

Computational Linguistics

Algorithms for Scope Underspecification

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Summer 2013

Today

- Refining the solver from the last lecture
- Hypernormally connected dominance graphs
- Tree Automata (in a nutshell)
- Redundancy elimination (very short)

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(Bodirsky & al., 2004)

Solving dominance graphs

solve($G = (V, E \cup D)$) = (*)

choose a free fragment F of G else fail

let G_1, \dots, G_k be the WCC's of $G \setminus F$

let $(V_i, E_i \cup D_i) = \text{solve}(G_i)$

return $(V, E \cup D_1 \cup \dots \cup D_k \cup D')$

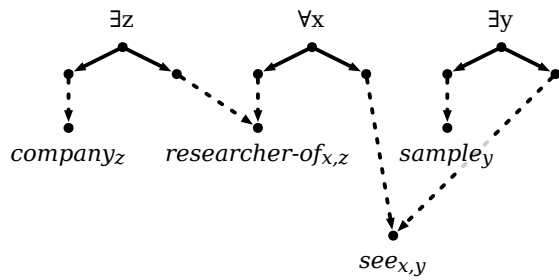
where D' are dominance edges that connect the holes of F
with the solved form of one of the corresponding G_i

(*) slightly simplified version, works only for connected normal dominance graphs

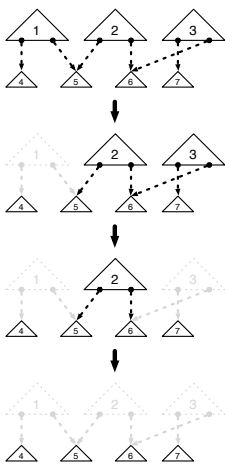
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An Example

- Every researcher of a company saw a sample.

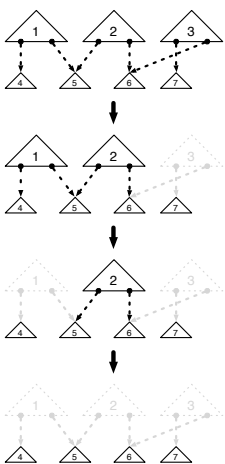


An Example: Run #1



subgraph	free	wccs
{1,...,7}	1	{4}, {2,3,5,6,7}
{2,3,5,6,7}	3	{7}, {2,5,6}
{2,5,6}	2	{5}, {6}

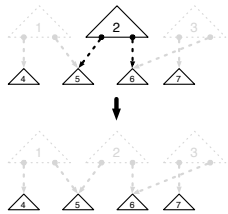
An Example: Run #2



subgraph	free	wccs
{1,...,7}	3	{7}, {1,2,4,5,6}
{1,2,4,5,6}	1	{4}, {2,5,6}
{2,5,6}	2	{5}, {6}

An Example

Problem: The algorithm is applied twice to the subgraph $\{2,5,6\}$



subgraph	free	wccs
$\{1, \dots, 7\}$	1	$\{4\}, \{2,3,5,6,7\}$
$\{2,3,5,6,7\}$	3	$\{7\}, \{2,5,6\}$
$\{2,5,6\}$	2	$\{5\}, \{6\}$

subgraph	free	wccs
$\{1, \dots, 7\}$	3	$\{7\}, \{1,2,4,5,6\}$
$\{1,2,4,5,6\}$	1	$\{4\}, \{2,5,6\}$
$\{2,5,6\}$	2	$\{5\}, \{6\}$

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Chart-based solver

- **Basic idea:** use dynamic programmic techniques and store intermediate results in a chart-like datastructure
- The chart records how graphs are decomposed into smaller subgraphs if free fragments are removed
 - The chart assigns each subgraph a “split.”
 - Splits consist of (references to) a free fragment F and the weakly connected components of $G \setminus F$.
 - Notation: $F(G_1, \dots, G_n)$

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(Koller & Thater, 2005)

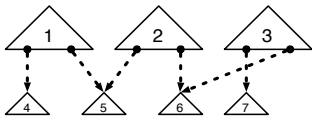
The algorithm

```

GRAPH-SOLVER-CHART( $G'$ )
1  if there is an entry for  $G'$  in the chart
2  then return true
3   $free \leftarrow$  FREE-FRAGMENTS( $G'$ )
4  if  $free = \emptyset$ 
5  then return false
6  if  $G'$  contains only one fragment
7  then return true
8
9  for each  $F \in free$ 
10 do  $split \leftarrow$  SPLIT( $G', F$ )
11   for each  $S \in WCCS(G' - F)$ 
12   do if GRAPH-SOLVER-CHART( $S$ ) = false
13   then return false
14   add ( $G', split$ ) to the chart
15 return true
    
```

