Computational Linguistics

Algorithms for Scope Underspecification

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Summer 2011

Today

- Refining the solver from the last lecture
- Hypernormally connected dominance graphs
- Tree Automata (in a nutshell)
- Redundancy elimination (very short)

(Bodirsky &al., 2004)

Solving dominance graphs

solve(G = ⟨V, E ⊎ D⟩) = (*)

choose a free fragment F of G else fail

let $G_1, \ldots, G_k$ be the WCC's of G\F

let $\langle V_i, E_i \uplus D_i \rangle = \text{solve}(G_i)$

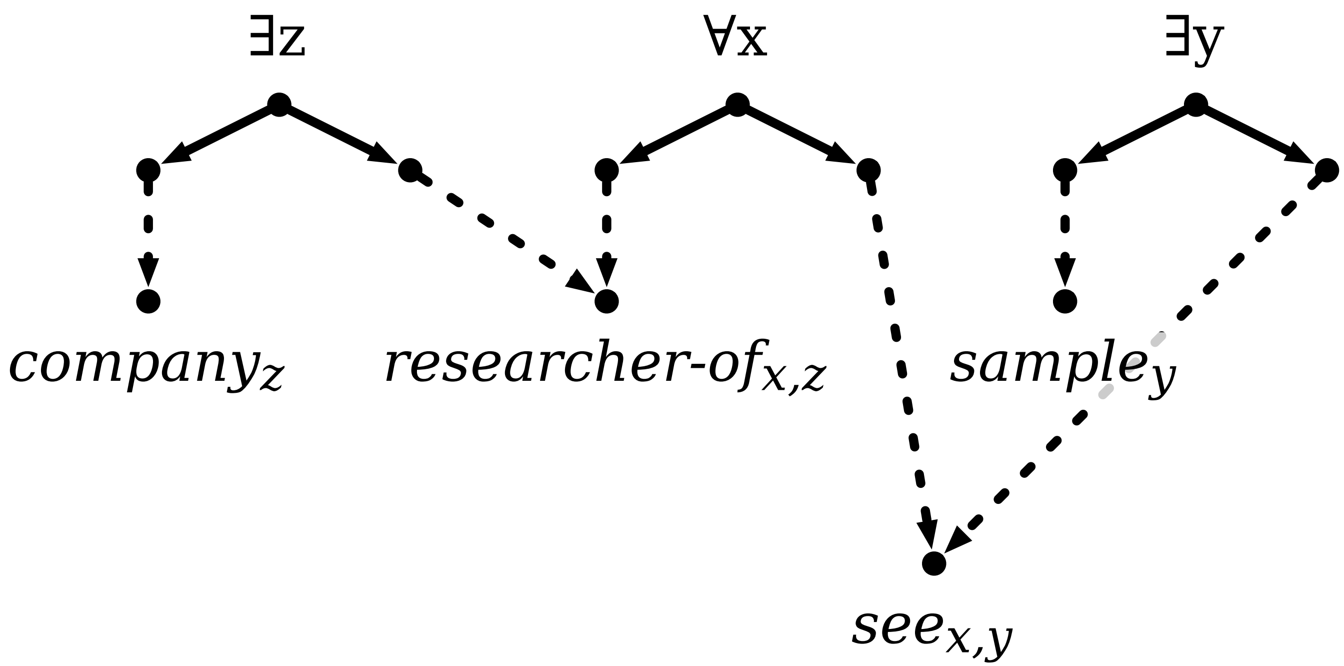
return $\langle V, E \uplus D_1 \uplus \cdots \uplus D_k \uplus D' \rangle$

where D' are dominance edges that connect the holes of F with the solved form of one of the corresponding G_i

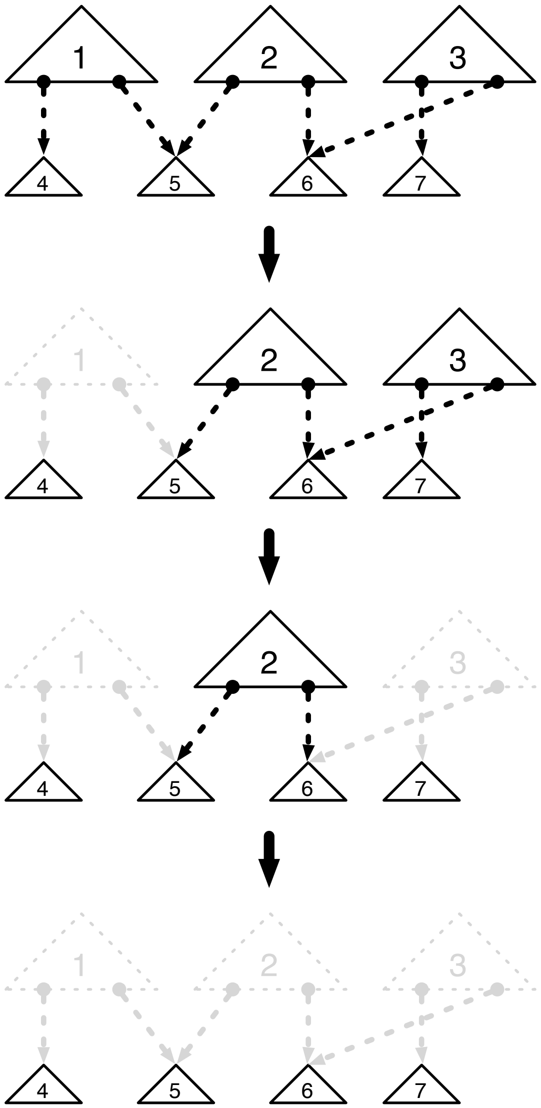
(*) slightly simplified version, works only for connected normal dominance graphs

An Example

- *Every researcher of a company saw a sample.*

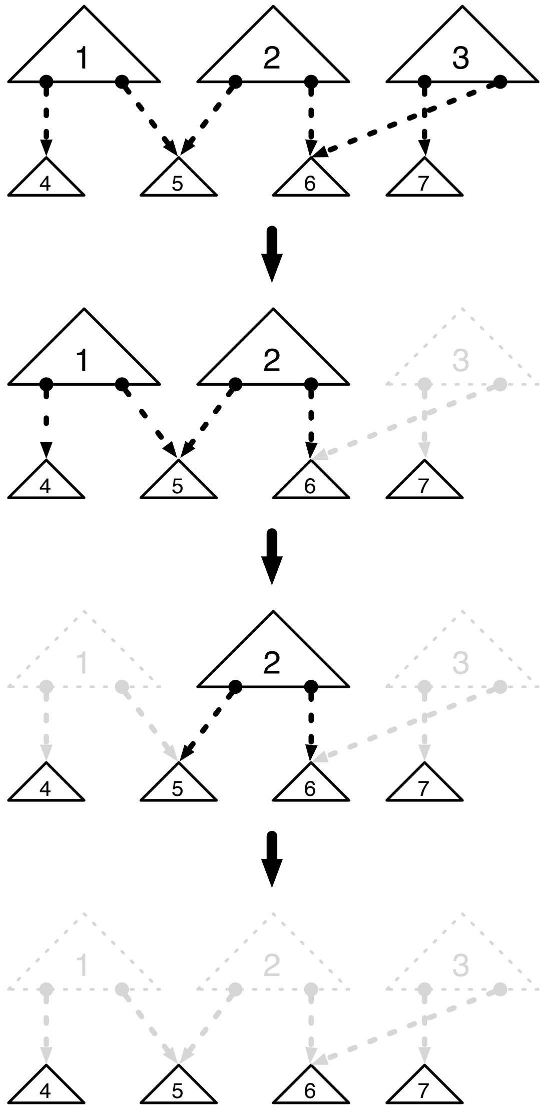


An Example: Run #1



subgraph	free	wccs
{1,...,7}	1	{4}, {2,3,5,6,7}
{2,3,5,6,7}	3	{7}, {2,5,6}
{2,5,6}	2	{5}, {6}

