

# Computational Linguistics

## Probabilistic Parsing

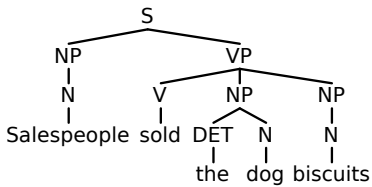
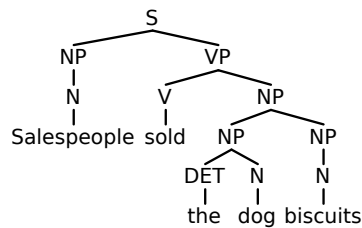
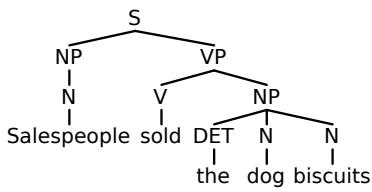
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(Charniak, 1997)

### *Salespeople sold the dog biscuits*



$S \rightarrow NP VP$	$NP \rightarrow NP NP$
$VP \rightarrow V NP$	$NP \rightarrow N$
$VP \rightarrow V NP NP$	$DET \rightarrow the$
$NP \rightarrow DET N$	$N \rightarrow dog$
$NP \rightarrow DET N N$	...

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## Ambiguity & Disambiguation

- **Probabilistic disambiguation**  
choose the one that is most derivation tree if the input sentence is ambiguous (has > 1 derivation trees)
- **We need ...**
  - a probabilistic model of (context-free) grammar
  - methods to estimate probabilities

## Further Motivation

- **Natural language is ambiguous**  
⇒ disambiguation
- **Grammar development**  
⇒ automatically induce grammars
- **Efficient search**  
⇒ compute the most likely parse tree first
- **Robustness**

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## Probabilistic Context-Free Grammars (PCFG)

- **Probabilistic context-free grammar (PCFG)**
  - a context-free grammar  $\langle V, \Sigma, R, S \rangle$
  - a function  $P$  assigning a value  $p \in [0, 1]$  to each rule
    - such that  $\sum_{\beta \in V^*} P(A \rightarrow \beta) = 1$
- $P(A \rightarrow \beta)$  = the conditional probability that symbol  $A$  is expanded to  $\beta$ 
  - Alternative notations:  $P(\beta \mid A)$ ,  $P(A \rightarrow \beta \mid A)$ ,  $A \rightarrow \beta [p]$

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## Derivation Trees (Recap)

- Derivation trees:
  - The root node is labeled with the start symbol  $S$
  - Leaf nodes are labeled with terminal symbols or  $\epsilon$
  - An inner node and their child nodes correspond to the rules that have been used in the derivation
- **Parsing:**  
Compute all derivation trees for a given input
- **Probabilistic parsing:**  
Compute the most likely derivation tree

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# Probabilistic Context-Free Grammar (PCFG)

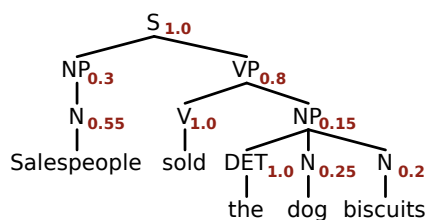
- A PCFG assigns a probability to each derivation tree of a sentence.
- **The probability of a derivation tree T** is defined as the product of the probabilities of all the rules that have been used to expand the nodes in T:
  - $P(T, w) = P(T) = \prod_{n \in T} P(R(n))$
  - R(n) is the rule that has been used to expand node n
  - Note:  $P(T, w) = P(T) P(w | T) = P(T)$ , because  $P(w | T) = 1$
- **The probability of a sentence w** is the sum of the probabilities of all its derivation trees:
  - $P(w) = \sum_T P(w, T)$ , for  $w \in L(G)$

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(Charniak, 1997)