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An Example



```

GRAPH-SOLVER-CHART( $G'$ )
1  if there is an entry for  $G'$  in the chart
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12   do if GRAPH-SOLVER-CHART( $S$ ) = false
13   then return false
14   add ( $G', split$ ) to the chart
15 return true
    
```

subgraph	splits
{1,...,7}	1({4}, {2,3,5,6,7})
	2({1,4,5}, {3,6,7})
	3({1,2,4,5,6}, {7})
{2,3,5,6,7}	2({5}, {3,6,7})
	3({2,5,6}, {7})
{1,2,4,5,6}	1({4}, {2,5,6})
	2({1,4,5}, {6})
	{2,5,6} 2({5}, {6})
	{3,6,7} 3({7}, {6})
	{1,4,5} 1({4}, {5})

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Complexity

- Let G be a dominance graph with n nodes and m edges
- The computation of free fragments takes time $O(n + m)$
- The time to compute the chart is $O(n(n + m) \text{wcs}(G))$
 - $\text{wcs}(G)$ = the number of weakly connected subgraphs of G
- Worst case complexity in the number of nodes is $O(n^2 2^n)$
 - \Rightarrow big improvement compared to $O(n^2 n!)$ of the basic solver
 - ($n!$ = upper bound for the number of solved forms)

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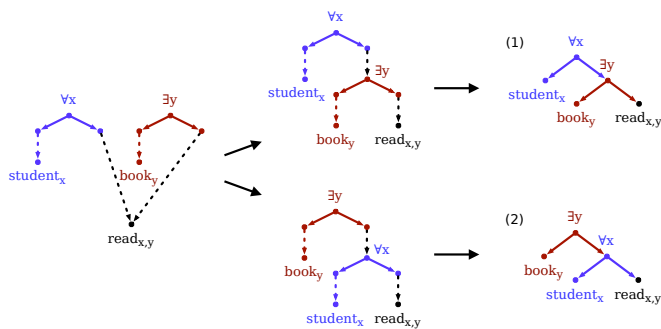
Hypernormally Connected Dominance Graphs

The big picture

- Every student reads a book

(1) $\forall x(\text{student}(x), \exists y(\text{book}(y), \text{read}(x, y)))$

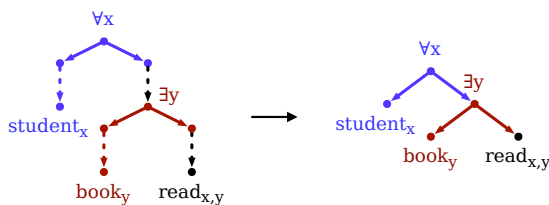
(2) $\exists y(\text{book}(y), \forall x(\text{student}(x), \text{read}(x, y)))$



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Simple solved forms

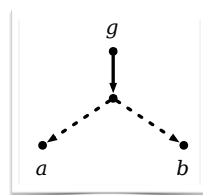
- We are usually interested only in solved forms in which every hole is related to exactly one root
 - let's call such solved forms "simple"
- Simple solved forms can be mapped to "proper" trees simply by "plugging" the hole with the root connected to it by a dominance edge.



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Not all solved forms are simple

- Problem:** Not all solved forms are simple
- Solution:** Identify a class of dominance graphs that only have simple solved forms



- ⇒ Hypernormally connected dominance graphs

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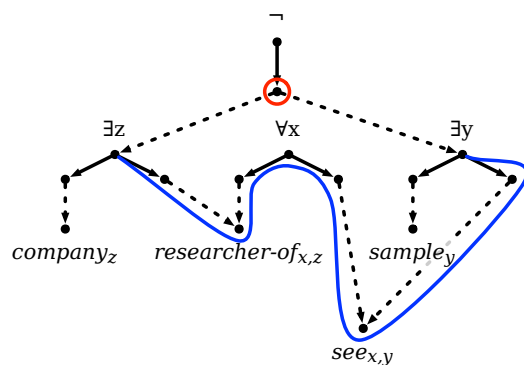
Hypernormally connected dominance graphs

- A **hypernormal path** in a (normal) dominance graph G is a path in the undirected version of G that does not use two dominance edges incident to the same hole.
- A (normal) dominance graph G is **hypernormally connected** if each pair of nodes is connected by some hypernormal path.

