



Finite State Automata

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including some slides by Julia Hockenmaier (Upenn)
and Reinhard Völler (U. Hamburg)



Idea

- A finite state automaton is a mathematical model of a system with in- and output
- Such a system is always in one of several possible states
- Examples:
 - A circuit with n gates always is in a certain state
 - Text editors or lexical analyzers of compilers can be modelled as FSA
 - Every computer is a FSA. However, this view is not very useful because of the large number of states.

Automata



- Automata are used, to test, analyze and process sequences of symbols or actions
- E.g. for the processing of user input

- With the help of automata one can test, whether
 - The input is correct
 - it 's components come in the right order
 - functions have the right number an type of arguments
 - Etc

Components of an automaton



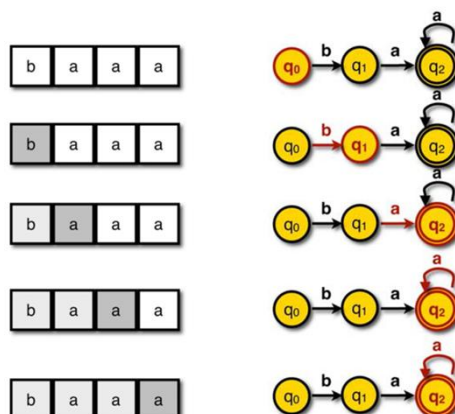
- Finite set of states
- One start state, one or more final states
- input: word of the language
- output: word of the language
- rules for all types of inputs
- Important! These rules have to be complete, all possible types of input have to be accounted for.
- An error state may be defined

Formal Definition



- A deterministic finite state automaton is a quintuple
 $A = (Z, E, \delta, z_0, F)$
- Z Set of States
- E Set of input symbols
- $\delta: Z \times E \rightarrow Z$ state transition functions
- $z_0 \in Z$ start state
- $F \subseteq Z$ Set of final states

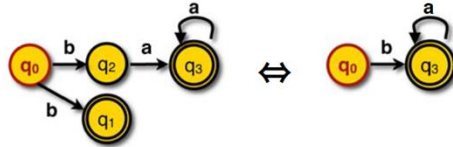
Example



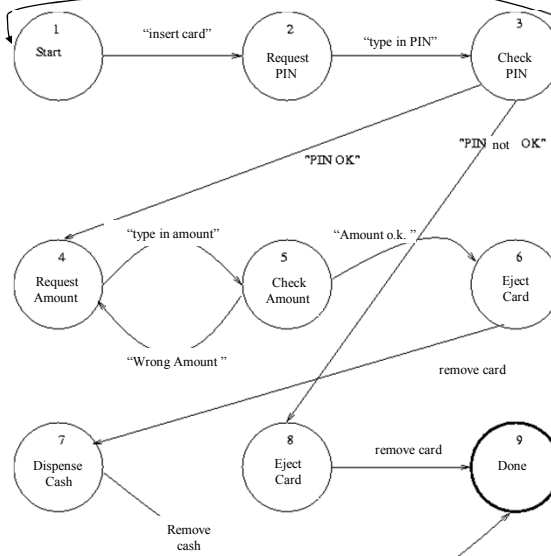
Finite State Automata (FSAs)



- Every NFA can be transformed into an equivalent DFA:



- Recognition of a string w with a DFA is linear in the length of w
- Finite-state automata define the class of regular languages
 - $L_1 = \{anbm\} = \{ab, aab, abb, aaab, abb, \dots\}$ is a regular language,
 - $L_2 = \{anbn\} = \{ab, aabb, aaabbb, \dots\}$ is not (it's context-free).
- You can't construct an FSA that accepts all the strings in L_2 and nothing else.



Example 1



□ Name List:

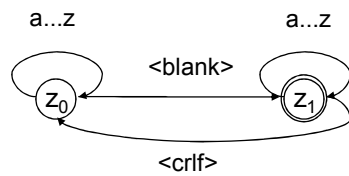
Peter Müller

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Doris Steckler



	z₀	z₁
a	z ₀	z ₁
.	z ₀	z ₁
.	z ₀	z ₁
.	z ₀	z ₁
z	z ₀	z ₁
<blank>	z ₁	
<crif>		z ₀

Definition



□ A nondeterministic finite state automaton is a quintuple
 $A = (Z, E, \delta, z_0, F)$

- Z Set of States
- E Set of input symbols
- $\delta: Z \times E \rightarrow 2^Z$ state transition functions
- $z_0 \in Z$ start state
- $F \subseteq Z$ Set of final states

Automata with output

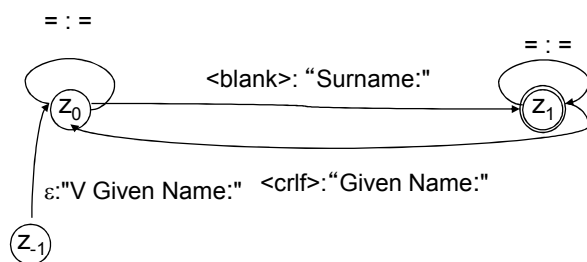


- Ein Mealey-Automaton is a sextuple $A = (Z, E, A, \delta, z_0, \lambda)$
- Z Set of States
- E Set of input symbols
- A Set of output symbols
- $\delta: Z \times E \rightarrow Z$ state transition functions
- $\lambda: Z \times E \rightarrow A$ output function
- $z_0 \in Z$ start state
- $F \subseteq Z$ Set of final states

