionality (Frege's Principle):

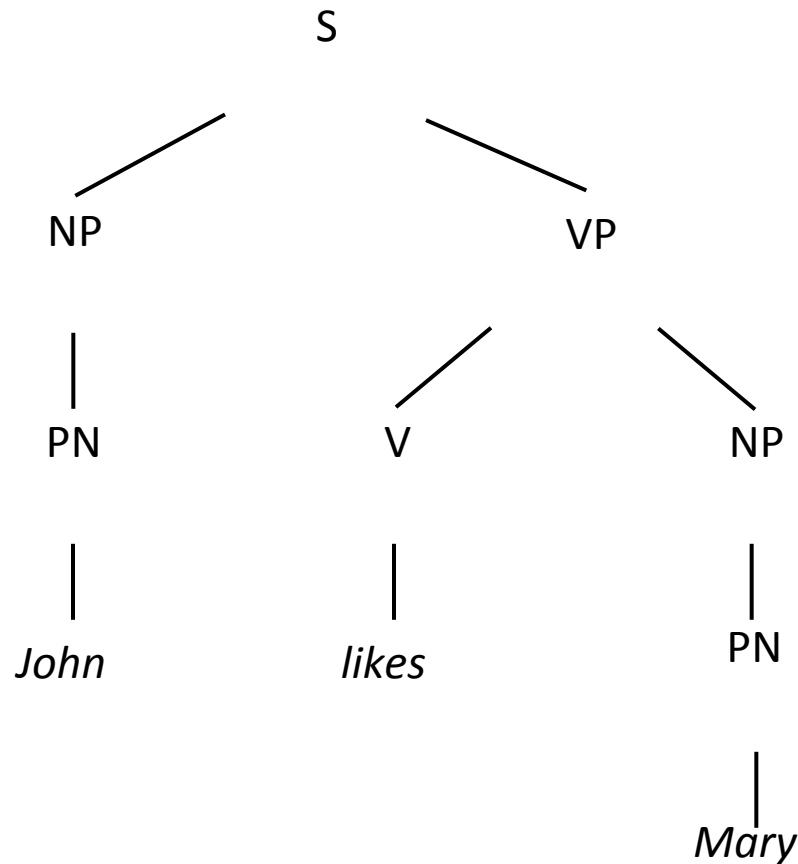
- The meaning of a complex expression is uniquely determined by the meanings of its sub-expressions and its syntactic structure.

Semantic Composition

□ *John likes Mary* \Rightarrow like(john, mary)

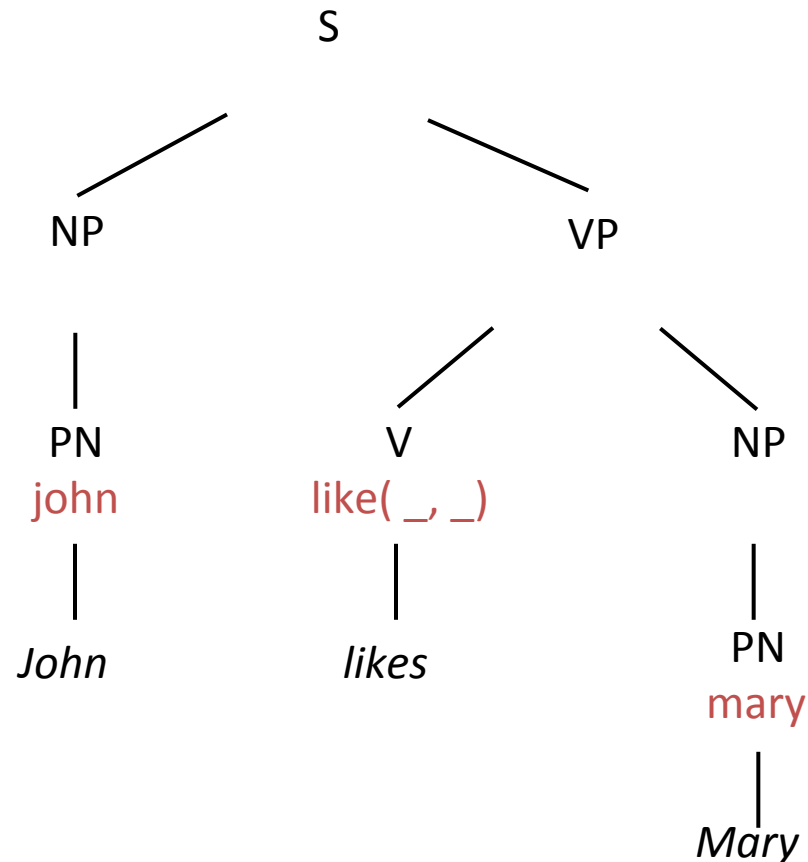
Semantic Composition

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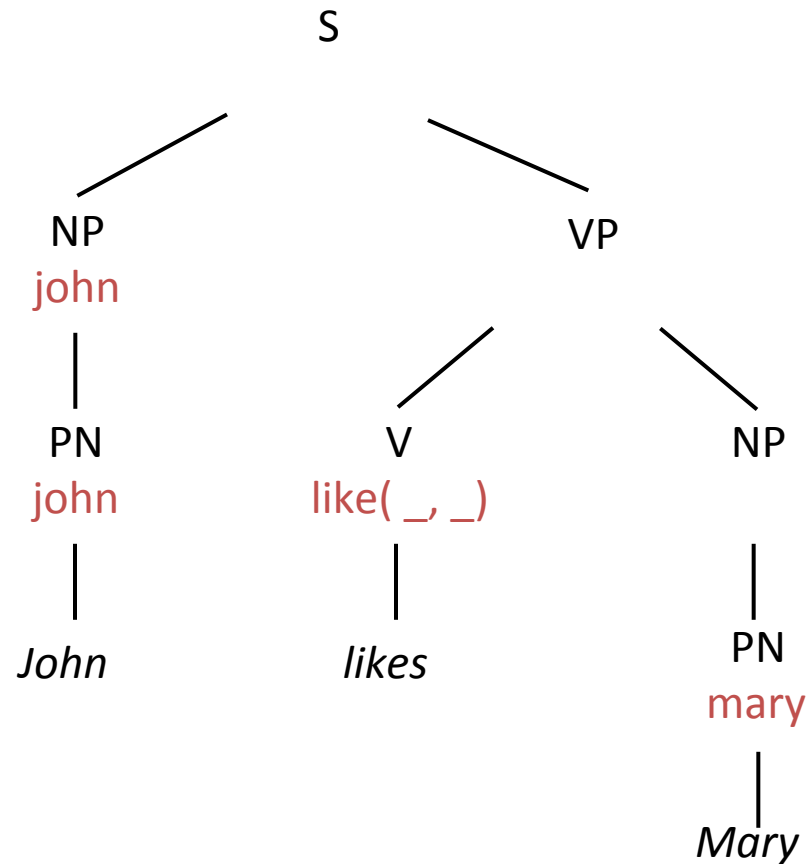
Semantic Composition

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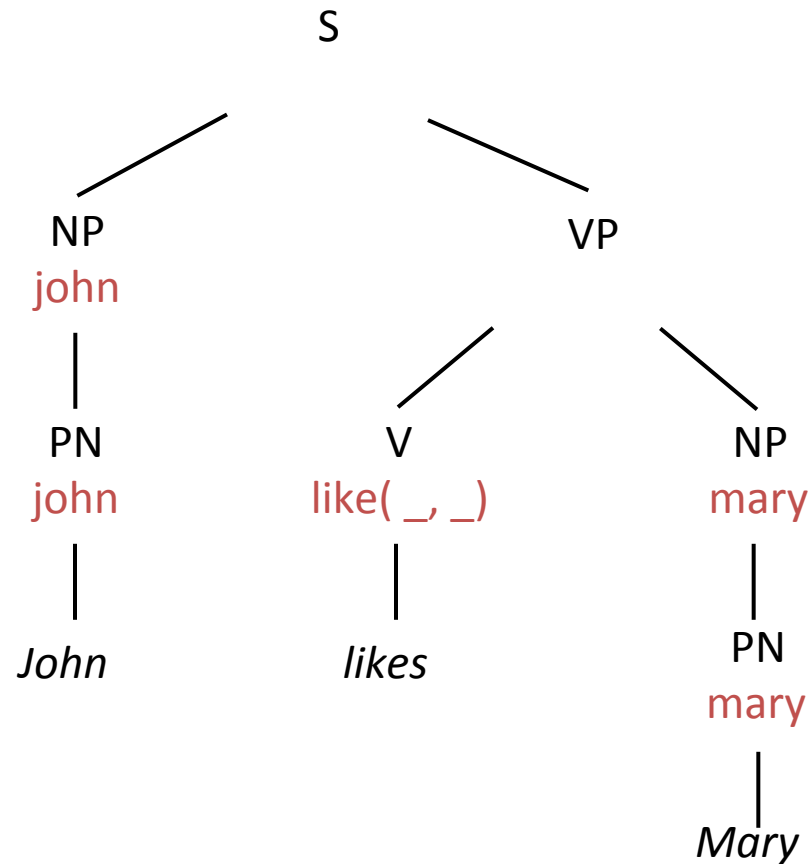
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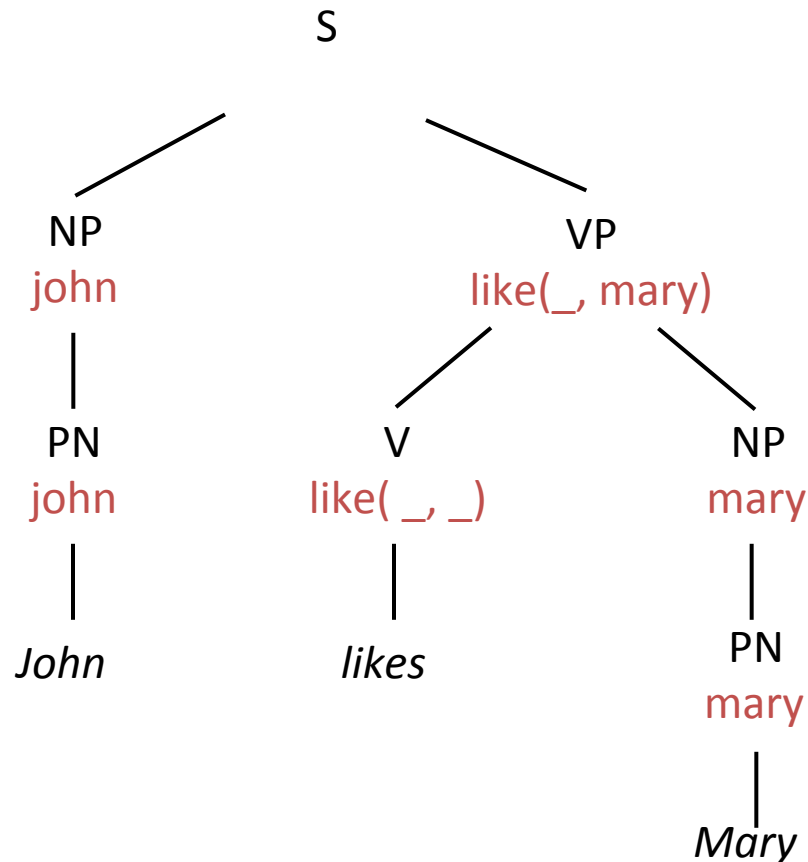
Semantic Composition