An Example: Run #2

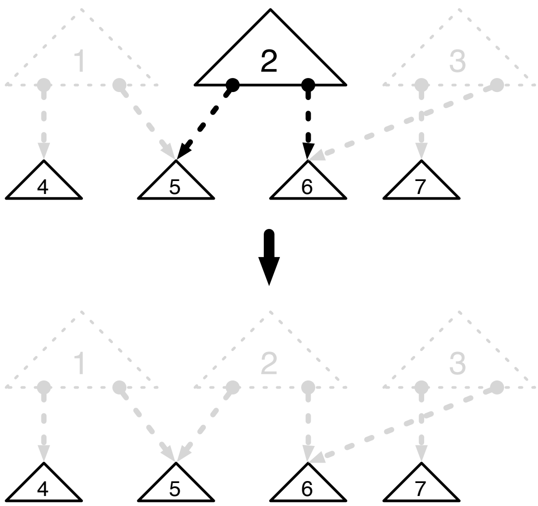


subgraph	free	wccs
{1,...,7}	3	{7}, {1,2,4,5,6}
{1,2,4,5,6}	1	{4}, {2,5,6}
{2,5,6}	2	{5}, {6}

An Example

Problem: The solved form for {2,5,6} is computed twice!

subgraph	free	wccs
{1,...,7}	1	{4}, {2,3,5,6,7}
{2,3,5,6,7}	3	{7}, {2,5,6}
{2,5,6}	**2**	**{5}, {6}**



subgraph	free	wccs
{1,...,7}	3	{7}, {1,2,4,5,6}
{1,2,4,5,6}	1	{4}, {2,5,6}
{2,5,6}	**2**	**{5}, {6}**

Chart-based solver

- **Basic idea:** use dynamic programmic techniques and store intermediate results in a chart-like datastructure
- The chart records how graphs are decomposed into smaller subgraphs if free fragments are removed
  - The chart assigns each subgraph a “split.”
  - Splits consist of (references to) a free fragment F and the weakly connected components of G\F.
    - Notation: $F(G_1, \ldots, G_n)$

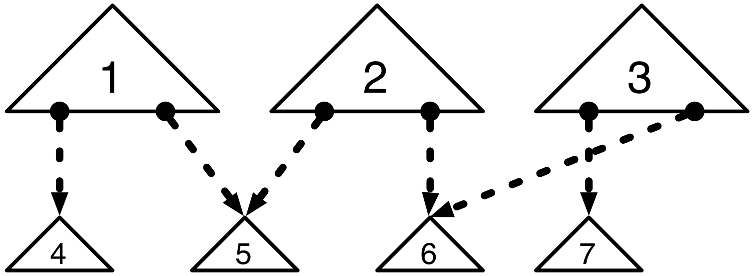
(Koller & Thater, 2005)

The algorithm

```
GRAPH-SOLVER-CHART(G′)
if there is an entry for G′ in the chart
   then return true
free ← FREE-FRAGMENTS(G′)
if free = ∅
   then return false
if G′ contains only one fragment
   then return true

for each F ∈ free
do split ← SPLIT(G′, F)
     for each S ∈ WCCS(G′ − F)
     do if GRAPH-SOLVER-CHART(S) = false
              then return false
     add (G′, split) to the chart
return true
```

An Example



GRAPH-SOLVER-CHART(G')

if there is an entry for G' in the chart
  then return true
free $\leftarrow$ FREE-FRAGMENTS(G')
if *free* $= \emptyset$
  then return false
if G' contains only one fragment
  then return true

for each $F \in$ *free*
do *split* $\leftarrow$ SPLIT(G', F)
  for each $S \in$ WCCS($G' - F$)
  do if GRAPH-SOLVER-CHART(S) = **false**
    then return false
  add (G', *split*) to the chart
return true

subgraph	splits
{1,...,7}	1({4}, {2,3,5,6,7})
	2(**{1,4,5}**, **{3,6,7}**)
	3({1,2,4,5,6}, {7}⟩
{2,3,5,6,7}	2({5}, **{3,6,7}**)
	3(**{2,5,6}**, {7})
{1,2,4,5,6}	1({4}, **{2,5,6}**)
	2(**{1,4,5}**, {6})
{2,5,6}	2({5}, {6})
{3,6,7}	3({7}, {6})
{1,4,5}	1({4}, {5})

Complexity

- Let G be a dominance graph with n nodes and m edges
- The computation of free fragments takes time $O(n + m)$
- The time to compute the chart is $O(n(n + m)\ wcsg(G))$
  - where wcsg(G) is the number of weakly connected subgraphs of G
- Worst case complexity in the number of nodes is $O(n^2 2^n)$
  - $\Rightarrow$ big improvement compared to $O(n^2 n!)$ of the basic solver ($n! =$ upper bound for the number of solved forms).