Example 1a



	z_{-1}	z_0	z_1
ϵ : "Given Name:"	z_0		
$= : =$		z_0	z_1
<blank>: "Surname:"		z_1	
<crLf> : "Given Name:"			z_0



Example 2



☐ Telefonliste

Abbecker, Marion	0681/343239
Becker, Klaus	0631/4457328
Bruck, Günter	06897/45322
.	
.	
.	
Zeltmacher, Doris	06898/3496

Example 2



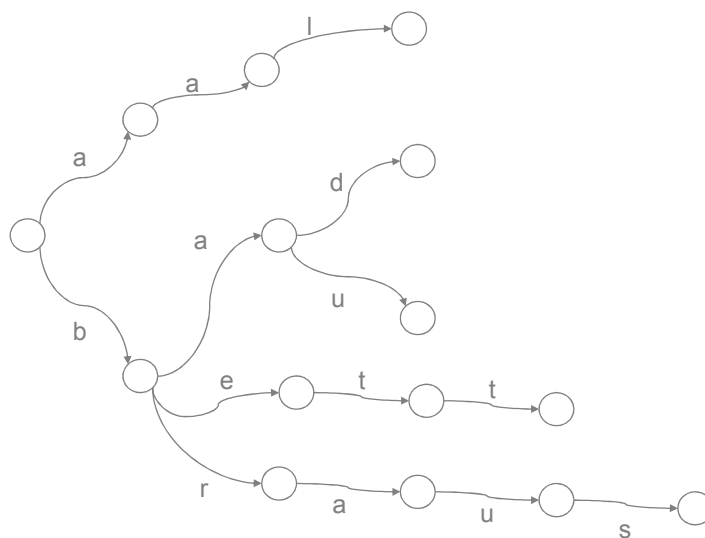
Given Name: Marion Surname: Abbecker
Given Name: Klaus Surname: Becker
Given Name: Günter Surname: Bruck

Morphology



- Inflection creates different forms of the same word:
 - Verbs: to be, being, I am, you are, he is, I was,
 - Nouns: one book, two books
- Derivation creates different words from the same lemma:
 - Grace, disgrace, disgraceful, disgracefully
- Compounding combines two words into a new word:
 - Cream, ice cream, ice cream cone, ice cream cone bakery
 - Eis, Speiseeis, Erdbeerspeiseeis Erdbeersspeiseeiskugel, Erdbeersspeiseeiskugelverkäufer
- Word formation is productive: New words are subject to all of these processes:
 - Google: Googler, to google, to ungoogle, to misgoogle, googlification,
 - ungooglification, googlified, Google Maps, Google Maps service,...

