THE THEORY OF PHONEMIC ANALYSIS*

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Alternate symbolizations which are proposed for a particular dialect of a language sometimes differ only in a trivial manner. Such symbolizations may not have a one-to-one correspondence, but there may exist some function which maps the symbols of one representation onto the symbols of the other. Some symbolizations, however, differ in non-trivial ways. These differences result from differences in underlying assumptions, i.e. in the theories upon which the symbolizations are based. While such differences may result from disagreements about the nature of the speech data to be symbolized, they generally result from differences of a theoretical nature which are independent of the details of the particular language to be symbolized. