Computational Linguistics, Summer 2011, Exercise “Underspecification”

1. Consider the following tree automaton

$A = \langle Q = \{q\}, \Sigma = \{\neg_{|1}, \exists x_{|2}, \exists y_{|2}, stud_{x|0}, book_{y|0}, read_{x,y|0}\}, Q_f = \{q\}, \Delta\rangle$

where Δ contains the following transition rules:

$$\begin{aligned}
\neg(q(x_1)) &\to q(\neg(x_1)) \\
\exists x(q(x_1), q(x_2)) &\to q(\exists x(x_1, x_2)) \\
\exists y(q(x_1), q(x_2)) &\to q(\exists y(x_1, x_2)) \\
stud_x &\to q(stud_x) \\
book_y &\to q(book_y) \\
read_{x,y} &\to q(read_{x,y})
\end{aligned}$$

This automaton simply accepts *all* trees over Σ.

(a) Modify (extend) the automaton so that it accepts all trees over Σ *except* those trees that contain ∃x(_, ∃y(_, _)) as a subtree (trees where ∃y occurs as the right child of ∃x).

(b) Intersect the automaton from (a) with the automaton on slide 24 from the lecture.

## Appendix: Intersection (see Comon &al. 2007, page 29)

Let $A_1 = \langle Q_1, \Sigma, Q_{f1}, \Delta_1\rangle$ and $A_2 = \langle Q_2, \Sigma, Q_{f2}, \Delta_2\rangle$ be two tree automata. An automaton that accepts $L(A_1) \cap L(A_2)$ can be defined as follows:

$A = \langle Q_1 \times Q_2, \Sigma, Q_{f1} \times Q_{f2}, \Delta\rangle$

$$\begin{aligned}
\Delta = \{\, & f(\langle p_1,q_1\rangle(x1), \ldots, \langle p_n,q_n\rangle(xn)) \to \langle p,q\rangle(f(x_1, \ldots, x_n)) \mid \\
& f(p_1(x_1), \ldots, p_n(x_n)) \to p(f(x_1, \ldots, x_n)) \in \Delta_1 \text{ and} \\
& f(q_1(x_1), \ldots, q_n(x_n)) \to q(f(x_1, \ldots, x_n)) \in \Delta_2 \}
\end{aligned}$$