Letter Tree

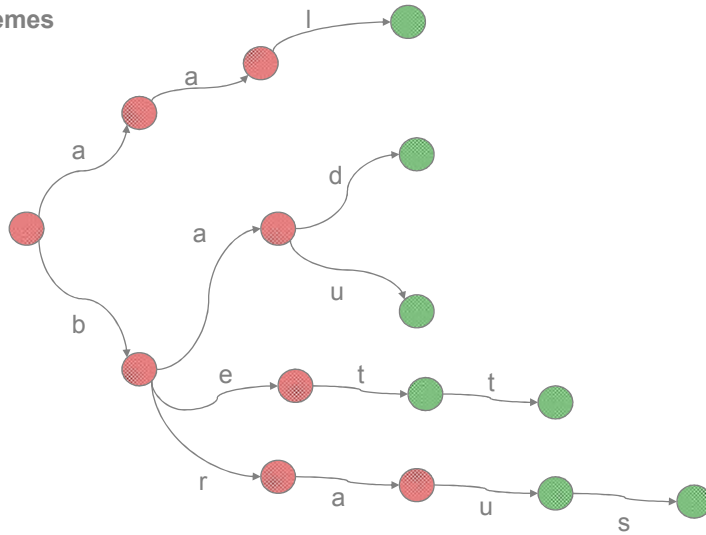


Letter Tree



Verb stem-morphemes

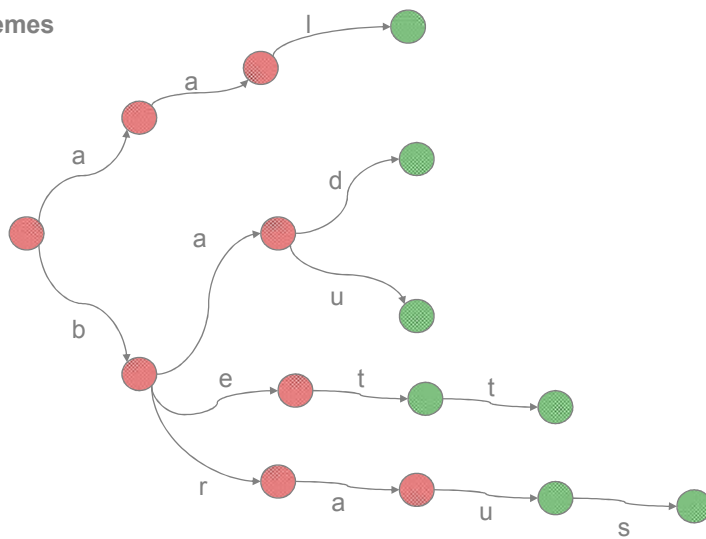
aal
bad
bau
bet
bett
brau
braus



Verb stem-morphemes

aal
bad
bau
bet
bett
brau
braus

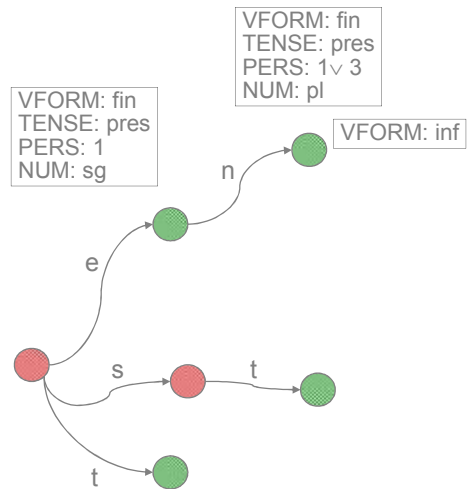
VSTEM: aal



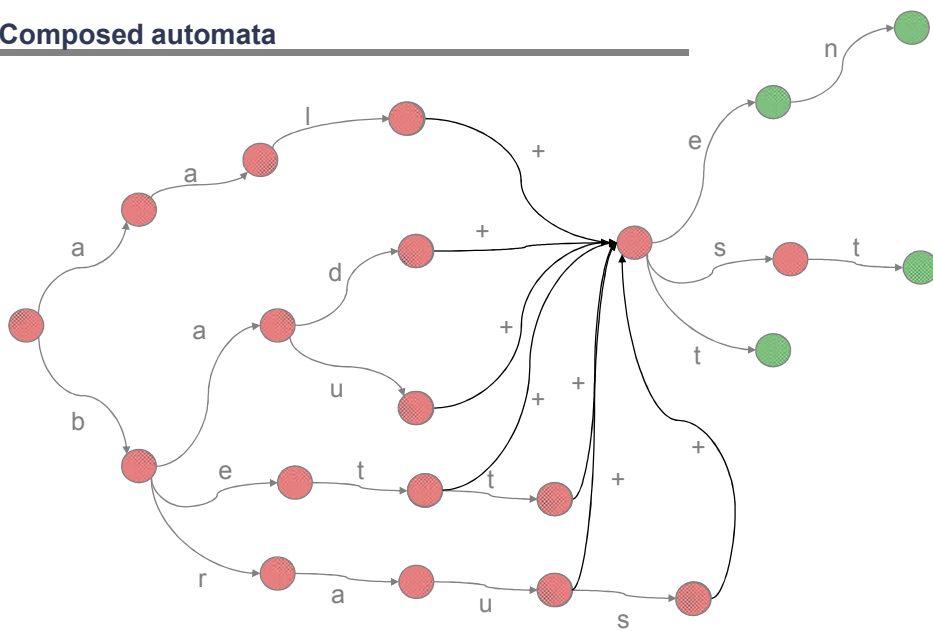
Letter Tree

Verb suffix - morpheme

-e
-en
-st
-t



Composed automata



Examples



- a a l + s t a a l + s t
- a a l s t a a l e s t
- b a d + s t b a d + s t
- b a d s t b a d e s t

Zwei-Ebenen Morphologie

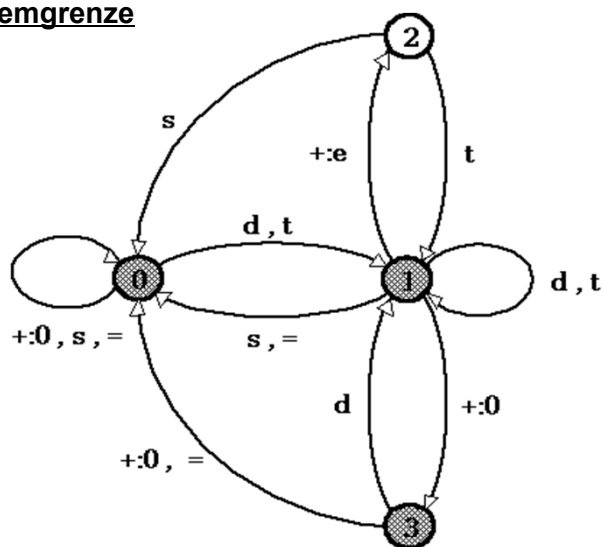


e-Epenthese an der Morphemgrenze (vereinfacht)

Regel: $+e \Leftrightarrow \{d, t\}_- \{s, t\}$

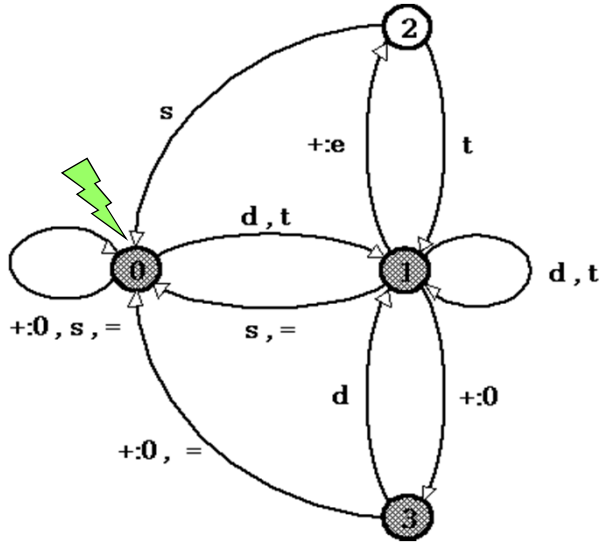
Übergangstabelle:

	0	1	2	3
$+e$	-2	-	-	
$+0$	0	3	-0	
$d:d$	1	1	-1	
$s:s$	0	0	0	-
$t:t$	1	1	1	-
$=:=$	0	0	-0	

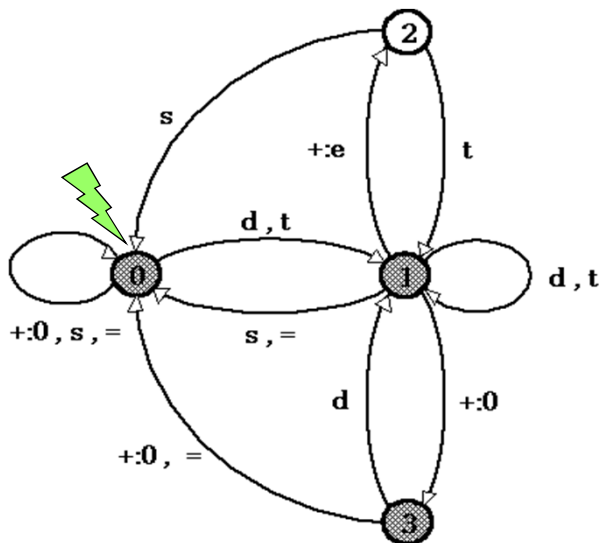


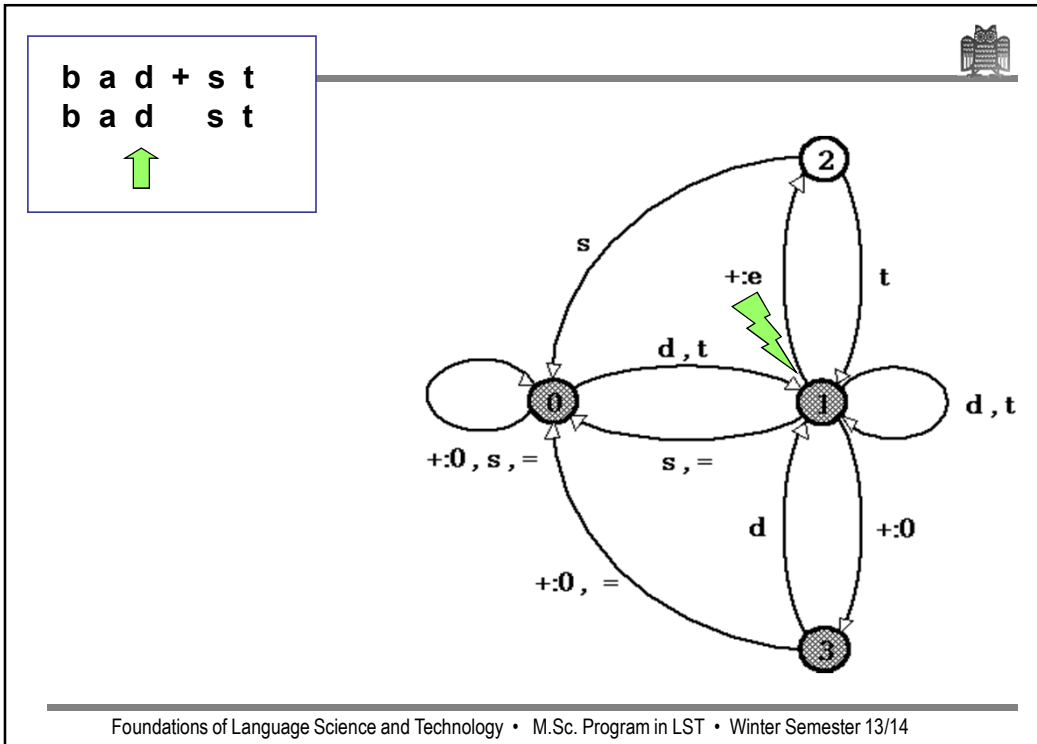
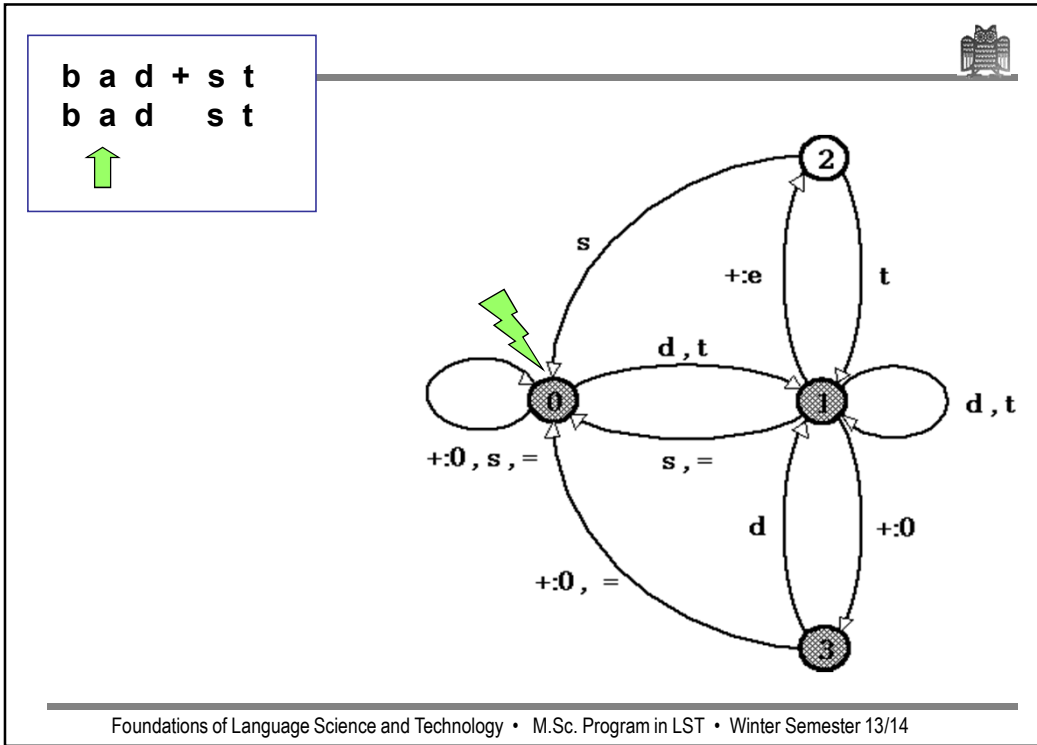
Abgewandeltes Beispiel von Prof. Dr. Harald Trost (Wien)

