FLST WS 2008/2009 – Semantics – Exercise sheet 1

Manfred Pinkal

1. Which of the following expressions are well-formed formulae? (p,q,r are propositional or sentence constants)

a. ¬(¬p ∨ q)
b. p ∨ (q)
c. p ∨ q
d. ¬(q)
e. (p → (q → (p → q)))
f. (p → ((p → q)))
g. (p ∧ (q ∧ r))
h. (p ∧ q ∧ r)

2. Formalise the following sentences in propositional logic! (Translate basic sentences like “it rains” or “Steve comes home late” to propositional constants p, q, r, …)

a. When it rains, it pours.
b. Sam wants a dog, but Alice prefers cats.
c. I will make the dishes if you cook.
d. I will make the dishes only if you cook
e. Marsha won't go out with John unless he shaves off his beard and stops drinking.
f. The stock market advances when public confidence in the economy is rising.
g. John and Bill are going to the movies, but not Tom.
h. If Mary hasn't got lost or had an accident, she will be here in 5 minutes.

3. Check with the truth-table method, whether the following formulae are logically valid, contradictory, or contingent (i.e. neither valid nor contradictory)!

a. ((p ∨ ¬q) ∧ q)
b. ((p ∧ q) → (p ∨ r))
c. (¬p ∧ ¬(p → q))

4. Check with the truth-table method whether entailment holds in the following cases:

d. { (p → ¬q), (r → q), (¬r → q) } ⊨ ¬p?
e. { (p → (q ∧ r)), ((q ∨ r) → s) } ⊨ (p → s) ?
f. { (p → (q ∨ r)), ((q ↔ s) } ⊨ (p → r) ?

5. Translate the following sentences to FOL.

a. John admires someone.
b. John admires himself.
c. Bill and Mary help each other.
d. A student reads an interesting book
e. Peter reads only interesting books.
f. No one is loved by everyone.

g. All but one student passed (the exam).
h. Only Peter flunked.
i. Exactly one student flunked.

5. Are the following formulae logically valid, contradictory, or contingent?

a. $\exists x(F(x) \wedge \neg F(x))$
b. $(\exists x\, F(x) \wedge \exists x \neg F(x))$
c. $(\exists x F(x) \vee \exists x \neg F(x))$
d. $(\forall x F(x) \vee \forall x \neg F(x))$
e. $\forall x(F(x) \vee \neg F(x))$
f. $(\forall x F(x) \rightarrow \exists x F(x))$