FLST WS 2014/15 – Semantics – Exercise sheet 1

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Exercises will be discussed on Friday, January 9th

1. Translate the following sentences to Predicate Logic. Try to retain as much of the structure as possible. Assume that "get from" translates to the predicate constant get_from. Ignore tense.

(1)   Mary has a rusty bicycle

(2)   Mary got a present from Bill, but she didn't get anything from John

(3)   Barking dogs don't bite

(4)   Only Bill passed

(5)   At most two students passed

2. Look at the following graphical representation of a model structure M with a universe made up of three entities. Denotations of a one-place predicate F and a two-place predicate R are indicated by the closed line and the dashed arrows, respectively.
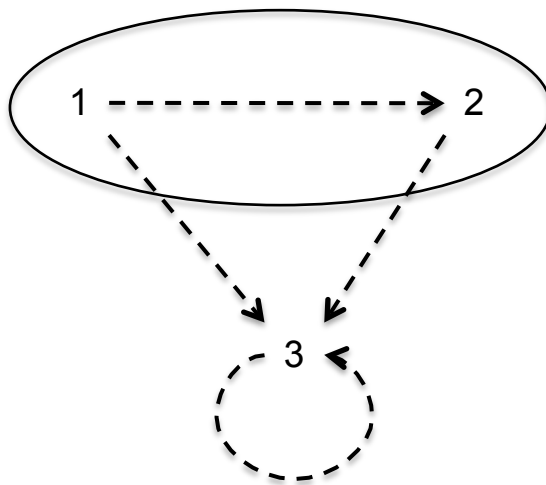
(a) Give a formal specification of the model structure.

(b) Compute the truth conditions for the following formulas, and determine the truth value for all of them.

(1) $\forall x(R(x,x) \rightarrow \neg F(x))$

(2) $\exists x \exists y(R(x,y) \wedge \neg F(x) \wedge \neg F(y))$

(3) $\forall x(R(x,x) \rightarrow \exists y(R(x,y) \wedge F(y)))$

3. Determine whether the following entailment relations hold or do not hold. To this purpose: Compute the truth conditions of the formulas occurring in premises and conclusions, and reformulate them in an appropriate way, if necessary. In positive entailment cases, give a set-theoretic proof/argument that the conclusion is true whenever the premise is true. In the negative cases, specify a model structure in which the premise is true, but the conclusion is not. (A, B stand for atomic formulas, F and G are one-place predicates.)

(7) $A \rightarrow B \models \neg A \rightarrow \neg B$

(8) $A \rightarrow B \models \neg B \rightarrow \neg A$

(9) $\forall x\, F(x) \models \exists x\, F(x)$

(10) $\forall x(F(x) \rightarrow G(x)) \models \exists x(F(x) \wedge G(x))$

4. Consider the following formulas:

(5) $\exists x\, get\_from(bill, x, mary) \rightarrow happy(bill)$

(6) $\exists x(get\_from(bill, x, mary) \rightarrow happy(bill))$

(7) $\forall x\, get\_from(bill, x, mary) \rightarrow happy(bill)$

(8) $\forall x(get\_from(bill, x, mary) \rightarrow happy(bill))$

(a) Compute the truth conditions for these four formulas. Which of them are equivalent? Which of them stand in an entailment relation?

(b) Which ones are appropriate representations of the following sentence, which ones aren't?

If Bill gets anything from Mary, he is happy.

Give arguments for your positive and negative decisions.