## Salespeople sold the dog biscuits

S → NP VP	[1.0]
VP → V NP	[0.8]
VP → V NP NP	[0.2]
NP → DET N	[0.5]
NP → N	[0.3]
NP → DET N N	[0.15]
NP → NP NP	[0.05]
DET → the	[1.0]
N → Salespeople	[0.55]
N → dog	[0.25]
N → biscuits	[0.2]
V → sold	[1.0]



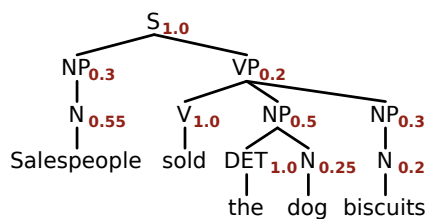
$$\begin{aligned}
 P(t) &= 1.0 \times 0.3 \times 0.55 \times \\
 &\quad 0.8 \times 1.0 \times 0.15 \times \\
 &\quad 1.0 \times 0.25 \times 0.2 \\
 &= 9.9 \times 10^{-4}
 \end{aligned}$$

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(Charniak, 1997)

## Salespeople sold the dog biscuits

S → NP VP	[1.0]
VP → V NP	[0.8]
VP → V NP NP	[0.2]
NP → DET N	[0.5]
NP → N	[0.3]
NP → DET N N	[0.15]
NP → NP NP	[0.05]
DET → the	[1.0]
N → Salespeople	[0.55]
N → dog	[0.25]
N → biscuits	[0.2]
V → sold	[1.0]

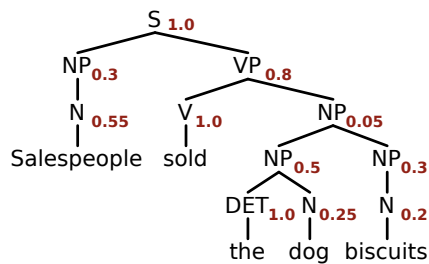


$$\begin{aligned}
 P(t) &= 1.0 \times 0.3 \times 0.55 \times \\
 &\quad 0.2 \times 1.0 \times 0.5 \times \\
 &\quad 1.0 \times 0.25 \times 0.3 \times 0.2 \\
 &= 2.475 \times 10^{-4}
 \end{aligned}$$

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## Salespeople sold the dog biscuits

$S \rightarrow NP VP$	[1.0]
$VP \rightarrow V NP$	[0.8]
$VP \rightarrow V NP NP$	[0.2]
$NP \rightarrow DET N$	[0.5]
$NP \rightarrow N$	[0.3]
$NP \rightarrow DET N N$	[0.15]
$NP \rightarrow NP NP$	[0.05]
$DET \rightarrow the$	[1.0]
$N \rightarrow Salespeople$	[0.55]
$N \rightarrow dog$	[0.25]
$N \rightarrow biscuits$	[0.2]
$V \rightarrow sold$	[1.0]



$$\begin{aligned}
 P(t) &= 1.0 \times 0.3 \times 0.55 \times 0.8 \times \\
 &\quad 1.0 \times 0.05 \times 0.5 \times 1.0 \times \\
 &\quad 0.25 \times 0.3 \times 0.2 \\
 &= 4.95 \times 10^{-5}
 \end{aligned}$$

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## Probabilistic Context-Free Grammar (PCFG)

- The probability of a sentence  $w$  is the sum of the probabilities of all its derivation trees:
  - $P(w) = \sum_T P(w, T)$ , for  $w \in L(G)$
- A PCFG  $G$  is **consistent** if  $\sum_{w \in L(G)} P(w) = 1$
- Recursion can lead to inconsistent grammars:
  - $S \rightarrow S S$  [0.6]
  - $S \rightarrow a$  [0.4]

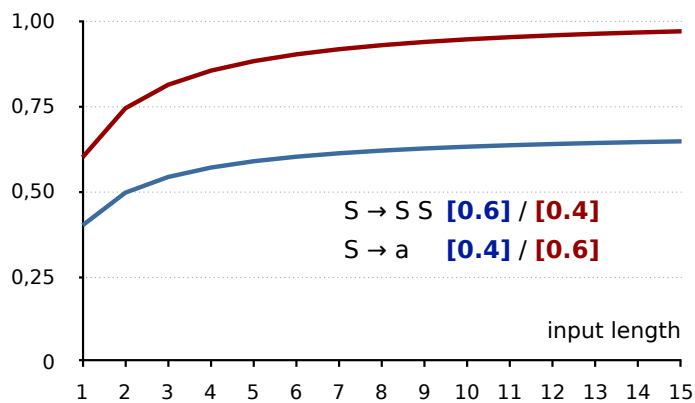
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## An inconsistent PCFG

- $S \rightarrow S S$  [0.6] / [0.4]
- $S \rightarrow a$  [0.4] / [0.6]
- $P(a^i) = \#trees(a^i) \times 0.6^{i-1} \times 0.4^i = 0.4$ 
  - $P(a) = 0.4, P(aa) = 0.096, P(aaa) = 0.0461, \dots$
- $P(a^i) = \#trees(a^i) \times 0.4^{i-1} \times 0.6^i = 0.4$ 
  - $P(a) = 0.6, P(aa) = 0.144, P(aaa) = 0.06912, \dots$
- Number of trees ( $\#trees$ ) for  $a^{i+1} = i$ -th Catalan number

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## An inconsistent PCFG



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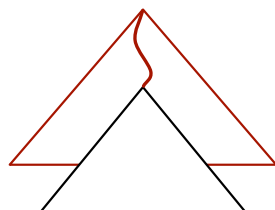
## Probabilistic Parsing

- **Language modelling (“inside probabilities”)**  
compute the probability that  $S \Rightarrow^* w$  for an input sentence  $w$ :
  - $P(w) = \sum_T P(w, T)$
- **Probabilistic parsing (“viterbi scores”)**  
compute the most likely derivation tree  $T(w)$  for an input sentence  $w$ :
  - $T(w) = \arg \max_T P(T | w)$   
 $= \arg \max_T \frac{P(T, w)}{P(w)}$   
 $= \arg \max_T P(T)$

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## Properties of PCFGs

- The probability of a (sub) tree is independent of
  - the context in which the tree occurs
  - the node(s) that dominates the tree



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## Probabilistic CYK Parsing

- Extend the CYK algorithm:
  - $T[i, j, A]$  = the probability that  $A \Rightarrow^* w_{i+1} \dots w_j$
- **Inside probabilities:**
  - $T[i, j, A]$  = sum of the probabilities of all derivation trees of the substring  $w_{i+1} \dots w_j$
- **Probability of a derivation tree (parsing)**
  - $T[i, j, A]$  = the probability of the most likely derivation
  - $B[i, j, A]$  = the corresponding derivation tree

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## CYK (without probabilities)

```
function CYK(G, w1 ... wn):
  for i in 1 ... n do
    T[i-1, i] = { A | A → wi ∈ R }
    for j in i - 2 ... 0 do
      T[j, i] = ∅
      for k in j + 1 ... i - 1 do
        T[j, i] = T[j, i] ∪
          { A | A → B C, B ∈ T[j, k], C ∈ T[k, i] }
      done
    done
  done
  if S ∈ T[0, n] then return True else return False
```

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## CYK (with probabilities)

```
function CYK(G, w1 ... wn):
  (initialize T and B)
  for i in 1 ... n do
    for all nonterminals A in G do
      T[i-1, i, A] = P(A → wi)
    for j in i - 2 ... 0 do
      for k in j + 1 ... i - 1 do
        for all A → B C do
          pr = T[j, k, B] × T[k, i, C] × P(A → B C)
          if pr > T[j, i, A] then
            T[j, i, A] = pr
            B[j, i, A] = (construct subtree)
  return (B[0, n, S] and T[0, n, S])
```

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## Learning PCFG Probabilities

- **Option #1**  
count frequencies of rules in syntactically annotated treebanks (such as the Penn Treebank)
- **Option #2**  
Inside-outside algorithm (not discussed here)

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## Learning PCFG Probabilities

- We are given a syntactically annotated corpus
  - annotated corpus = a set of derivation trees
- We can construct a grammar from the treebank by identifying the rules with all “subtrees” of height 1
- **Estimating rule probabilities:**
  - $P(A \rightarrow \alpha) = \frac{\text{count}(A \rightarrow \alpha)}{\sum_{\beta} \text{count}(A \rightarrow \beta)}$
  - $\text{count}(A \rightarrow \alpha)$  = the number of times the rule  $A \rightarrow \alpha$  has been used in all trees in the corpus

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(Example: Webber/Keller)

## Learning PCFG Probabilities

- **A very small treebank:**
  - S<sub>1</sub>: [s [NP grass] [VP grows]]
  - S<sub>2</sub>: [s [NP grass] [VP grows] [AP fast]]
  - S<sub>3</sub>: [s [NP grass] [VP grows] [AP slowly]]
  - S<sub>4</sub>: [s [NP bananas] [VP grow]]
- **Rules & rule probabilities:**
  - S → NP VP    2/4
  - S → NP VP AP    2/4
  - NP → grass    3/4
  - ...

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## Learning PCFG Probabilities

	Rule	$P(A \rightarrow \alpha)$
$r_1$	$S \rightarrow NP VP$	$2/4$
$r_2$	$S \rightarrow NP VP AP$	$2/4$
$r_3$	$NP \rightarrow \text{grass}$	$3/4$
$r_4$	$NP \rightarrow \text{bananas}$	$1/4$
$r_5$	$VP \rightarrow \text{grows}$	$3/4$
$r_6$	$VP \rightarrow \text{grow}$	$1/4$
$r_7$	$AP \rightarrow \text{fast}$	$1/2$
$r_8$	$AP \rightarrow \text{slowly}$	$1/2$

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## Learning PCFG Probabilities

### ■ Probabilities of the sentences:

- $P(S_1) = P(r_1) \times P(r_3) \times P(r_5) = 2/4 \times 3/4 \times 3/4 = 0.28125$
- $P(S_2) = P(r_2) \times P(r_3) \times P(r_5) \times P(r_7) = 0.140625$
- $P(S_3) = P(r_2) \times P(r_3) \times P(r_5) \times P(r_7) = 0.140625$
- $P(S_4) = P(r_1) \times P(r_4) \times P(r_6) = 0.03125$

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## Evaluation

- **Coverage:** How many sentences are well-formed according to the grammar?
- **Accuracy:** How many sentences are correctly parsed?
  - measured as “relative correctness” wrt. to category label, start and end position (yield) of all constituents (subtrees)
  - **Labelled precision:** percentage of correct subtrees in the parser output
  - **Labelled recall:** percentage of correct subtrees in the gold standard (test corpus)

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## Evaluation

- **Labelled Precision** =  $C / M$
- **Labelled Recall** =  $C / N$
- where
  - $C$  = number of correct constituents produced by the parser
  - $M$  = total number of constituents produced by the parser
  - $N$  = total number of constituents in reference corpus

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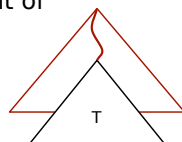
## Binarization

- Replace rules of the form  $A \rightarrow A_1 A_2 A_3 \dots A_k [p]$  by
  - $A \rightarrow \langle A_1, \dots, A_{k-1} \rangle A_k [p]$
  - $\langle A_1, \dots, A_{k-1} \rangle \rightarrow A_1 \dots A_{k-1} [1.0]$
- ... or binarize trees in the treebank before “reading off” the grammar from the trees.