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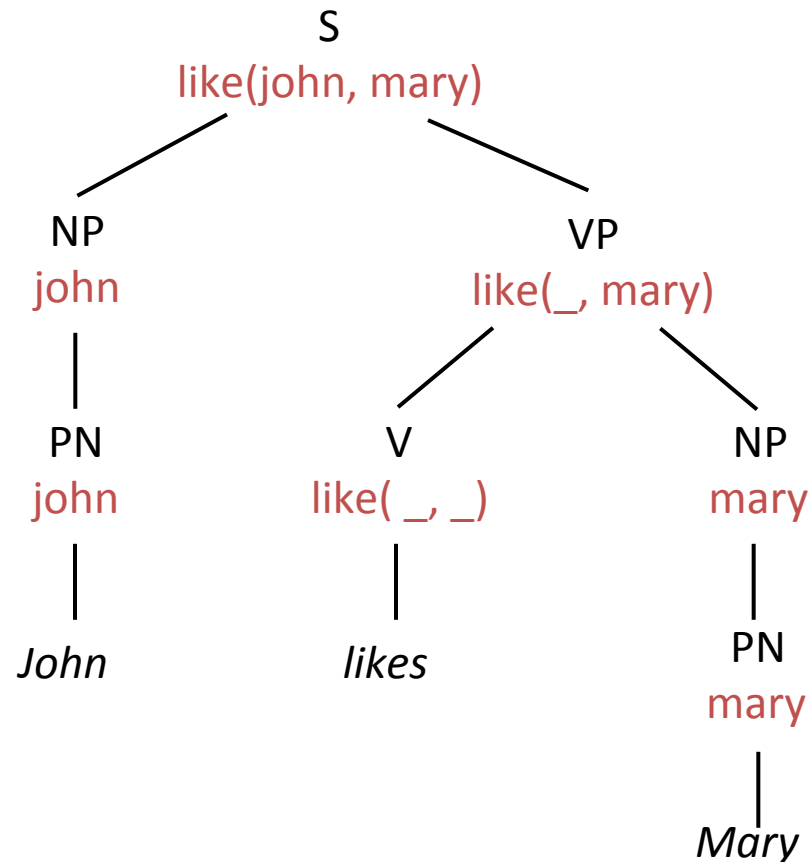
Semantic Composition

□ *John likes Mary* \Rightarrow like(john, mary)



Semantic Composition

□ *John likes Mary* \Rightarrow like(john, mary)



Semantic Composition

- ❑ How do meanings of syntactic complements find their appropriate argument positions in the composition process?
- ❑ The answer is: λ -Abstraction

Mary

λ -Abstraction

- ❑ **student**: a one-place predicate
- ❑ **student(x)**: a formula containing a free variable
- ❑ **$\lambda x[\text{student}(x)]$** : a one-place-predicate again: „to be a student“
- ❑ **$\lambda x[\text{student}(x)](\text{john})$** : a formula: application of a one-place predicate (the λ -expression) to the individual constant "john",
- ❑ which is equivalent to **student(john)**

Interpretation of λ -expressions

$$\square \llbracket \lambda x A \rrbracket^{M,g} = \{a \in U_M \mid \llbracket A \rrbracket^{M,g[x/a]} = 1\}$$

$$\square \llbracket \lambda x [\text{student}(x)] \rrbracket^{M,g} = \{a \in U_M \mid \llbracket \text{student}(x) \rrbracket^{M,g[x/a]} = 1\}$$
$$= \{a \in U_M \mid a \in V_M(\text{student})\}$$

i.e., the set of individuals who are students,
that is $V_M(\text{student})$

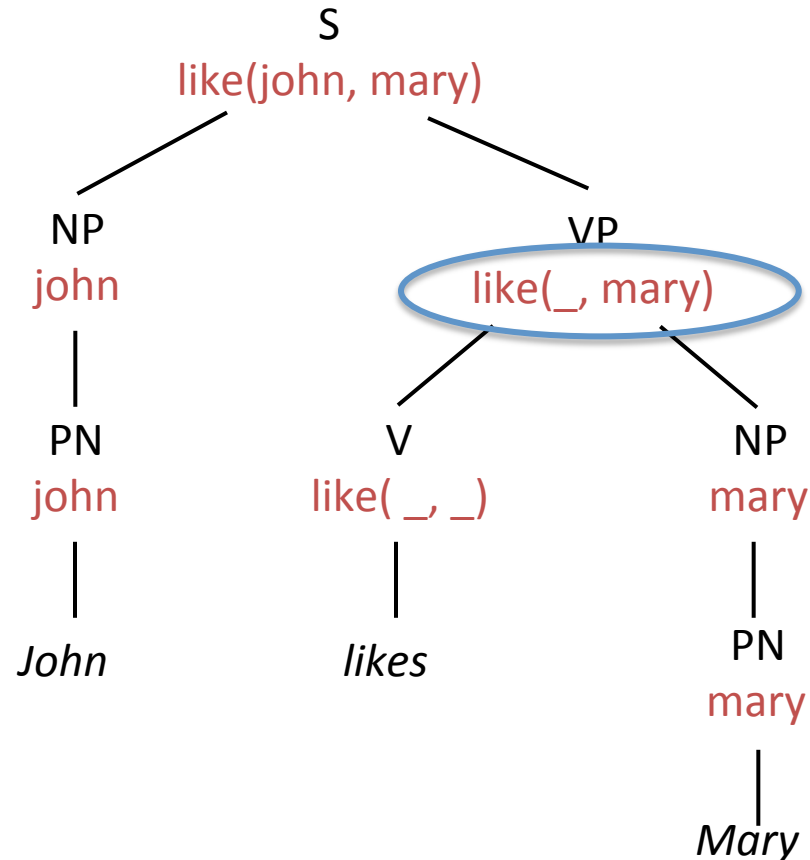
$$\square \llbracket \lambda x [\text{like}(x, \text{mary})] \rrbracket^{M,g} = \{a \in U_M \mid \llbracket \text{like}(x, \text{mary}) \rrbracket^{M,g[x/a]} = 1\}$$
$$= \{a \in U_M \mid \langle a, V_M(\text{mary}) \rangle \in V_M(\text{like})\},$$

i.e., the set of individuals who like Mary.