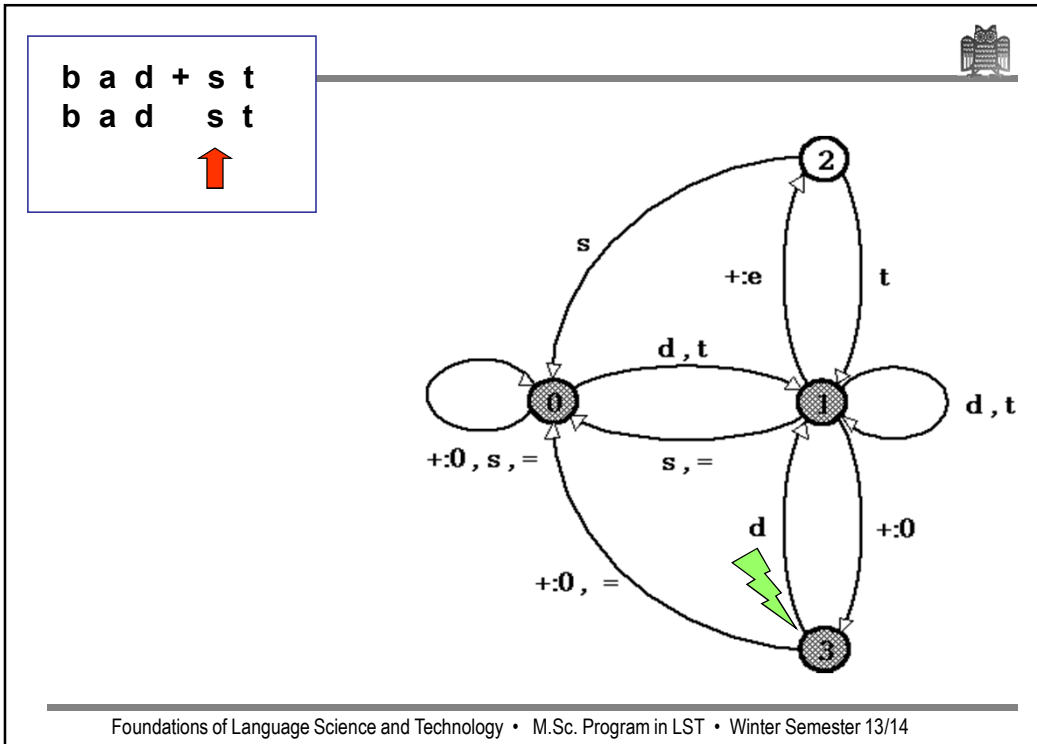
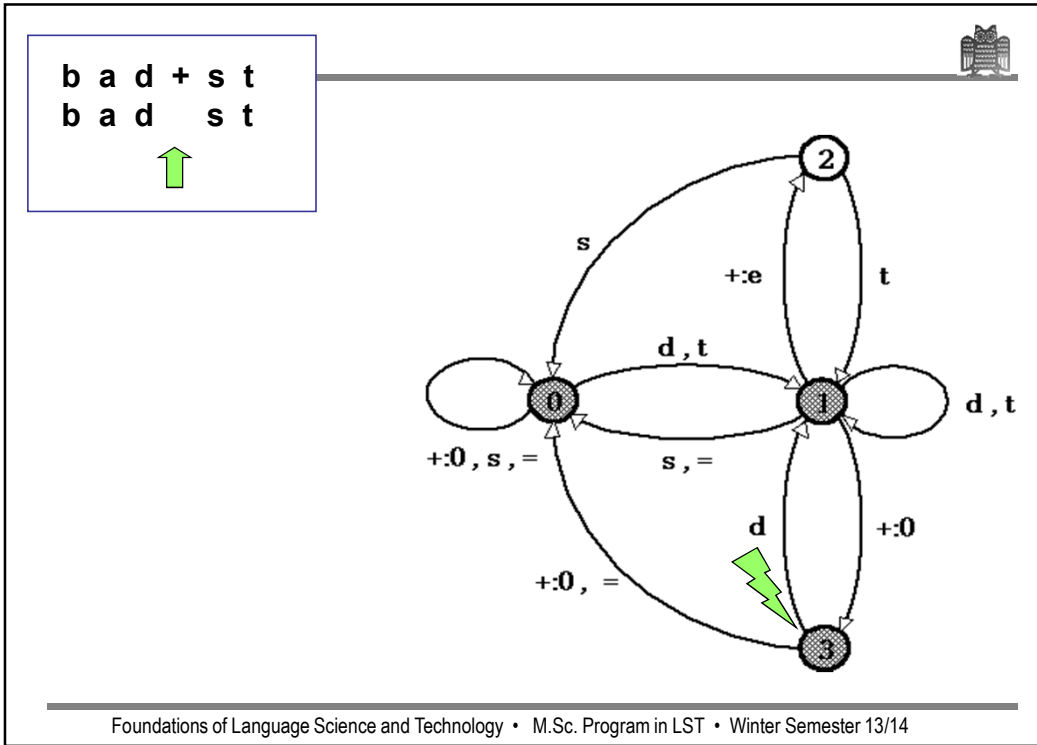
b a d + s t
 b a d s t
 ↑



