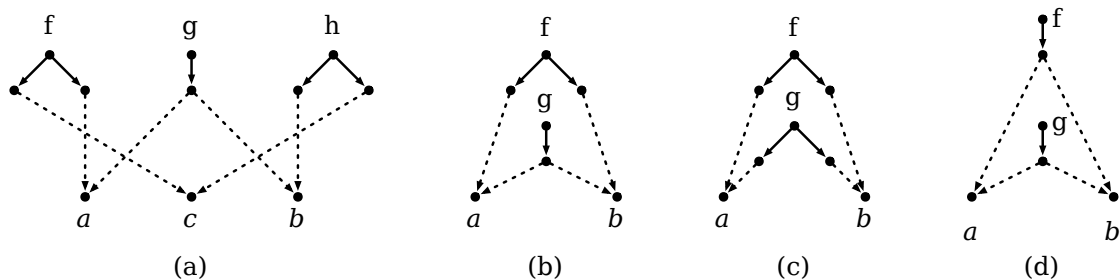


1. Consider the following dominance graphs:



Which of the graphs are hypernormally connected?

2. Optional / Bonus: Consider the following tree automaton

$A = \langle Q = \{q\}, \Sigma = \{\neg|1, \exists x|2, \exists y|2, \text{stud}_{x|0}, \text{book}_{y|0}, \text{read}_{x,y|0}\}, Q_f = \{q\}, \Delta \rangle$

where  $\Delta$  contains the following transition rules:

- $\neg(q(x_1)) \rightarrow q(\neg(x_1))$
- $\exists x(q(x_1), q(x_2)) \rightarrow q(\exists x(x_1, x_2))$
- $\exists y(q(x_1), q(x_2)) \rightarrow q(\exists y(x_1, x_2))$
- $\text{stud}_x \rightarrow q(\text{stud}_x)$
- $\text{book}_y \rightarrow q(\text{book}_y)$
- $\text{read}_{x,y} \rightarrow q(\text{read}_{x,y})$

This automaton simply accepts *all* trees over  $\Sigma$ .

- (a) Modify (extend) the automaton so that it accepts all trees over  $\Sigma$  *except* those trees that contain  $\exists x(\_, \exists y(\_, \_))$  as a subtree (trees where  $\exists y$  occurs as the right child of  $\exists x$ ).
- (b) Intersect the automaton from (a) with the automaton on slide 24 from the lecture.

**Appendix: Intersection (see Comon &al. 2007, page 29)**

Let  $A_1 = \langle Q_1, \Sigma, Q_{f1}, \Delta_1 \rangle$  and  $A_2 = \langle Q_2, \Sigma, Q_{f2}, \Delta_2 \rangle$  be two tree automata. An automaton that accepts  $L(A_1) \cap L(A_2)$  can be defined as follows:

$A = \langle Q_1 \times Q_2, \Sigma, Q_{f1} \times Q_{f2}, \Delta \rangle$

$\Delta = \{ f(\langle p_1, q_1 \rangle(x_1), \dots, \langle p_n, q_n \rangle(x_n)) \rightarrow \langle p, q \rangle(f(x_1, \dots, x_n)) \mid$   
 $f(p_1(x_1), \dots, p_n(x_n)) \rightarrow p(f(x_1, \dots, x_n)) \in \Delta_1$  and  
 $f(q_1(x_1), \dots, q_n(x_n)) \rightarrow q(f(x_1, \dots, x_n)) \in \Delta_2 \}$

To be turned in Thursday, 2015-06-11