FLST: Semantics I

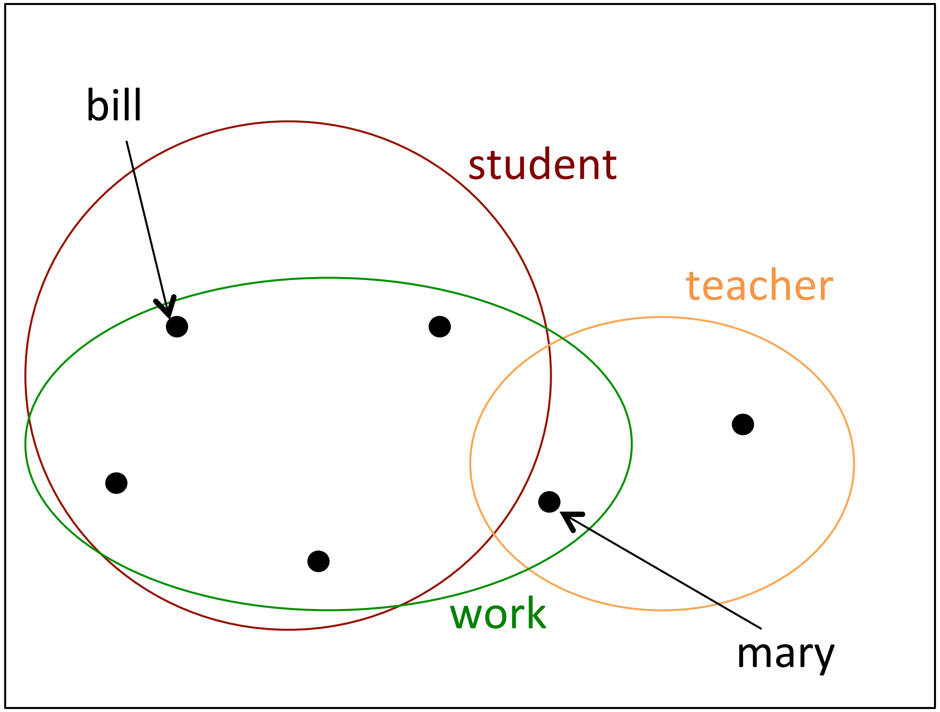
Vera Demberg
vera@coli.uni-sb.de

http://www.coli.uni-saarland.de/courses/FLST/2011
(all slides based on earlier years' slides by Manfred Pinkal)

Predicate Logic – Interpretation

- FOL expressions are interpreted with respect to certain situations or states of the world.
- These are schematically represented by relational structures which we call model structures.
- Different types or FOL expressions (terms, relation symbols, formulae) are assigned appropriate constructs from the model structure, their "denotations", by an interpretation function.
- In particular, formulae denote truth values.
- The truth conditions of a formula are considered its meaning.

Model Structure, Example



Model Structures

- Model structure: $M = \langle U_M, V_M \rangle$
  - U_M is non-empty set – the “universe”
  - V_M is an interpretation function assigning individuals ($\in U_M$) to individual constants and n-ary relations over U_M to n-place predicate symbols:
    - $V_M(P) \subseteq U_M^n$ if P is an n-place predicate symbol
    - $V_M(c) \in U_M$ if c is an individual constant
- Assignment function for variables $g: VAR \rightarrow U_M$

Computing Truth Conditions (1)

- Input sentence is: *"Bill works"*

- Semantic construction returns the formula work(bill)

- Predicate logic interpretation gives the truth conditions:

  $[\![\text{work(bill)}]\!]^{M,g} = 1$

  iff $[\![\text{bill}]\!]^{M,g} \in V_M(\text{work})$

  iff $V_M(\text{bill}) \in V_M(\text{work})$

- "work(bill)" is true in a model structure M iff the object denoted by "bill" in M is member of the set denoted by "work" in M

Computing Truth Values (1)