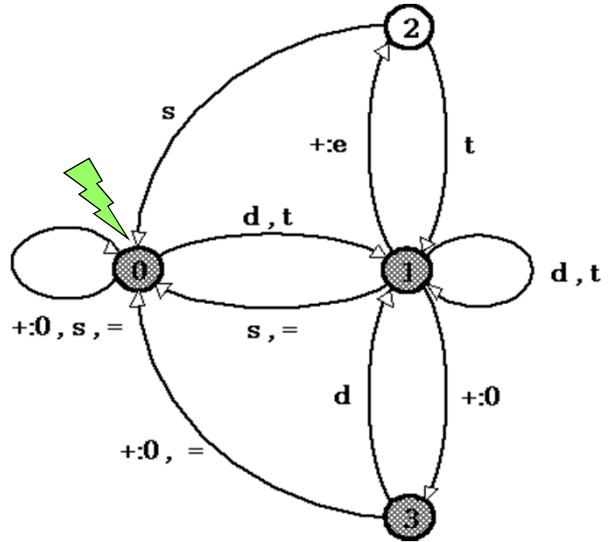
b a d + s t
 b a d s t
 ↑



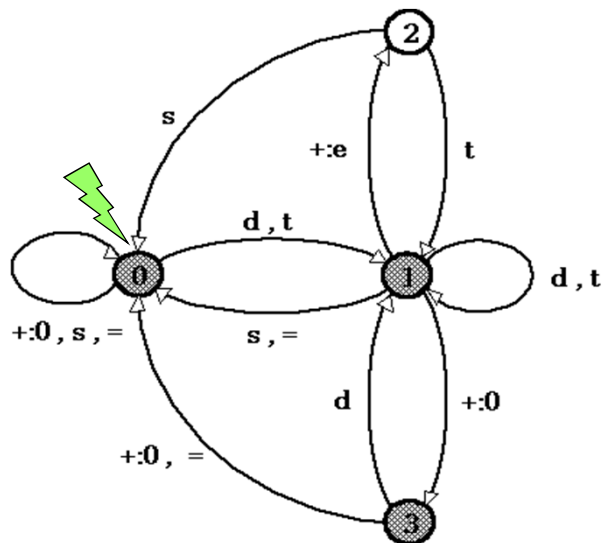


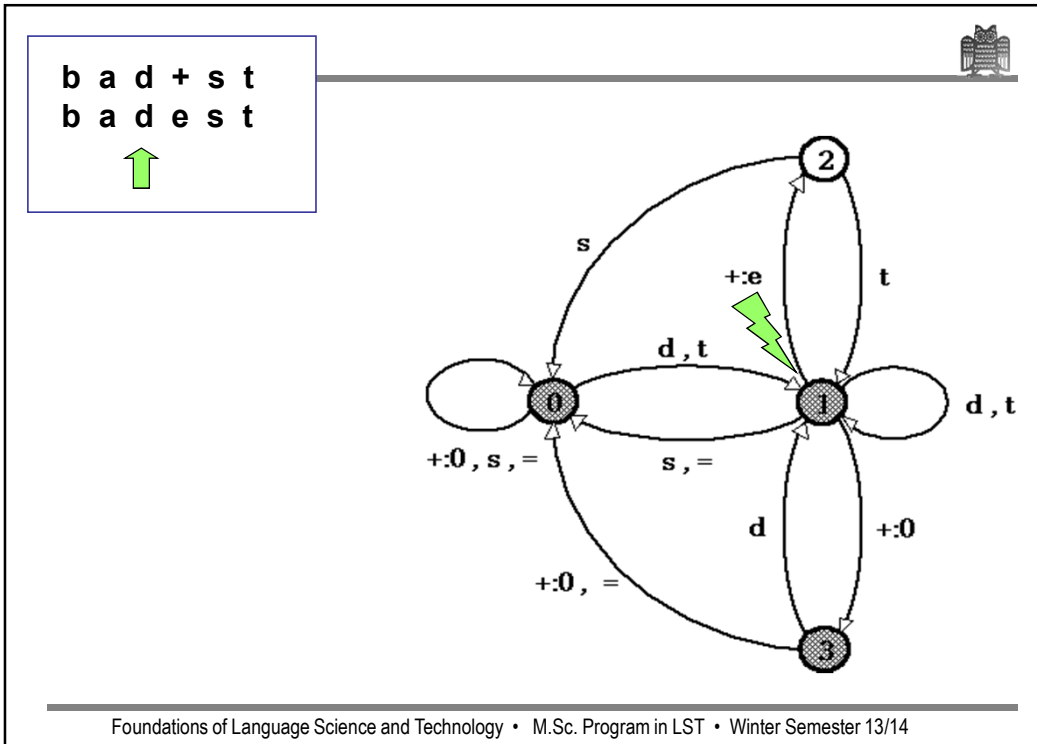
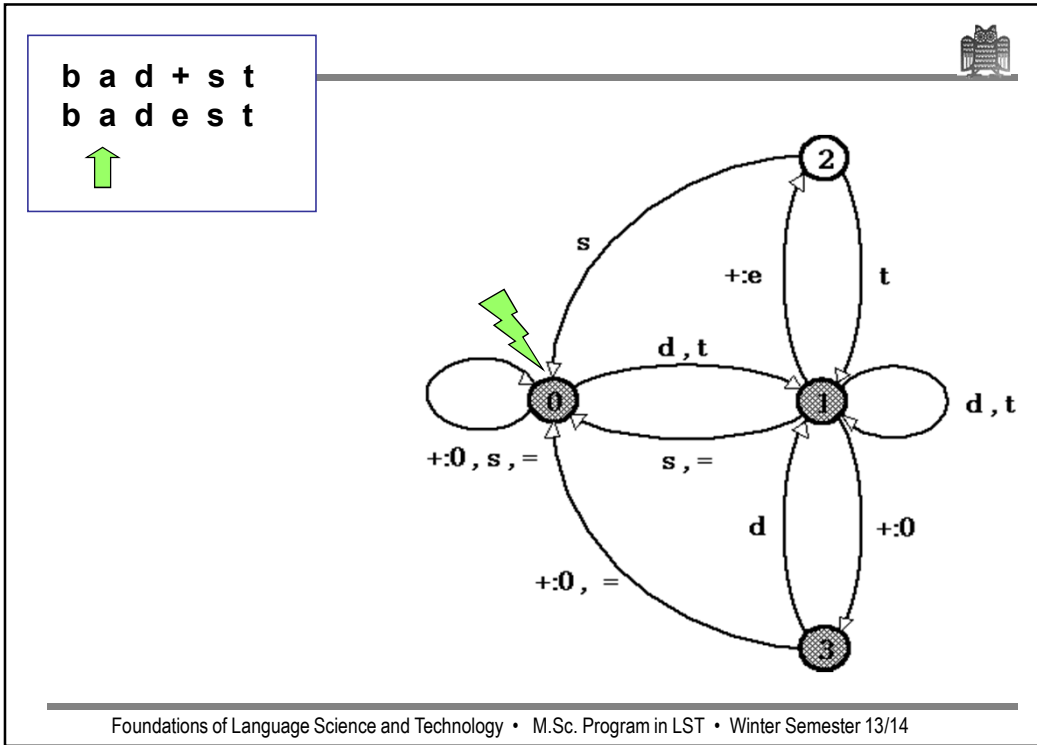