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## Problems

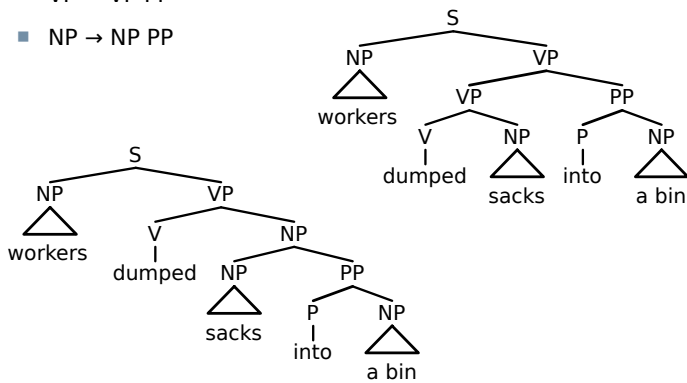
- The probability of a (sub) tree is independent of
  - the context in which the tree occurs
  - the node(s) that dominates the tree
- **Problems:** we *want* to capture ...
  - Lexical dependencies
  - Structural dependencies



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# Lexical Dependencies

- The two trees differ only in one rule:
  - VP → VP PP
  - NP → NP PP



# Lexical Dependencies

- The two trees differ only in one rule:
  - VP → VP PP
  - NP → NP PP
- ⇒ the grammar will either
  - always prefer the 1st rule (VP attachment) or
  - always prefer the 2nd rule (NP-attachment)
- **But ...**
  - *Workers dumped sacks into a bin*
  - *Fishermen caught tons of herring*
- ⇒ Lexikalized PCFG

(Manning & Schütze)

# Lexical Dependencies

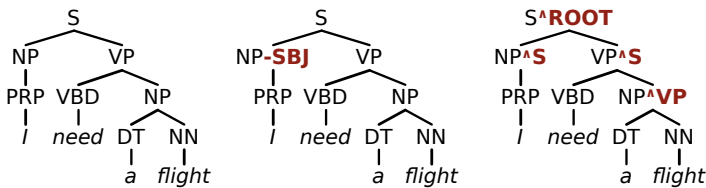
	come	take	think	want
VP → V	9,5%	2,6%	4,6%	5,7%
VP → V NP	1,1%	32,1%	0,2%	13,9%
VP → V PP	35,5%	3,1%	7,1%	0,3%
VP → V SBAR	6,6%	0,3%	73,0%	0,2%
VP → V S	2,2%	1,3%	4,8%	70,8%
VP → V NP S	0,1%	5,7%	0,0%	0,3%
VP → V PRT NP	0,3%	5,8%	0,0%	0,0%
VP → V PRT PP	6,1%	1,5%	0,2%	0,0%
...	...	...	...	...

# Structural dependencies

- **Structural independencies:**
  - The (probability of an) application of a rule is independent of all other rules in the derivation tree
  - NP → Pronoun vs. NP → Det Noun  
same probabilities for all occurrences of NP
- **But ...** (Francis & al, 1999)
  - Subject-NP: 91% pronouns, 9% non-pronouns
  - Object-NP: 34% pronouns, 66% non-pronouns
  - (Switchboard corpus, spoken language)
- ⇒ Parent annotation

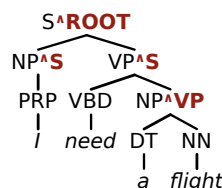
# Structural dependencies

- Some dependencies can be “built into” the category symbols.



# Structural dependencies

- **Parent Annotation:** nodes are annotated with the label of their parent nodes
- Similar effect compared to conditional probabilities
  - $P(\text{NP}^S \rightarrow \text{PRP})$
  - $P(\text{NP} \rightarrow \text{PRP} \mid \text{S})$
- Compare:
  - $P(\text{NP-SBJ} \rightarrow \text{PRP})$  – no correspondence to conditional probabilities



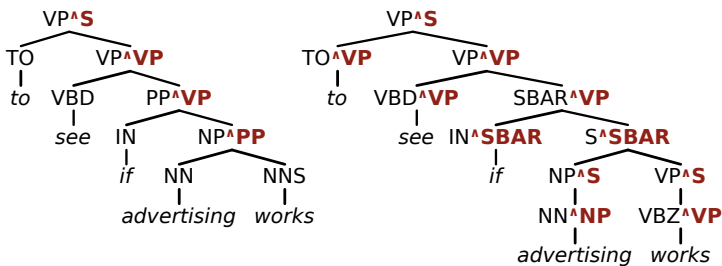
## Structural dependencies

- Parent annotation can also be useful for preterminal nodes
- Most frequent adverbs with parent ...
  - ADVP - *also, now*
  - VP - *not, n't*
  - NP - *only, just*
- Penn Treebank - no distinction (same POS) between
  - subordinating conjunctions (*while, as, if*),
  - complementizers (*that, for*)
  - prepositions (*of, in, from*)

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## Structural dependencies

- Parent annotation can also be useful for preterminal nodes



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## Structural dependencies

- **Parent annotation - drawbacks**
  - the grammar gets larger
  - fewer training data for each rule
  - reduced generalization ("overfitting")

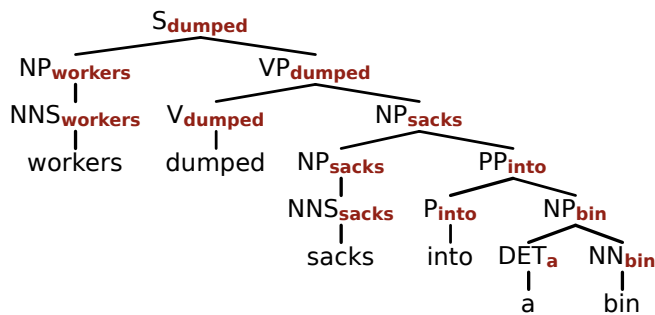
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# Lexical dependencies

- **The head** of a constituent is the “central” word of a phrase
  - Noun - NP
  - Verb - VP, S
  - Adjektive - AP
  - Preposition - PP

# Lexical dependencies

- **Lexicalized parsing:** annotate nodes with their lexical heads



# Lexical dependencies

	Rule	$P(A \rightarrow \alpha)$
r <sub>1</sub>	S <sub>dumped</sub> → NP <sub>workers</sub> VP <sub>dumped</sub>	1/1
r <sub>2</sub>	NP <sub>workers</sub> → NNS <sub>workers</sub>	1/1
r <sub>3</sub>	NP <sub>sacks</sub> → NNS <sub>sacks</sub>	1/2
r <sub>4</sub>	NP <sub>sacks</sub> → NP <sub>sacks</sub> PP <sub>into</sub>	1/2
r <sub>5</sub>	NP <sub>bin</sub> → DT <sub>a</sub> NN <sub>bin</sub>	1/1
...	...	...

## Lexical dependencies

- **Problems:**
  - this leads to much larger grammars
  - its hard to estimate the rule probabilities

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## Lexicalized parsing

- **Complexity (CYK)**
  - Runtime:  $O(|rules|n^3)$ ,
  - Worst case:  $|rules| = |nonterminals|^3$
- **Lexicalized grammars**
  - Worst case:  $|rules| = |nonterminals|^3 \cdot |terminals|^2$
  - $|terminals|$  usually much larger than  $|nonterminals|$
  - $\Rightarrow O(n^5)$  runtime for typical grammars and input sentences

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## Literature

- Jurafsky & Martin (2009) Speech and Language Processing Kapitel 14.
- Manning & Schütze (1999). Foundations of Statistical Natural Language Processing. Kapitel 11 & 12.
- Eugene Charniak (1993). Statistical Language Learning. Kapitel 5.

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