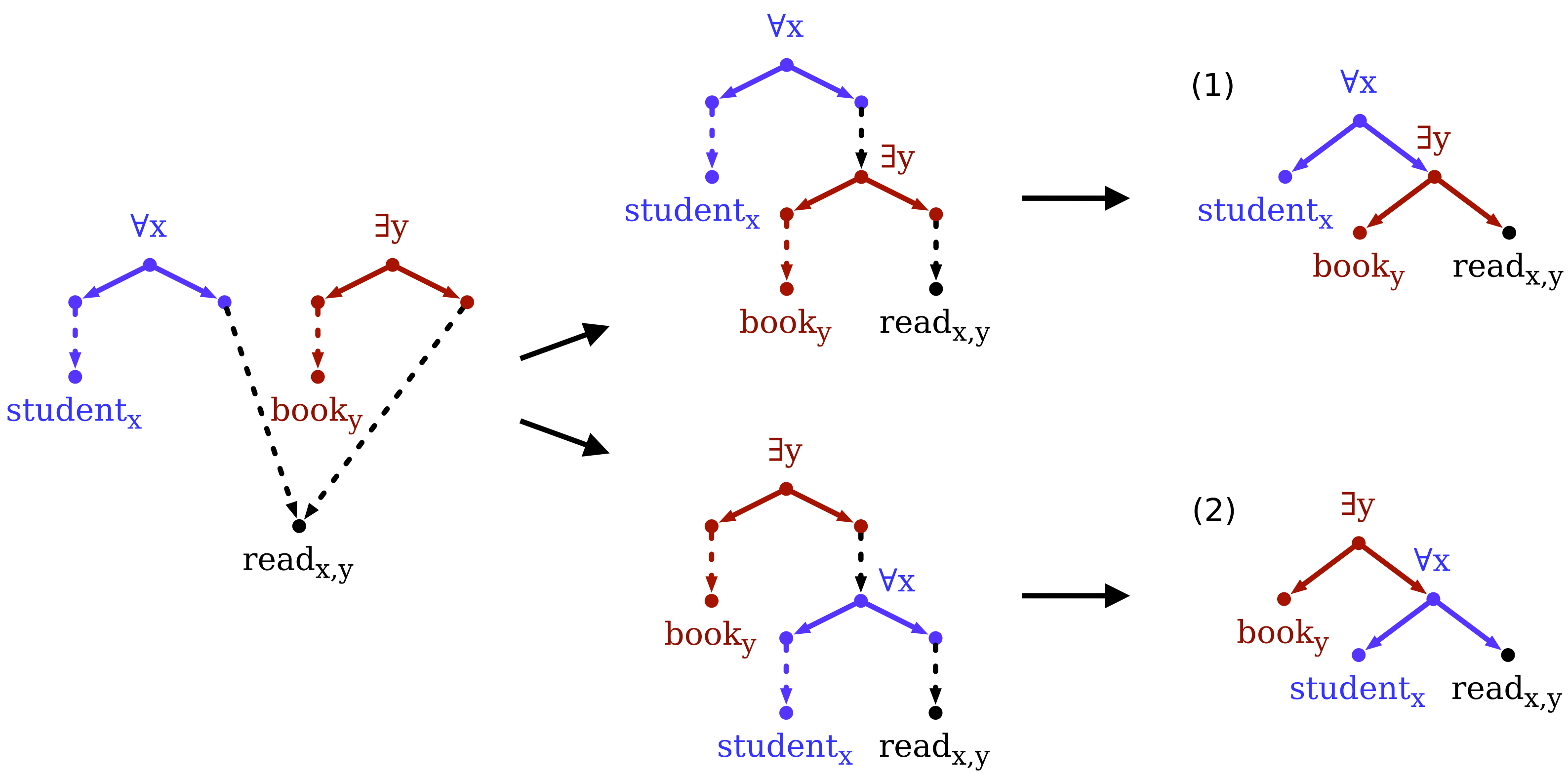
Hypernormally Connected Dominance Graphs

The big picture

- *Every student reads a book*

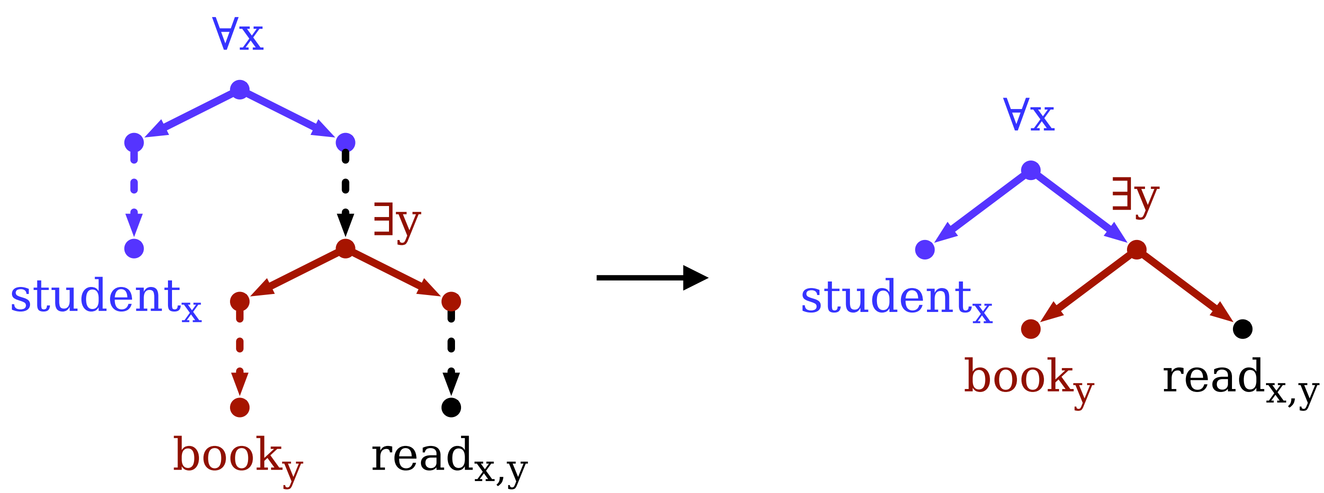
  (1) ∀x(student(x), ∃y(book(y), read(x, y))

  (2) ∃y(book(y), ∀x(student(x), read(x, y))



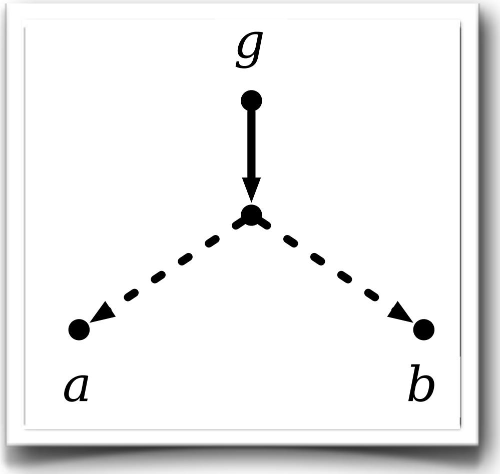
Simple solved forms

- We are usually interested only in solved forms in which every hole is related to exactly one root
  - let's call such solved forms "simple"
- Simple solved forms can be mapped to "proper" trees simply by "plugging" the hole with the root connected to it by a dominance edge.



Not all solved forms are simple

- **Problem:**
  Not all solved forms are simple
- **Solution:**
  Identify a class of dominance graphs that only have simple solved forms
- ⇒ Hypernormally connected dominance graphs