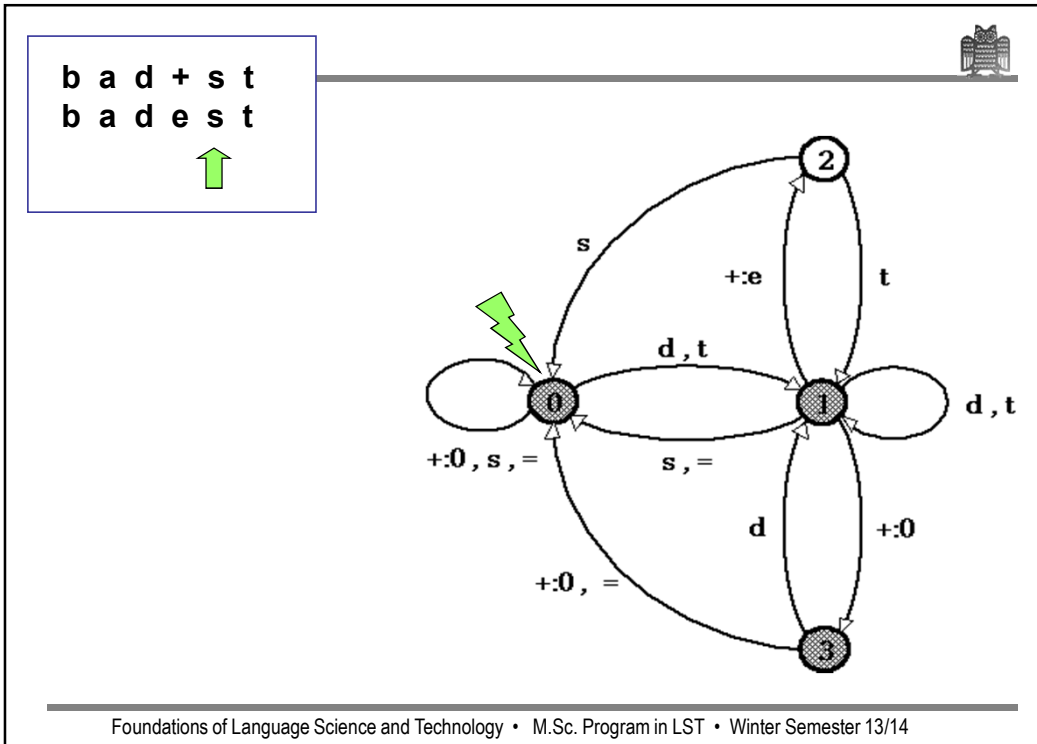
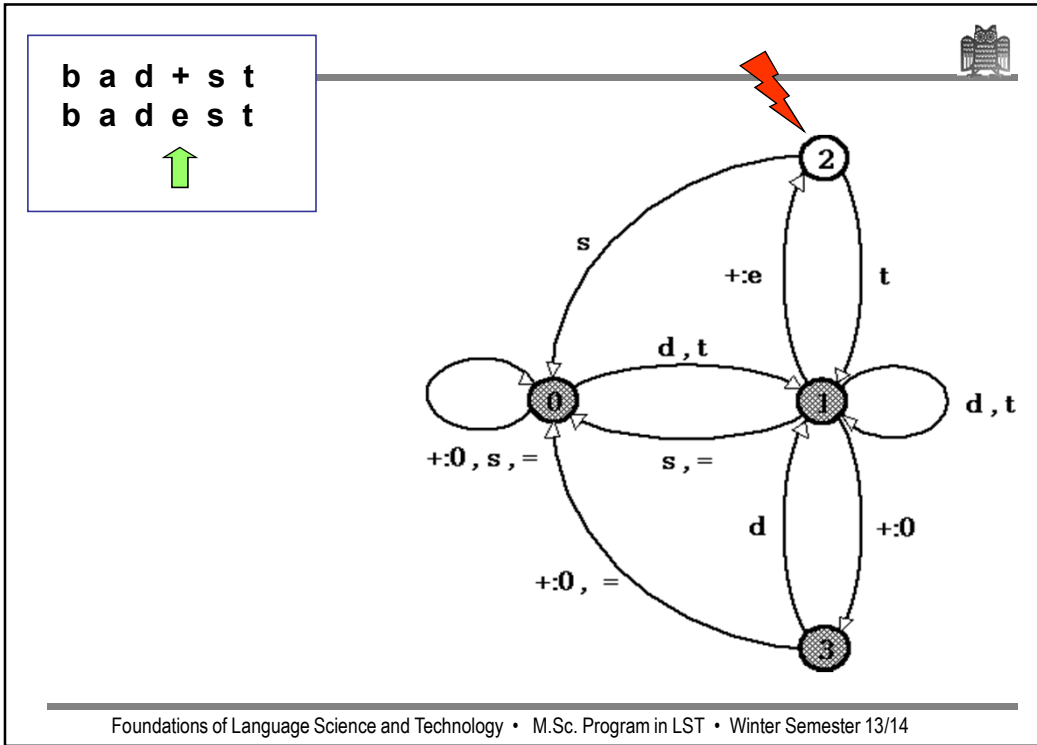
b a d + s t
b a d e s t
 ↑



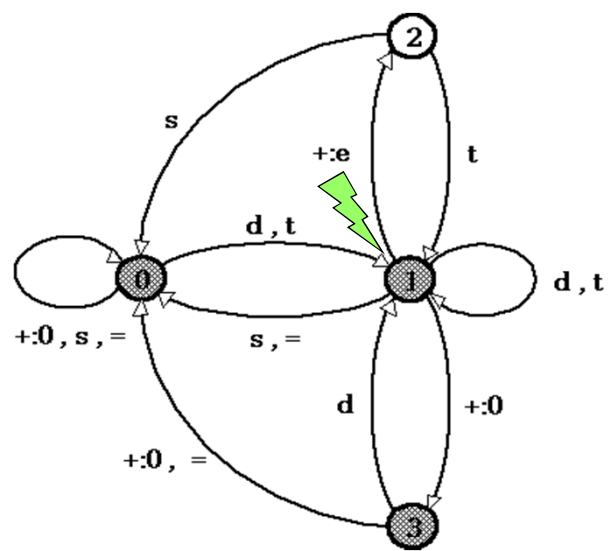
b a d + s t
b a d e s t
 ↑







b a d + s t
b a d e s t
↑



Exercises

1. Design a transducer, that prevents a triple consonant before a vowel at the morpheme boundary (old German orthography rule: Schiffahrt vs. Ballettruppe)!
2. Design a transducer that prevents the plural form “kiss~~s~~” but allows “kiss~~e~~s” and also prevents “wish~~s~~” but not “wish~~e~~s”!
3. Design a transducer that converts a number of Euro cents into a proper German Euro amount (“3200000” → “EUR 32.000,00”).
4. a) Build a transducer that abbreviates first and middle names:
 “William Brown” → “W. Brown”
 “Peter Albert Tucker” → “P. A. Tucker”
 b) Can you build a transducer that does the following transformations
 “William Brown” → “Brown, W.”
 “Kate Plumsfield” → “Plumsfield, K.”
 and what about “Peter Albert Tucker” → “Tucker, P. A.”