$[\![work(bill)]\!]^{M,g} = 1$ iff $V_M(bill) \in V_M(work)$

Let M=M1:

$V_{M1}(bill) \in V_{M1}(work)$,

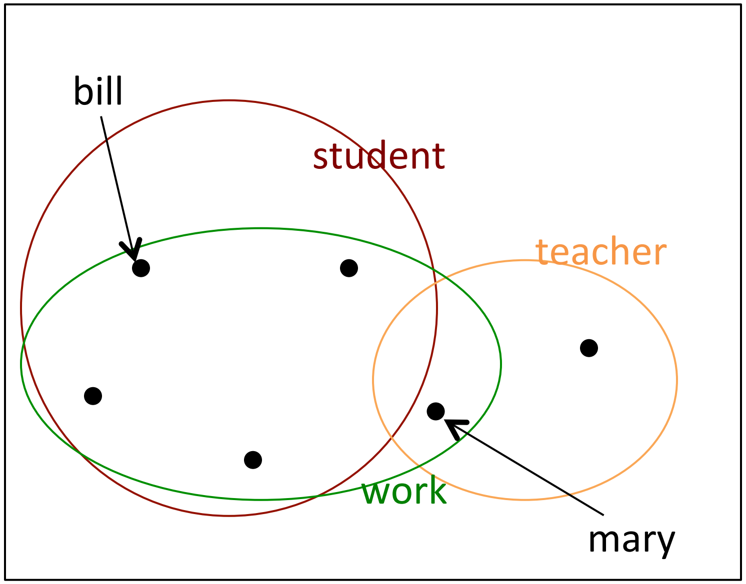
so $[\![work(bill)]\!]^{M1,g} = 1$

Let M=M2:

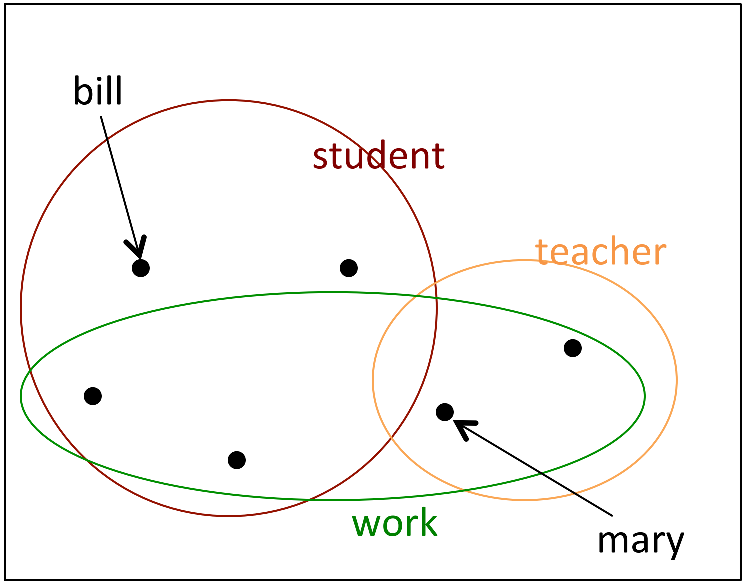
$V_{M2}(bill) \notin V_{M2}(work)$,

so $[\![work(bill)]\!]^{M2,g} = 0$

M1



M2



Computing Truth Conditions (2)

- *"Every student works"* $\Rightarrow \forall x(\text{student}(x) \rightarrow \text{work}(x))$
- $[\![\forall x(\text{student}(x) \rightarrow \text{work}(x))]\!]^{M,g} = 1$

  iff $[\![\text{student}(x) \rightarrow \text{work}(x)]\!]^{M,g[x/a]} = 1$ for every $a \in U_M$