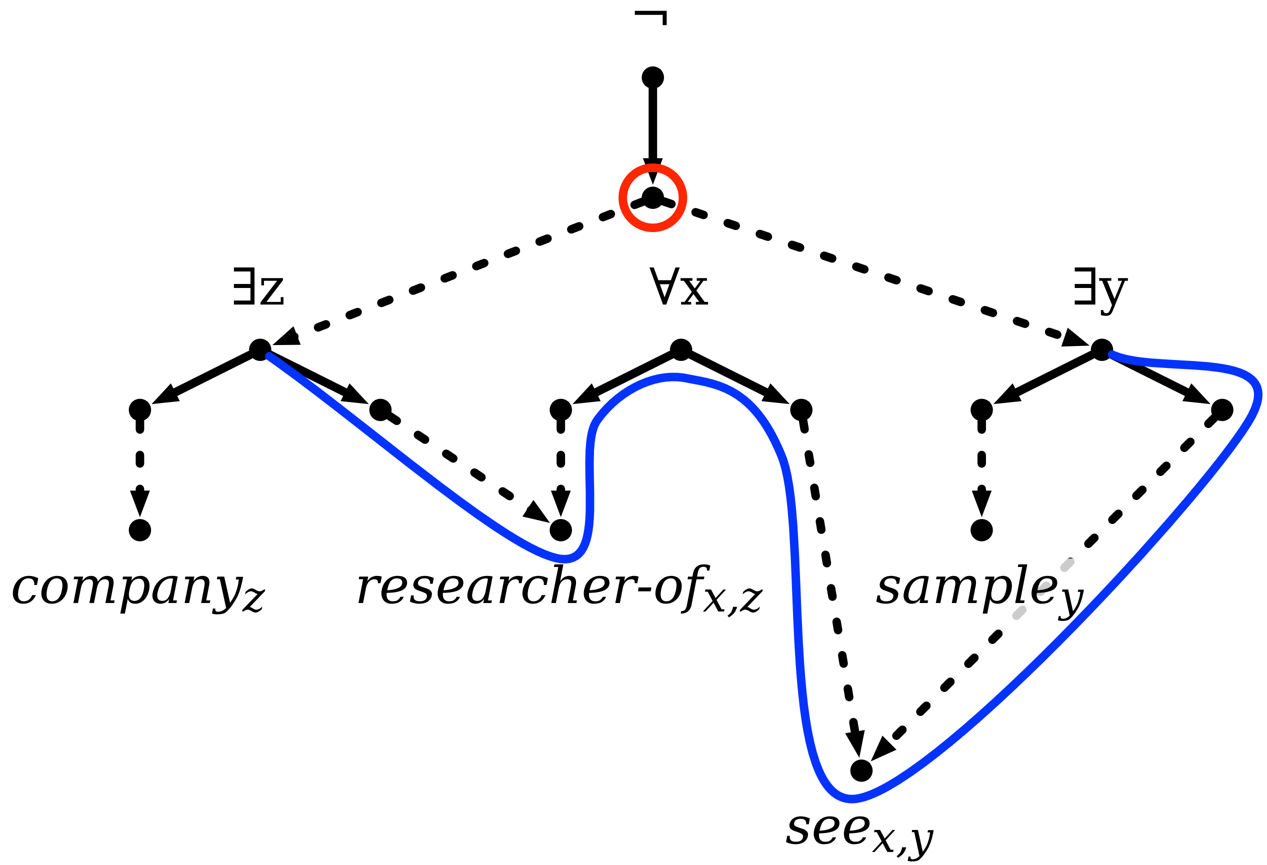


Hypernormally connected dominance graphs

- A **hypernormal path** in a (normal) dominance graph G is a path in the undirected version of G that does not use two dominance edges incident to the same hole.
- A (normal) dominance graph G is **hypernormally connected** if each pair of nodes is connected by some hypernormal path.

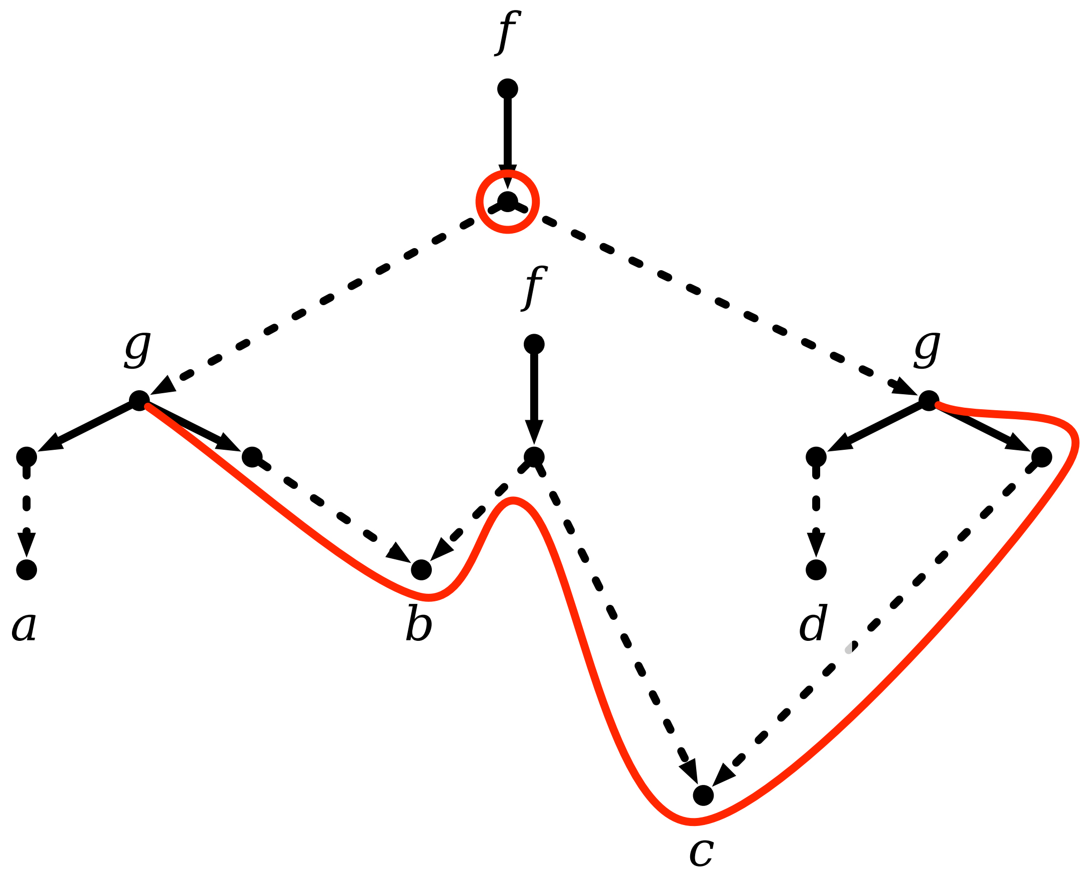
Hypernormally connected dominance graphs

- Hypernormally connected:



Hypernormally connected dominance graphs

- Not hypernormally connected:



Hypernormally connected dominance graphs

- **Lemma:** if G is a hypenormally connected normal dominance graph with free fragment F, then all WCCs of G\F are hypernormally connected.
- **Proposition:** if a normal dominance graph is hypernormally connected, then all its (minimal) solved forms are simple.

Tree Automata

(see Comon &al., 2007)

Bottom-up Tree Automaton

- **A tree automaton** is a tuple $A = \langle Q, \Sigma, Q_f, \Delta \rangle$
  - Σ a finite ranked signature
  - Q a finite set of states
  - $Q_f \subseteq Q$ a finite set of final (accepting) states
  - Δ a finite set of transition rules
- **Transition rules:**
  - $f(q_1(x_1), \ldots, q_n(x_n)) \rightarrow q(f(x_1, \ldots, x_n))$
    - $f \in \Sigma$
    - $q, q_1, \ldots, q_n \in Q$
    - $x_1, \ldots, x_n$ different variables

An Example Computation

- $Q = \{q_3, q_4, \ldots\}$, $\Sigma = \{\exists x_{|2}, book_{y|0}, \ldots\}$, $Q_f = \{q_{12345}\}$