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Hypernormally connected dominance graphs

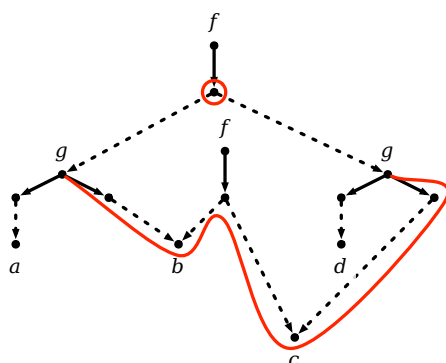
- Hypernormally connected:



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Hypernormally connected dominance graphs

- Not hypernormally connected:



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Hypernormally connected dominance graphs

- **Lemma:** if G is a hypernormally connected normal dominance graph with free fragment F , then all WCCs of $G \setminus F$ are hypernormally connected.
- **Proposition:** if a normal dominance graph is hypernormally connected, then all its (minimal) solved forms are simple.

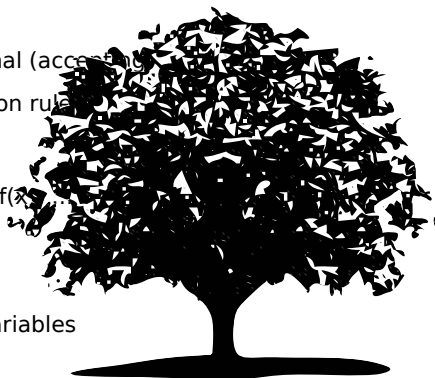
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Tree Automata

(see Comon & al., 2007)

Bottom-up Tree Automaton

- A **tree automaton** is a tuple $A = \langle Q, \Sigma, Q_f, \Delta \rangle$
 - Σ a finite ranked signature
 - Q a finite set of states
 - $Q_f \subseteq Q$ a finite set of final (accepting) states
 - Δ a finite set of transition rules
- **Transition rules:**
 - $f(q_1(x_1), \dots, q_n(x_n)) \rightarrow q(f(x_1, \dots, x_n))$
 - $f \in \Sigma$
 - $q, q_1, \dots, q_n \in Q$
 - x_1, \dots, x_n different variables



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An Example Computation

- $Q = \{q_3, q_4, \dots\}, \Sigma = \{\exists x|_2, \text{book}_y|_0, \dots\}, Q_f = \{q_{12345}\}$

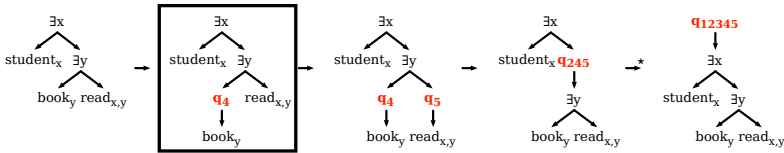
$$\exists x(q_3(x_1), q_{245}(x_2)) \rightarrow q_{12345}(\exists x(x_1, x_2))$$

$$\exists y(q_4(x_1), q_5(x_2)) \rightarrow q_{245}(\exists y(x_1, x_2))$$

$$\text{student}_x \rightarrow q_3(\text{student}_x)$$

$$\text{book}_y \rightarrow q_4(\text{book}_y)$$

$$\text{read}_{x,y} \rightarrow q_5(\text{read}_{x,y})$$



Bottom-up Tree Automaton

- A tree t is **accepted** by an automaton $A = \langle Q, \Sigma, Q_f, \Delta \rangle$ if
 - $t \rightarrow^* q(t)$
 - $q \in Q_f$
- The language $L(A)$** of trees recognized by A is the set of trees accepted by A .

Another Example

- Automaton (rules)**

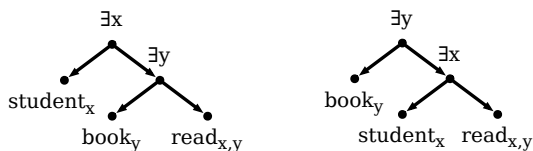
$$\exists x(q_3(x_1), q_{245}(x_2)) \rightarrow q_{12345}(\exists x(x_1, x_2)) \quad \text{student}_x \rightarrow q_3(\text{student}_x)$$

$$\exists y(q_4(x_1), q_{135}(x_2)) \rightarrow q_{12345}(\exists y(x_1, x_2)) \quad \text{book}_y \rightarrow q_4(\text{book}_y)$$

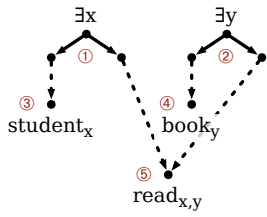
$$\exists y(q_4(x_1), q_5(x_2)) \rightarrow q_{245}(\exists y(x_1, x_2)) \quad \text{read}_{x,y} \rightarrow q_5(\text{read}_{x,y})$$

$$\exists x(q_3(x_1), q_5(x_2)) \rightarrow q_{135}(\exists x(x_1, x_2))$$

- Accepted Trees:**



Back to dominance charts



dominance chart	
{1,2,3,4,5}	1({3}, {2,4,5})
	2({4}, {1,3,5})
{2,4,5}	2({4}, {5})
{1,3,5}	1({3}, {5})

Compare

tree automaton	dominance chart
$\exists x(q_3(x_1), q_{245}(x_2)) \rightarrow q_{12345}(\exists x(x_1, x_2))$	{1,2,3,4,5} 1({3}, {2,4,5})
$\exists y(q_4(x_1), q_{135}(x_2)) \rightarrow q_{12345}(\exists y(x_1, x_2))$	2({4}, {1,3,5})
$\exists y(q_4(x_1), q_5(x_2)) \rightarrow q_{245}(\exists y(x_1, x_2))$	{2,4,5} 2({4}, {5})
$\exists x(q_3(x_1), q_5(x_2)) \rightarrow q_{135}(\exists x(x_1, x_2))$	{1,3,5} 1({3}, {5})
student _x → q ₃ (student _x)	
book _y → q ₄ (book _y)	
read _{x,y} → q ₅ (read _{x,y})	

Charts = Tree automata

- Dominance charts can be seen as (or translated into) tree automata
 - ... provided that the original dominance graph is hypernormally connected (why?)

Charts = Tree automata

- The class of recognizable languages is closed under
 - Union
 - Complement
 - Intersection
- \Rightarrow We can model certain inferences on the level of dominance charts by intersecting regular tree languages
 - Redundancy elimination
 - Weakest readings

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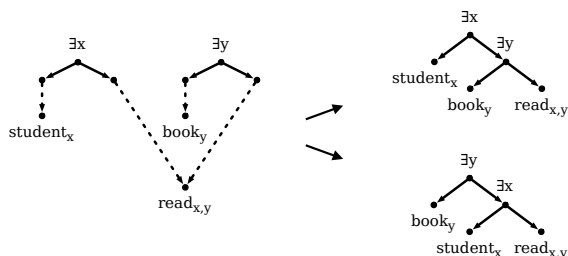
Redundancy Elimination (very short)

Ambiguity, revisited

- *A student reads a book*

(1) $\exists x(\text{student}(x), \exists y(\text{book}(y), \text{read}(x, y)))$

(2) $\exists y(\text{book}(y), \exists x(\text{student}(x), \text{read}(x, y)))$



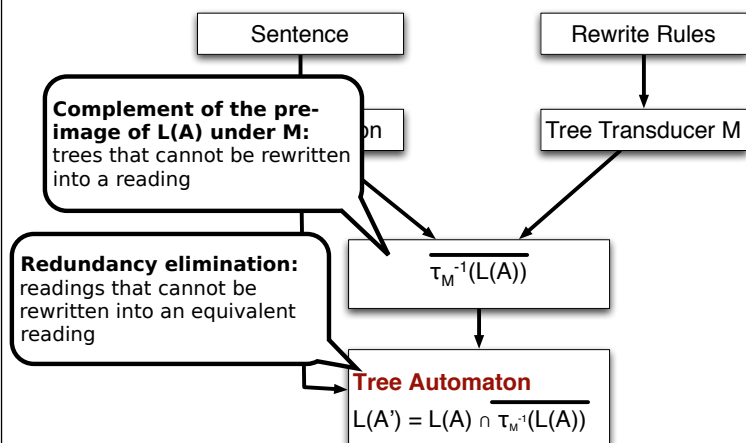
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Redundancy Elimination

- *A student reads a book*
 - (1) $\exists x(\text{student}(x), \exists y(\text{book}(y), \text{read}(x, y)))$
 - (2) $\exists y(\text{book}(y), \exists x(\text{student}(x), \text{read}(x, y)))$
- Readings (1) and (2) are logically equivalent!
- Basic idea:
 - Model the relation between (1) and (2) by rewrite rules
 - Translate the rewrite rules into a tree automaton
 - Redundancy elimination = Intersection of regular tree languages

(Koller & Thater, 2010)

Redundancy Elimination



References

- Alexander Koller and Stefan Thater (2005). The evolution of dominance constraint solvers [PDF]. In Proceedings of the ACL Workshop on Software.
- Alexander Koller and Stefan Thater (2010). Computing Weakest Readings [PDF]. In Proceedings of the 48th Annual Meeting of the Association for Computational Linguistics.
- Hubert Comon, Max Dauchet, Remi Gilleron, Florent Jacquemard, Denis Lugiez, Christof Löding, Sophie Tison, Marc Tommasi. Tree Automata, Techniques and Applications. [PDF]