  $[\![\text{student}(x)]\!]^{M,g[x/a]} = 0$ or $[\![\text{work}(x)]\!]^{M,g[x/a]} = 1$

  iff $[\![x]\!]^{M,g[x/a]} \notin V_M(\text{student})$ or $[\![x]\!]^{M,g[x/a]} \in V_M(\text{work})$

  iff $g[x/a](x) \notin V_M(\text{student})$ or $g[x/a](x) \in V_M(\text{work})$

  iff $a \notin V_M(\text{student})$ or $a \in V_M(\text{work})$
- $\forall x(\text{student}(x) \rightarrow \text{work}(x))$ is true in M iff for every $a \in U_M$

  $a \notin V_M(\text{student})$ or $a \in V_M(\text{work})$
- which is equivalent to: $V_M(\text{student}) \subseteq V_M(\text{work})$

Computing Truth Values (2)

$[\![\forall x(\text{student}(x) \rightarrow \text{work}(x))]\!]^{M,g} = 1$ iff $V_M(\text{student}) \subseteq V_M(\text{work})$

Let M=M1:

$V_{M1}(\text{student}) \subseteq V_{M1}(\text{work})$,

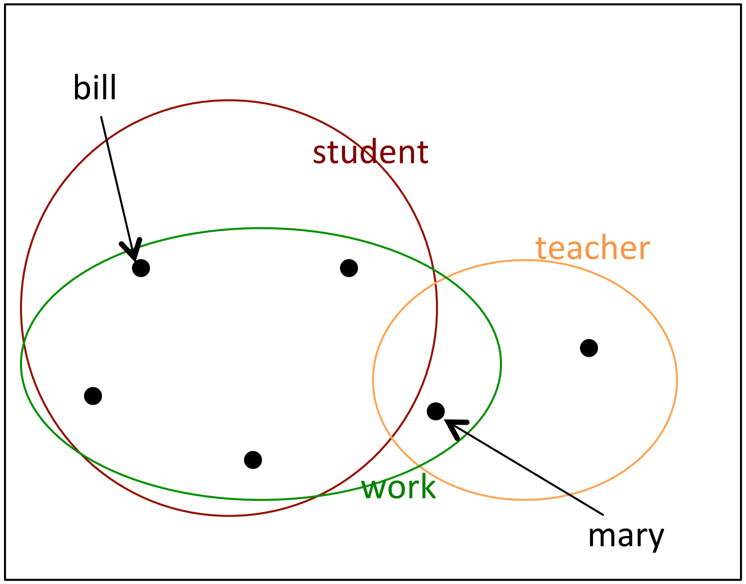
so $[\![\forall x(\text{student}(x) \rightarrow \text{work}(x))]\!]^{M1,g} = 1$

Let M=M2:

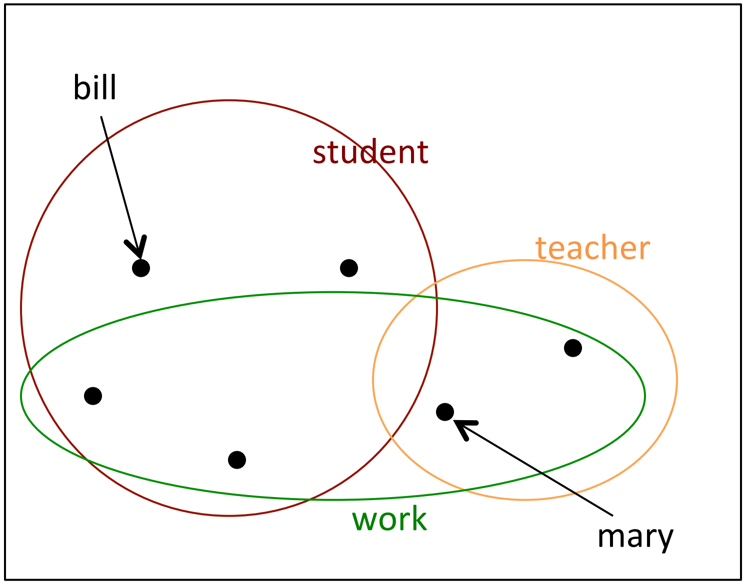
$V_{M2}(\text{student}) \not\subseteq V_{M2}(\text{work})$,

so $[\![\forall x(\text{student}(x) \rightarrow \text{work}(x))]\!]^{M2,g} = 0$

M1



M2



In-class exercise

"A child smiles."