This is not necessarily identical to the denotation
of any predicate constant.

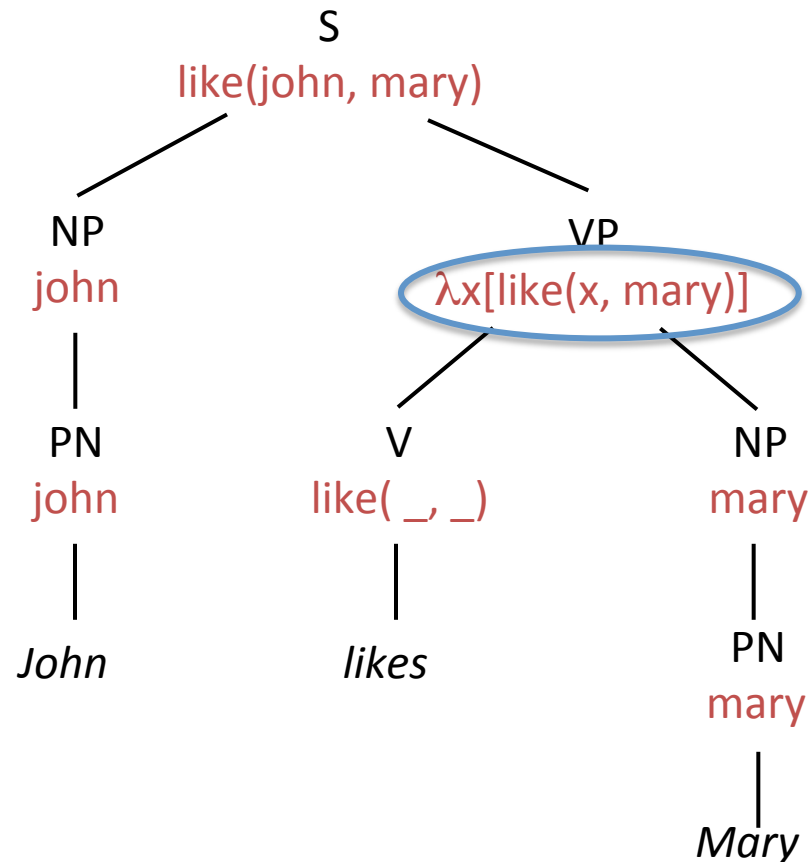
Semantic Composition

□ *John likes Mary* \Rightarrow like(john, mary)



Semantic Composition

□ *John likes Mary* \Rightarrow like(john, mary)



Application of λ -Expressions

John \Rightarrow john

likes Mary $\Rightarrow \lambda x[\text{like}(x, \text{mary})]$

John likes Mary $\Rightarrow \lambda x[\text{like}(x, \text{mary})](\text{john})$

$\Leftrightarrow \text{like}(\text{john}, \text{mary})$

$\llbracket \lambda x[\text{like}(x, \text{mary})](\text{john}) \rrbracket^{M,g} = 1$

iff $\llbracket \text{john} \rrbracket^{M,g} \in \llbracket \lambda x[\text{like}(x, \text{mary})] \rrbracket^{M,g}$

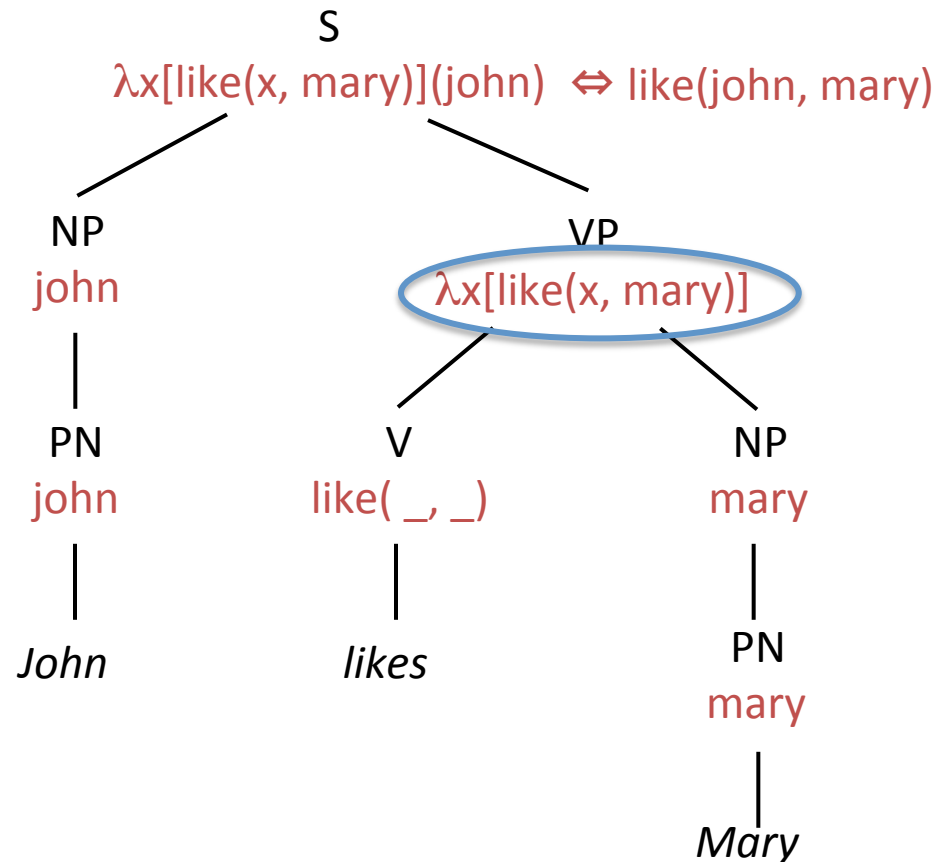
iff $V_M(\text{john}) \in \{a \in U_M \mid \langle a, V_M(\text{mary}) \rangle \in V_M(\text{like})\}$

iff $\langle V_M(\text{john}), V_M(\text{mary}) \rangle \in V_M(\text{like})$

iff $\llbracket \text{like}(\text{john}, \text{mary}) \rrbracket^{M,g} = 1$

Semantic Composition

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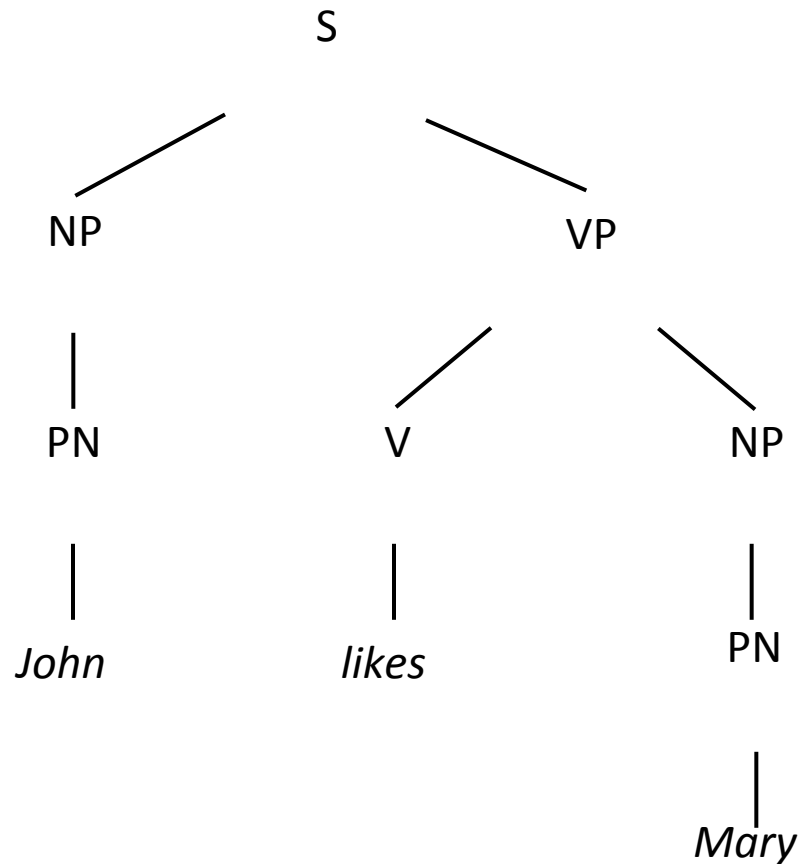


λ -Conversion

- $\lambda x[\text{student}(x)](\text{john})$ and $\text{student}(\text{john})$ are equivalent, and so are $\lambda x[\text{like}(x, \text{mary})](\text{john})$ and $\text{like}(\text{john}, \text{mary})$.
- In general: $\lambda xA(b) \Leftrightarrow A[x/b]$, where $A[x/b]$ is the result of replacing all free occurrences of variable x in A with b . This equivalence holds independent of the choice of A and b .
- Thus, we can rewrite any application of a λ -expression λxA to an argument b by the result of substituting all free occurrences of the λ -variable x in A with b (without considering truth conditions).
- $\lambda xA(b) \Rightarrow A[x/b]$ as a rewrite rule is called the rule of λ -conversion or λ -reduction.

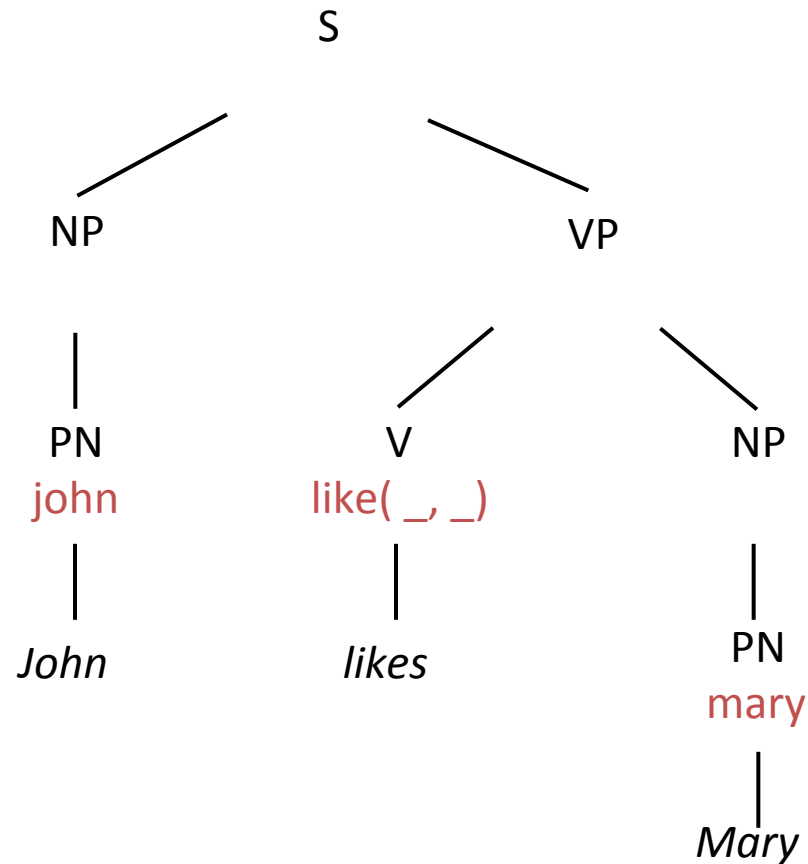
Semantic Composition

□ *John likes Mary* \Rightarrow like(john, mary)