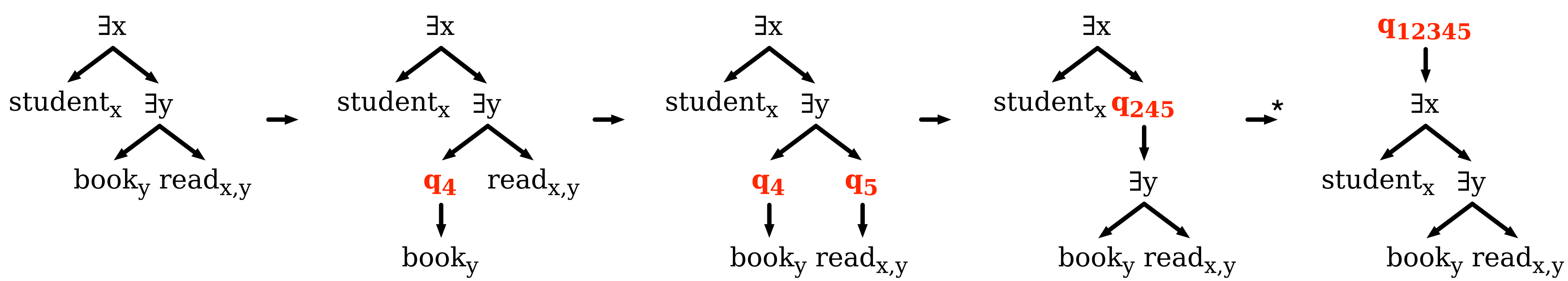
$$\exists x(q_3(x_1), q_{245}(x_2)) \rightarrow q_{12345}(\exists x(x_1, x_2))$$

$$\exists y(q_4(x_1), q_5(x_2)) \rightarrow q_{245}(\exists y(x_1, x_2))$$

$$student_x \rightarrow q_3(student_x)$$

$$book_y \rightarrow q_4(book_y)$$

$$read_{x,y} \rightarrow q_5(read_{x,y})$$



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$$\text{student}_x \rightarrow q_3(\text{student}_x)$$

$$\text{book}_y \rightarrow q_4(\text{book}_y)$$

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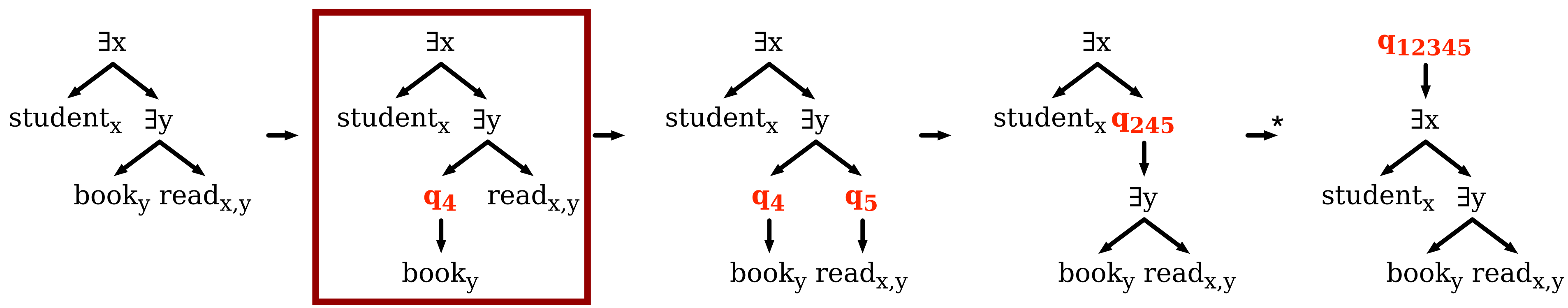

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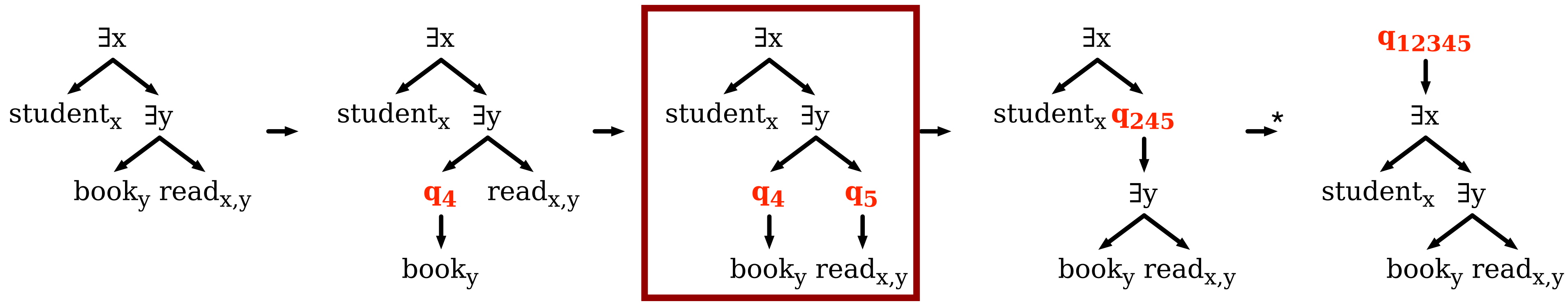

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$$\exists x(q_3(x_1), q_{245}(x_2)) \to q_{12345}(\exists x(x_1, x_2))$$

$$\boxed{\exists y(q_4(x_1), q_5(x_2)) \to q_{245}(\exists y(x_1, x_2))}$$

$$\text{student}_x \to q_3(\text{student}_x)$$

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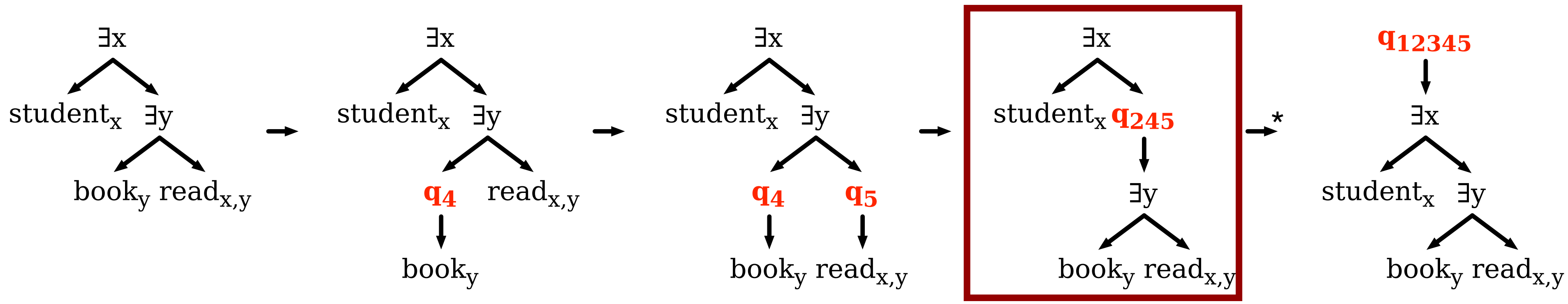

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$$\exists y(q_4(x_1), q_5(x_2)) \rightarrow q_{245}(\exists y(x_1, x_2))$$

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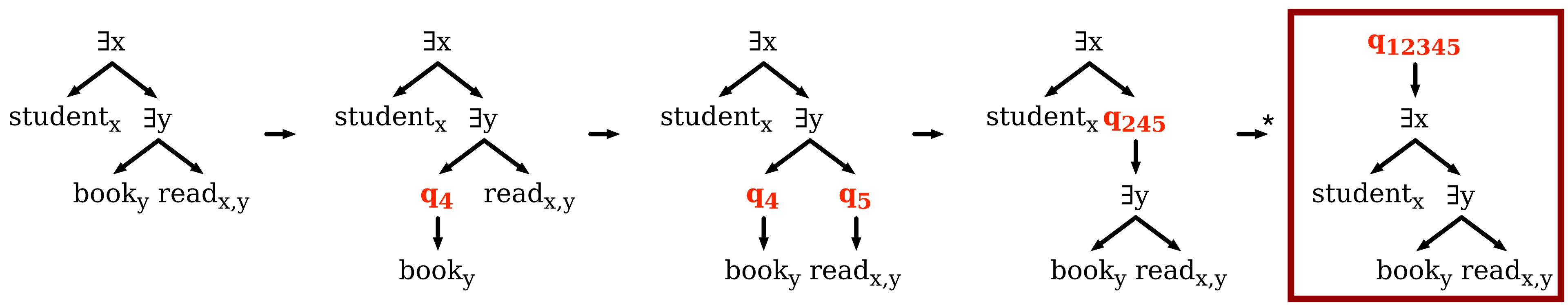


Bottom-up Tree Automaton

- A tree t is **accepted** by an atomaton $A = \langle Q, \Sigma, Q_f, \Delta \rangle$ if
  - $t \rightarrow^* q(t)$
  - $q \in Q_f$
- **The language L(A)** of trees recognized by A is the set of trees accpeted by A.

Another Example

- **Automaton** (rules)

$\exists x(q_3(x_1), q_{245}(x_2)) \rightarrow q_{12345}(\exists x(x_1, x_2))$

$\exists y(q_4(x_1), q_{135}(x_2)) \rightarrow q_{12345}(\exists y(x_1, x_2))$

$\exists y(q_4(x_1), q_5(x_2)) \rightarrow q_{245}(\exists y(x_1, x_2))$

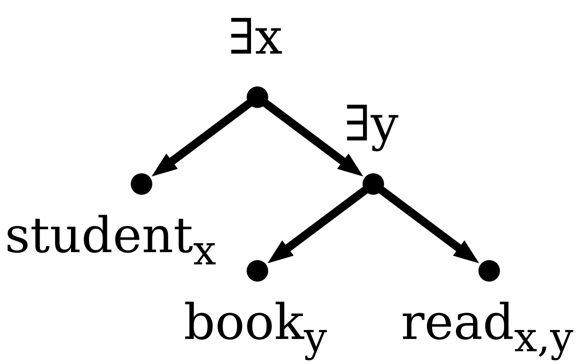
$\exists x(q_3(x_1), q_4(x_2)) \rightarrow q_{135}(\exists y(x_1, x_2))$

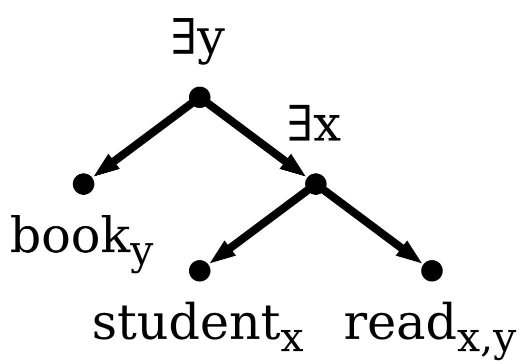
$student_x \rightarrow q_3(student_x)$

$book_y \rightarrow q_4(book_y)$

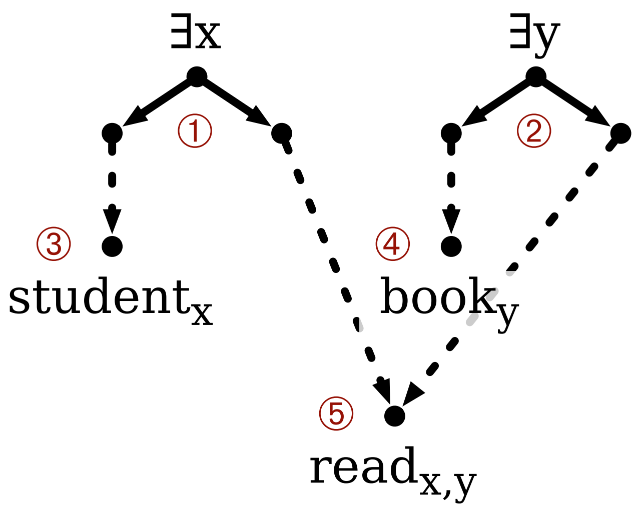
$read_{x,y} \rightarrow q_5(read_{x,y})$

- **Accepted Trees:**





Back to dominance charts



dominance chart	
{1,2,3,4,5}	1({3}, {4,5,6})
	2({4}, {1,3,5})
{2,4,5}	2({4}, {5})
{1,3,5}	1({3}, {5})

Compare

tree automaton	dominance chart
$\exists x(q_3(x_1), q_{245}(x_2)) \rightarrow q_{12345}(\exists x(x_1, x_2))$	$\{1,2,3,4,5\}$ $1(\{3\}, \{4,5,6\})$
$\exists y(q_4(x_1), q_{135}(x_2)) \rightarrow q_{12345}(\exists y(x_1, x_2))$	$2(\{4\}, \{1,3,5\})$
$\exists y(q_4(x_1), q_5(x_2)) \rightarrow q_{245}(\exists y(x_1, x_2))$	$\{2,4,5\}$ $2(\{4\}, \{5\})$
$\exists x(q_3(x_1), q_4(x_2)) \rightarrow q_{135}(\exists y(x_1, x_2))$	$\{1,3,5\}$ $1(\{3\}, \{5\})$
$student_x \rightarrow q_3(student_x)$	
$book_y \rightarrow q_4(book_y)$	
$read_{x,y} \rightarrow q_5(read_{x,y})$	

Charts = Tree automata

- Dominance charts can be seen as (or translated into) tree automata
  - ... provided that the original dominance graph is hypernormally connected (why?)

Charts = Tree automata

- The class of recognizable languages is closed under
  - Union
  - Complement
  - Intersection
- ⇒ We can model certain inferences on the level of dominance charts by intersecting regular tree languages
  - Redundancy elimination
  - Weakest readings

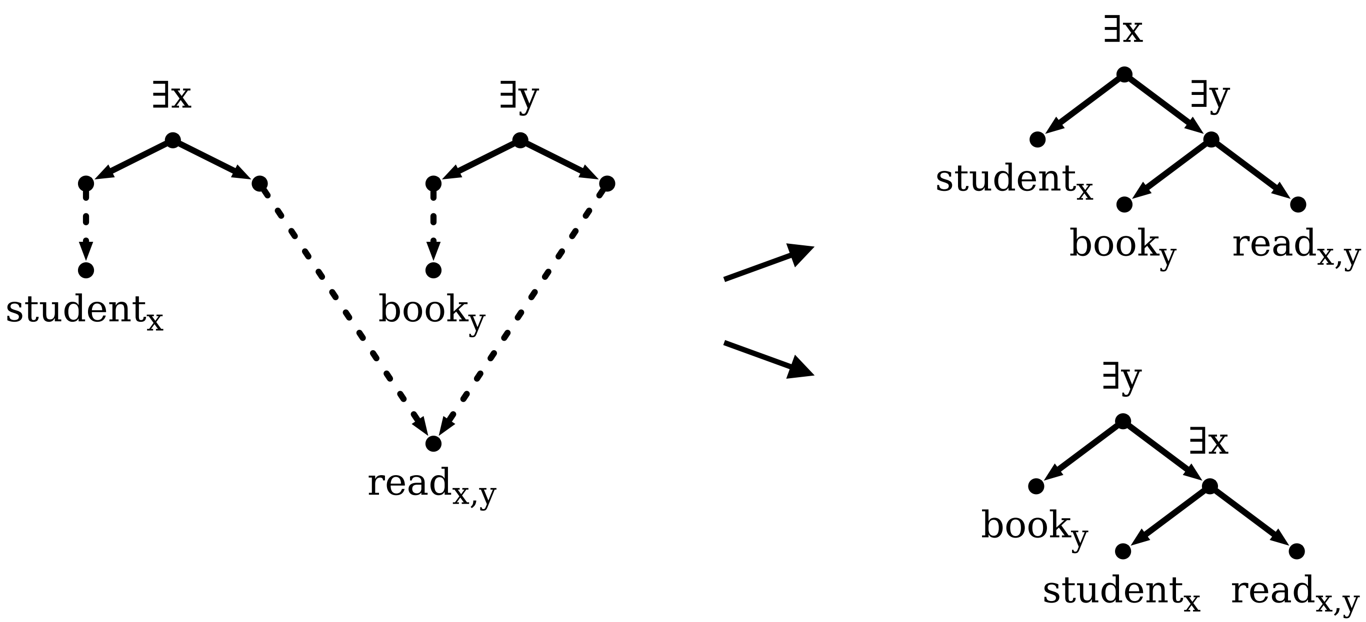
Redundancy Elimination (very short)

Ambiguity, revisited

- *A student reads a book*

  (1) ∃x(student(x), ∃y(book(y), read(x, y))

  (2) ∃y(book(y), ∃x(student(x), read(x, y))



Redundancy Elimination

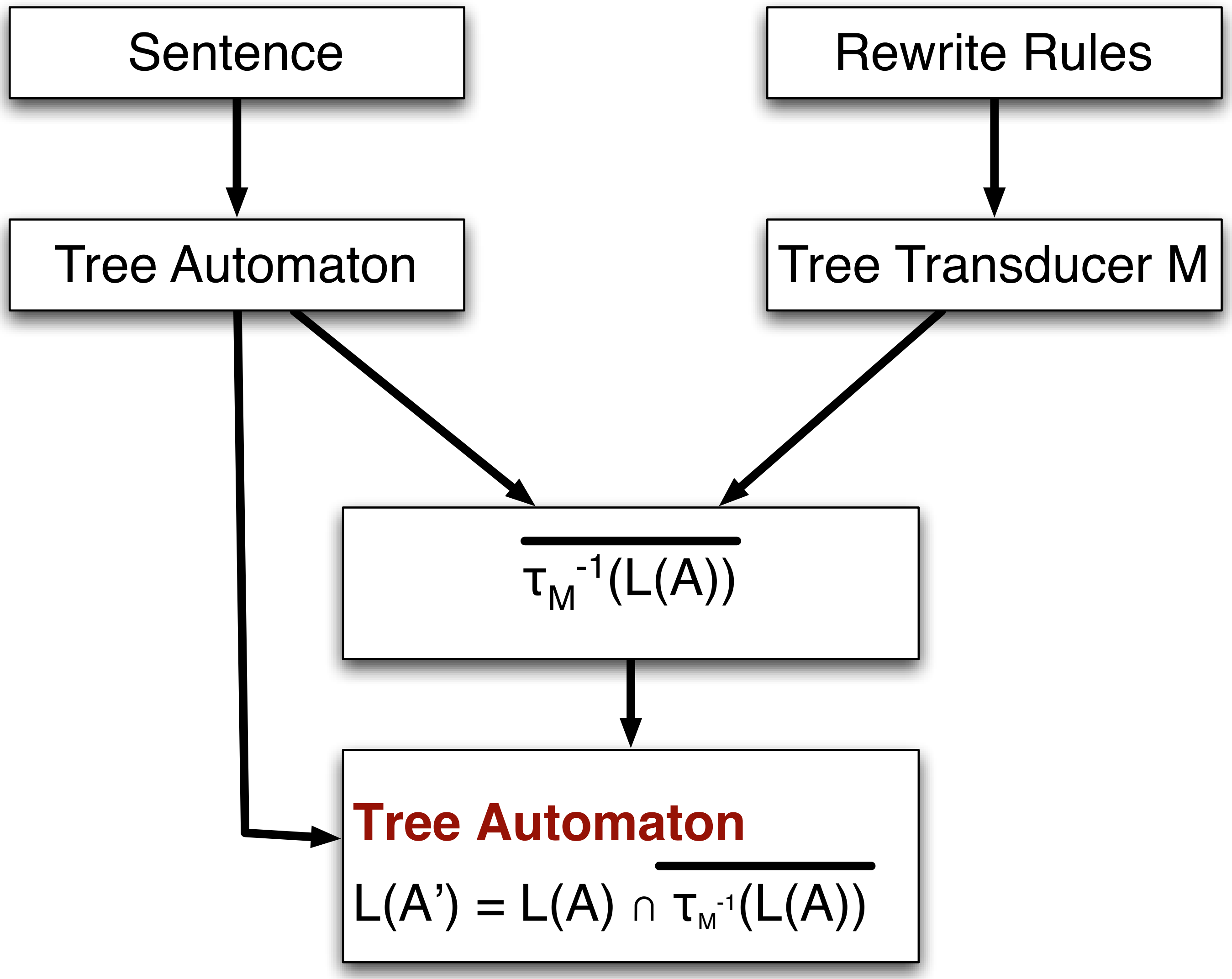
- *A student reads a book*

  (1) $\exists x(\text{student}(x), \exists y(\text{book}(y), \text{read}(x, y))$

  (2) $\exists y(\text{book}(y), \exists x(\text{student}(x), \text{read}(x, y))$

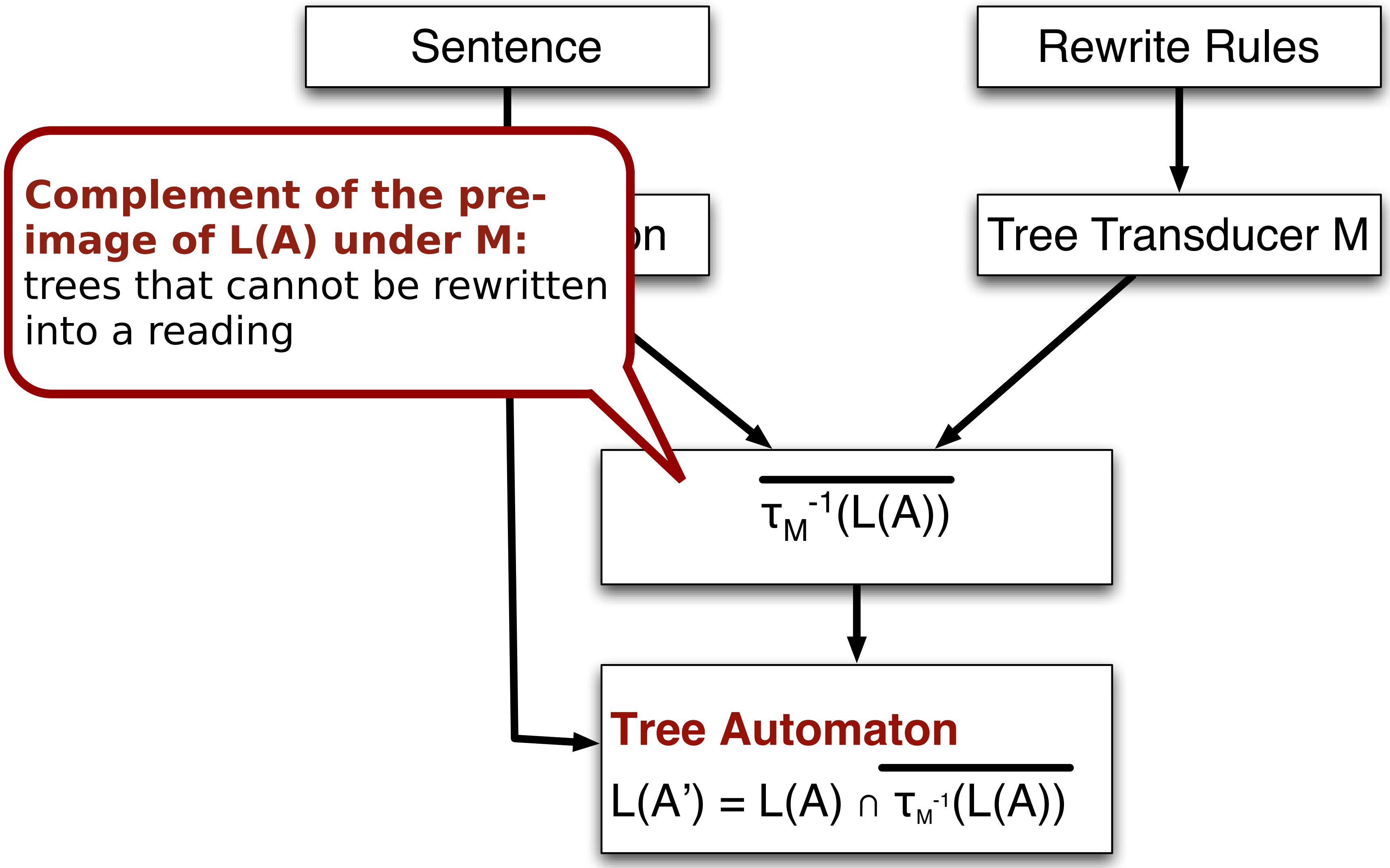
- Readings (1) and (2) are logically equivalent!
- Basic idea:
  - Model the relation between (1) and (2) by rewrite rules
  - Translate the rewrite rules into a tree automaton
  - Redundancy elimination = Intersection of regular tree languages

Redundancy Elimination



(Koller & Thater, 2010)

Redundancy Elimination



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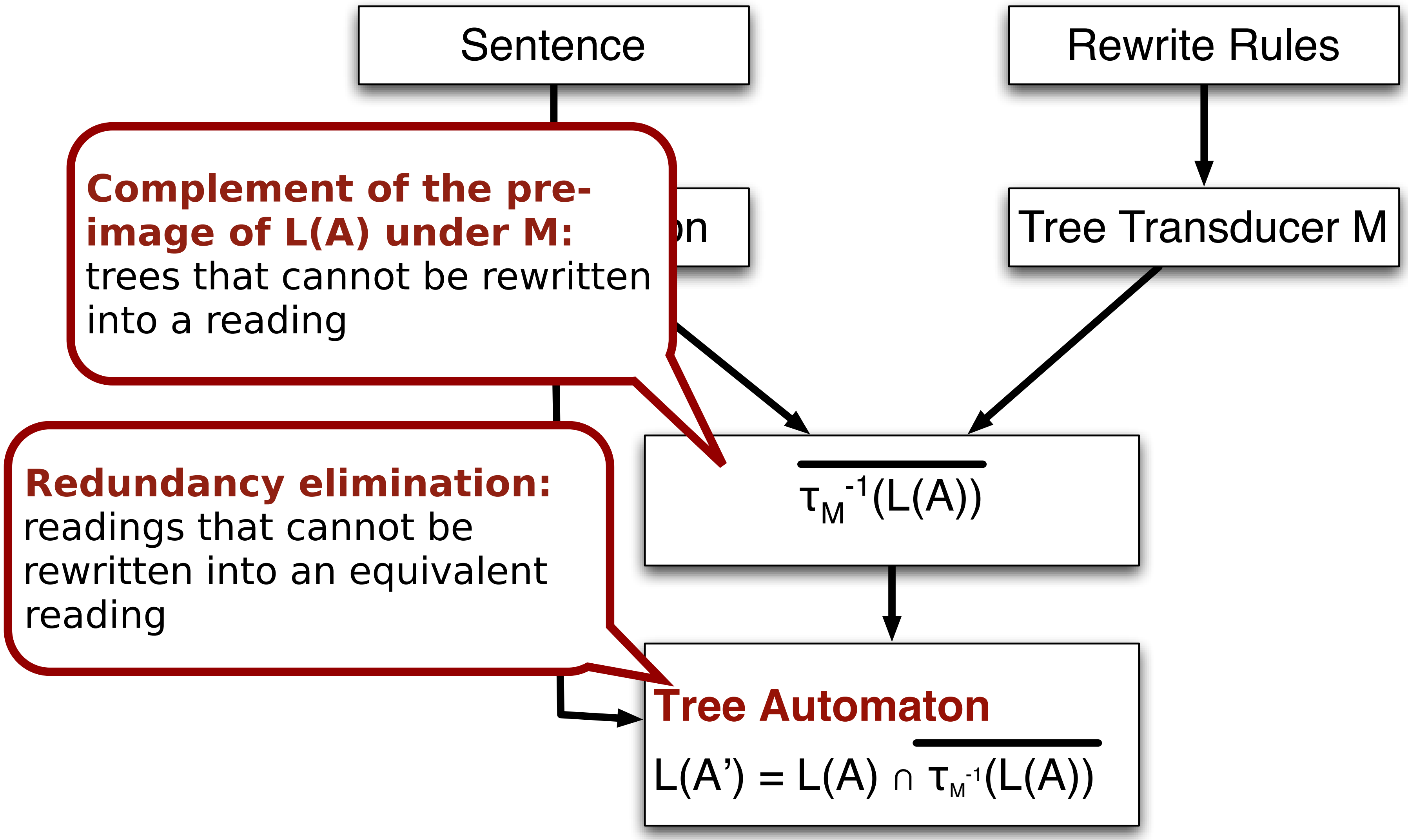
Redundancy Elimination



References

- Alexander Koller and Stefan Thater (2005). The evolution of dominance constraint solvers [PDF]. In Proceedings of the ACL Workshop on Software.
- Alexander Koller and Stefan Thater (2010). Computing Weakest Readings [PDF]. In Proceedings of the 48th Annual Meeting of the Association for Computational Linguistics.
- Hubert Comon, Max Dauchet, Remi Gilleron, Florent Jacquemard, Denis Lugiez, Christof Löding, Sophie Tison, Marc Tommasi. Tree Automata, Techniques and Applications. [PDF]