- How can this be expressed in FOL?
- compute truth-conditions
- draw a model structure where this is correct
- specify the model structure formally
- draw a model world where this is incorrect (and specify it formally)

Reminder:

Interpretation of formulas with respect to a model structure M and variable assignment g:

- $[\![R(t_1, ..., t_n)]\!]^{M,g} = 1$ iff $\langle [\![t_1]\!]^{M,g}, \ldots, [\![t_n]\!]^{M,g} \rangle \in V_M(R)$
- $[\![t_1 = t_2]\!]^{M,g} = 1$ iff $[\![t_1]\!]^{M,g} = [\![t_2]\!]^{M,g}$
- $[\![\neg\varphi]\!]^{M,g} = 1$ iff $[\![\varphi]\!]^{M,g} = 0$
- $[\![\varphi \land \psi]\!]^{M,g} = 1$ iff $[\![\varphi]\!]^{M,g} = 1$ and $[\![\psi]\!]^{M,g} = 1$
- $[\![\varphi \lor \psi]\!]^{M,g} = 1$ iff $[\![\varphi]\!]^{M,g} = 1$ or $[\![\psi]\!]^{M,g} = 1$
- $[\![\varphi \rightarrow \psi]\!]^{M,g} = 1$ iff $[\![\varphi]\!]^{M,g} = 0$ or $[\![\psi]\!]^{M,g} = 1$
- $[\![\varphi \leftrightarrow \psi]\!]^{M,g} = 1$ iff $[\![\varphi]\!]^{M,g} = [\![\psi]\!]^{M,g}$
- $[\![\exists x\varphi]\!]^{M,g} = 1$ iff there is a $d \in U_M$ such that $[\![\varphi]\!]^{M,g[x/d]} = 1$
- $[\![\forall x\varphi]\!]^{M,g} = 1$ iff for all $d \in U_M$, $[\![\varphi]\!]^{M,g[x/d]} = 1$

The Most Certain Principle of Semantics

- "For two sentences A *and B, if in some possible situation A is true and B is false, A and B must have different meanings*." (M. Cresswell, 1975)

Applied to logical representations of NL sentences:

- For a logical formula α and a sentence A: If in some possible situation corresponding to a model structure M A is true, and α is not, or vice versa, then α is not an appropriate meaning representation for A.

Examples

- *Dolphins live in pods*

- $\forall d\ (dolphin(d) \rightarrow \exists p\ (pod(p) \wedge live\text{-}in\ (d,p)))$
- $\forall p\ (pod(p) \rightarrow \exists d\ (dolphin(d) \wedge live\text{-}in\ (d,p)))$

- *Germans drink beer*

Semantics: The Logical Paradigm

- Validation of semantic representations via truth-conditional interpretation
- Semantically controlled inference
- A model of compositionality

Truth, Validity, Entailment

- A formula A is true in model structure M
  iff $[\![A]\!]^{M,g} = 1$ for every variable assignment g.
- A formula A is valid ($\models A$)
  iff A is true in every model structure.
- A set of formulas Γ entails formula A ($\Gamma \models A$) iff A is true in in every model structure M in which all $A \in \Gamma$ are true
  - The members of Γ are called premises or hypotheses
  - A is called the conclusion

Determining Entailment

- student(bill), ∀x(student(x) → work(x)) $\overset{?}{\models}$ work(bill)
- For every M :
  student(bill) is true in M iff V_M(bill) ∈ V_M(student)
  ∀x(student(x) → work(x)) is true in M iff V_M(student) ⊆ V_M(work)
- From V_M(bill) ∈ V_M(student) and V_M(student) ⊆ V_M(work), it follows that V_M(bill) ∈ V_M(work) (basic set-theoretic inference)
- Now, V_M(bill) ∈ V_M(work) is just the truth condition for work(bill).
- Therefore: In every model structure M satisfying student(bill) and ∀x(student(x)→work(x)), the formula work(bill) is true: Valid entailment.