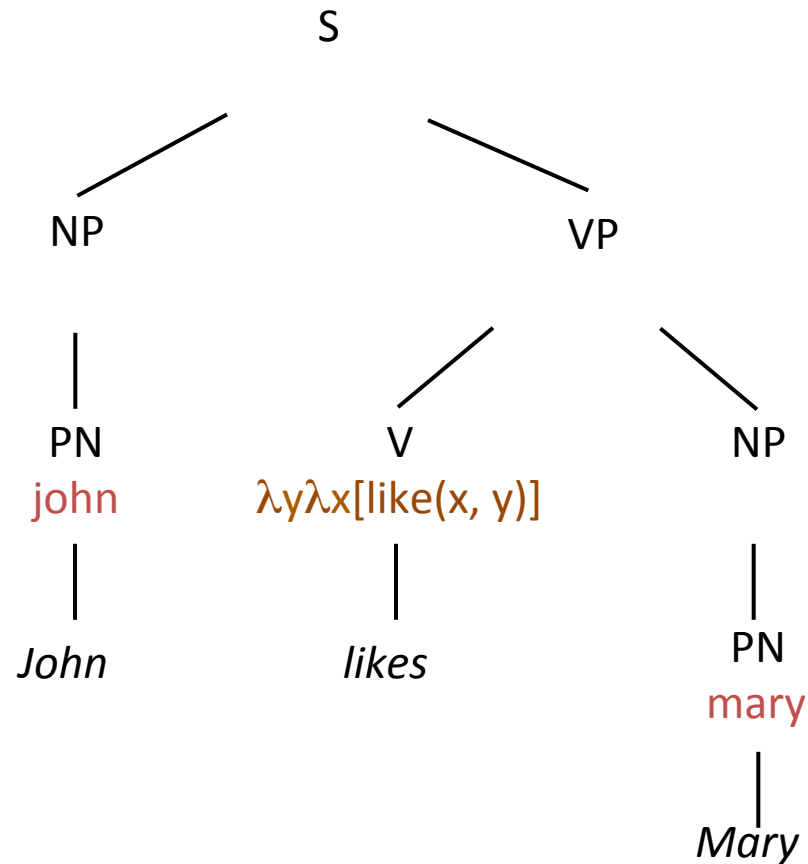
Semantic Composition

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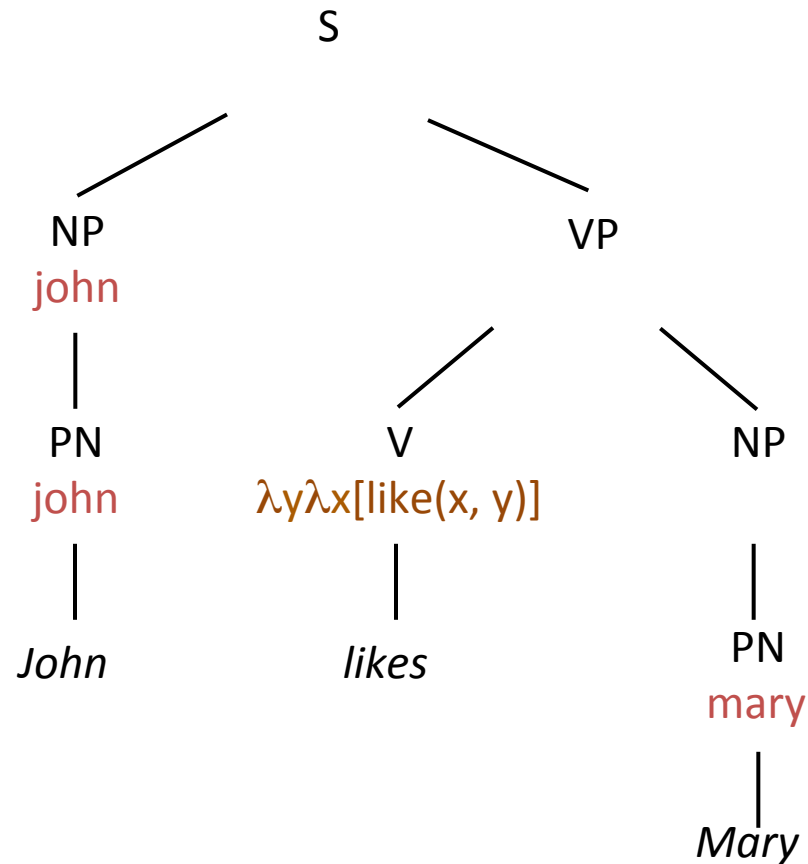
Semantic Composition: Lexical Information

□ *John likes Mary* \Rightarrow like(john, mary)



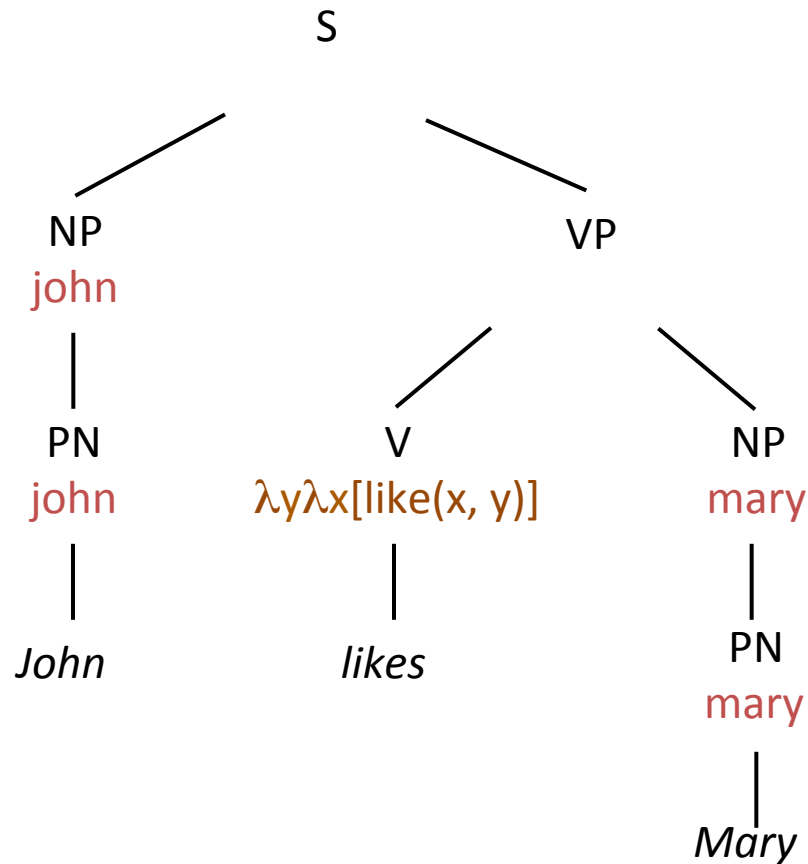
Semantic Composition: Projection

□ *John likes Mary* \Rightarrow like(john, mary)



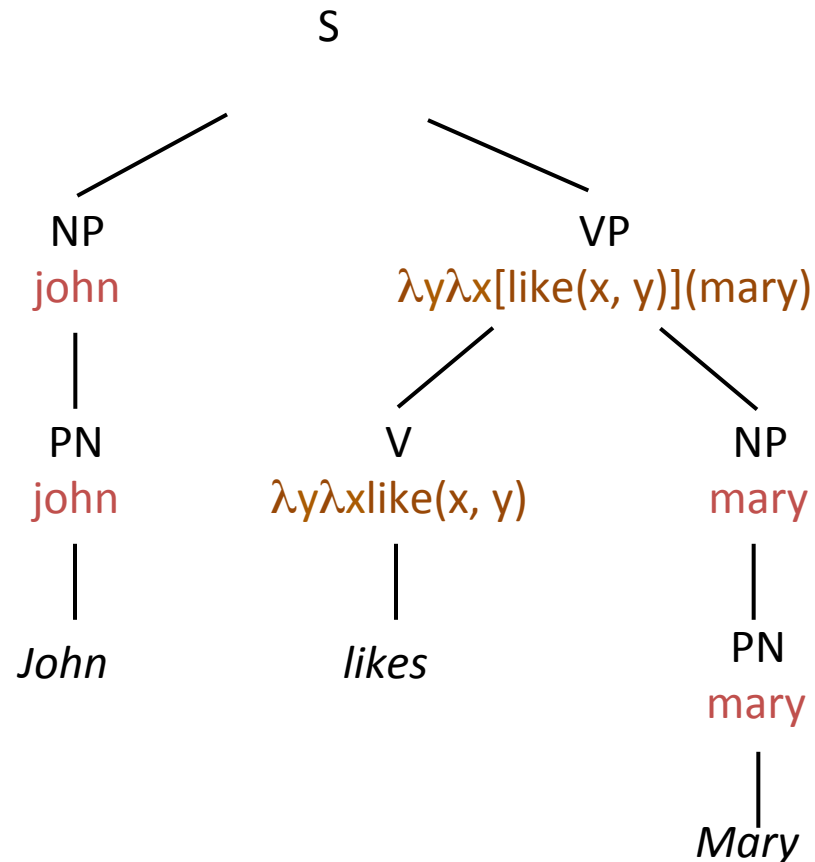
Semantic Composition: Projection

□ *John likes Mary* \Rightarrow like(john, mary)



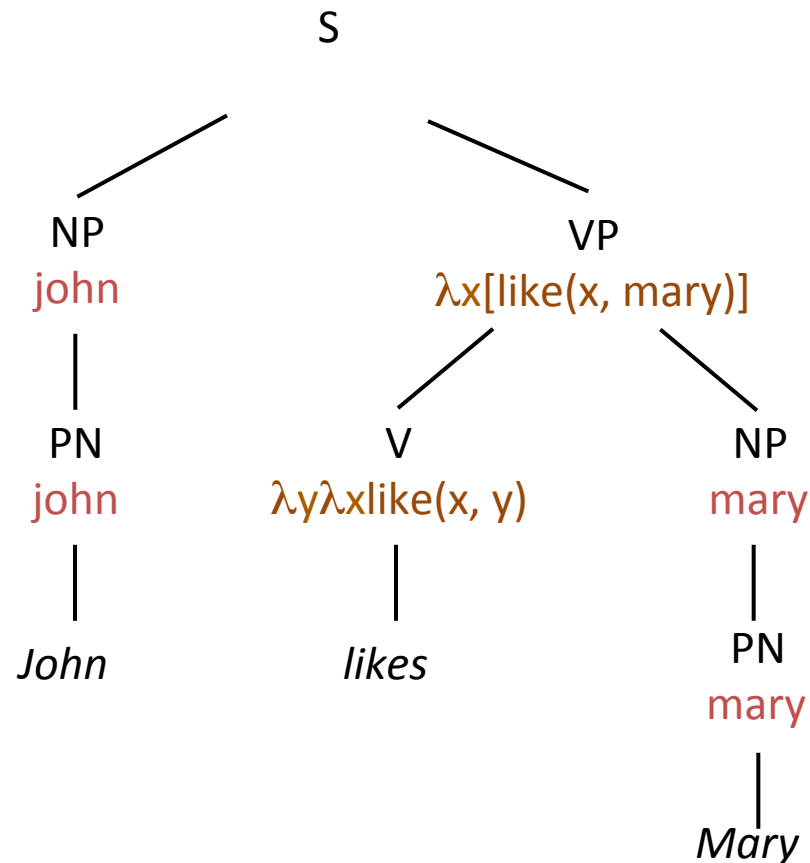
Semantic Composition: Application

□ *John likes Mary* \Rightarrow like(john, mary)