Entailment and Deduction

- We just have proved:

  $student(bill), \forall x(student(x) \rightarrow work(x)) \vDash work(bill)$

- We did this independent of the choice of non-logical constants. Thus, the result can be generalized to arbitrary instantiations of the following entailment scheme (b standing for any individual constant, and F for any one-place predicate).

  $F(b), \forall x(F(x) \rightarrow G(x)) \vDash G(b)$

Entailment and Deduction

- The proof of the entailment $F(b), \forall x(F(x) \rightarrow G(x)) \vDash G(b)$ justifies the safe application of the following deduction rule:

  $F(b), \forall x(F(x) \rightarrow G(x)) \vdash G(b)$

  which is actually one of the „Aristotelian syllogisms“

- Application of a deduction rule $\Gamma \vdash A$: Whenever we find a set of formulas in a (FOL) database that match Γ,
  - we may safely („salva veritate“) add the appropriate instantiation of A to the database. We may do this without looking into the semantic interpretation of the involved formulas.

Determining Entailment (2)

- $work(john) \rightarrow work(bill), \neg work(bill) \overset{?}{\models} \neg work(john)$
- To check entailment, it is sufficient to inspect four different variants of model structures, i.e., those where
  - $[\![work(john)]\!]^{M,g} = 1$ and $[\![work(bill)]\!]^{M,g} = 1$
  - $[\![work(john)]\!]^{M,g} = 1$ and $[\![work(bill)]\!]^{M,g} = 0$
  - $[\![work(john)]\!]^{M,g} = 0$ and $[\![work(bill)]\!]^{M,g} = 1$
  - $[\![work(john)]\!]^{M,g} = 0$ and $[\![work(bill)]\!]^{M,g} = 0$

Determining Entailment (2)

- With the truth-table method, we show that whenever $[\![work(john) \rightarrow work(bill)]\!]^{M,g} = [\![\neg work(bill)]\!]^{M,g} = 1$, also $[\![\neg work(john)]\!]^{M,g} = 1$: Valid entailment.

- The result is independent of the specific atomic formulas. Therefore we can generalise the result to:
  - $A \rightarrow B, \neg B \models \neg A$, which justifies $A \rightarrow B, \neg B \vdash \neg A$ as a deduction rule ("Modus Tollens").

Deduction Calculi

- Computing entailment via semantic interpretation is inefficient and in many cases infeasible.

- Deduction calculi (or proof theoretic systems) provide a strictly syntactic way of inference modeling, through rewrite of logical formulae.

- We say that A is derivable from a set of formulas Γ ($\Gamma \vdash A$) in a given deduction system, iff one can obtain A starting from Γ, by using deduction rules (and possibly axioms) of that deduction system.

Soundness and Completeness

- Truth-conditional interpretation of the logical formalism enable us to determine whether a given deduction system is
  - sound, i.e., derives only those formula A from a set of premises Γ which are entailed by Γ.
  - complete, i.e., allows to derive all formulas entailed by Γ.
- In short:
  - Soundness: If $\Gamma \vdash A$, then $\Gamma \vDash A$.
  - Completeness: If $\Gamma \vDash A$, then $\Gamma \vdash A$.
- Sound and complete deduction systems derive all and only the truth-conditionally entailed formulas from any set of premises.
- There are many possible deduction systems (choices between collections of deduction rules), but there is only one entailment concept for First-Order Predicate Logic.

Theorem Provers

- The problem of FOL entailment checking is very hard: It is even undecidable.

- However, automated deduction systems have been optimized through decades, and have become very efficient. They are called theorem provers, because their original motivation was mathematical theorem proving.