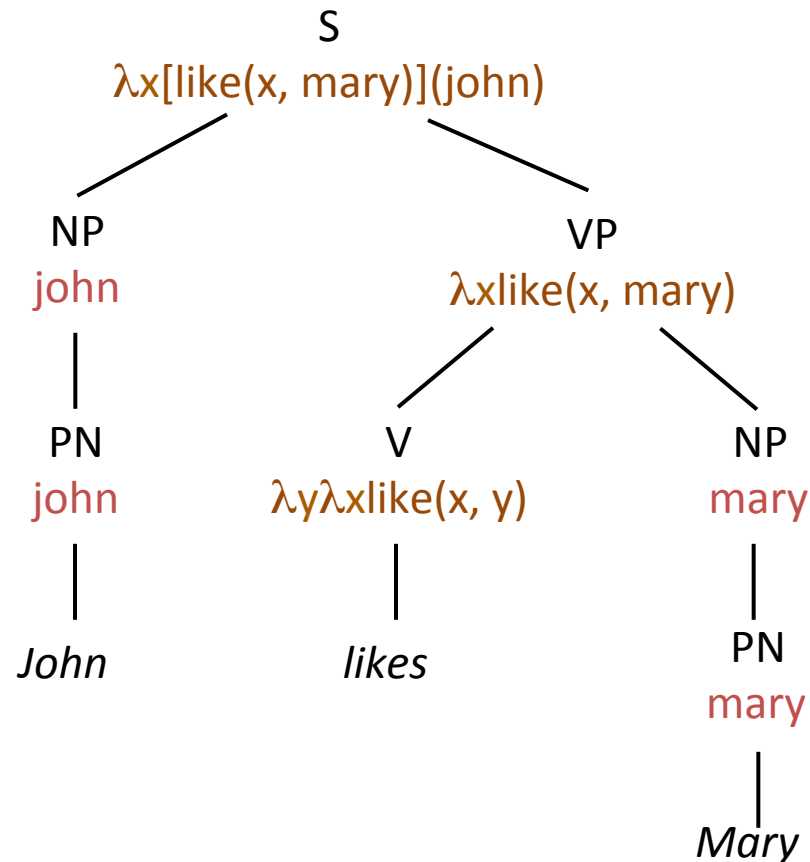
Semantic Composition: Reduction

□ *John likes Mary* \Rightarrow like(john, mary)



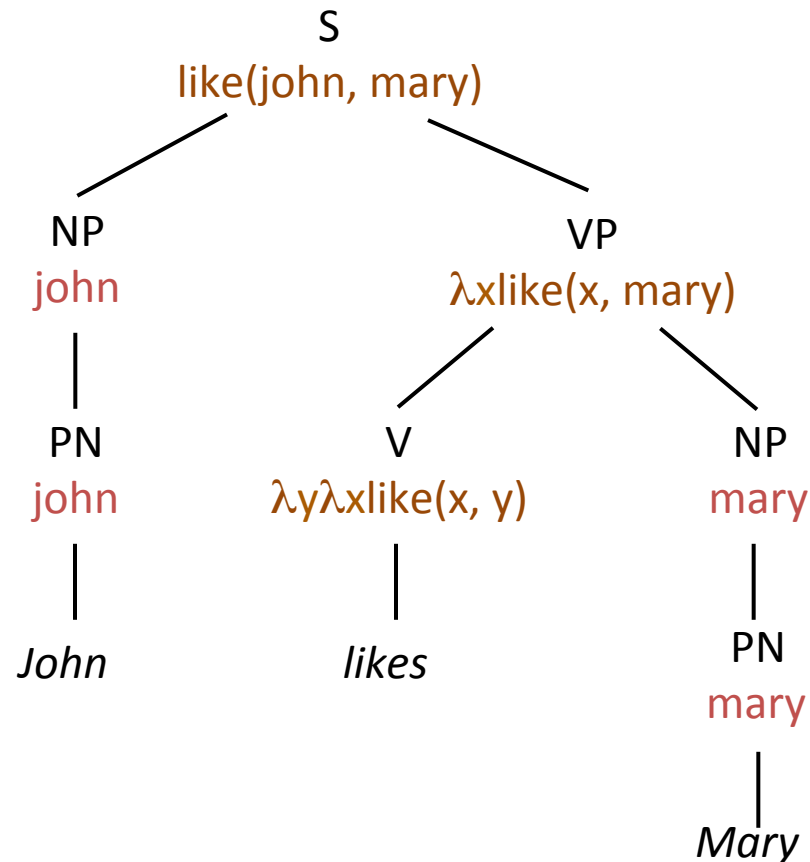
Semantic Composition: Application

□ *John likes Mary* \Rightarrow like(john, mary)



Semantic Composition: Reduction

□ *John likes Mary* \Rightarrow like(john, mary)



More λ -Expressions

“to like Mary”

$\lambda x[\text{like}(x, \text{mary})]$

“to be liked by Mary”

$\lambda x[\text{like}(\text{mary}, x)]$

“to like oneself”

$\lambda x[\text{like}(x, x)]$

“to sing and dance”

$\lambda x[\text{sing}(x) \wedge \text{dance}(x)]$

“to be somebody, whom everyone likes”

$\lambda x[\forall y \text{ like}(y, x)]$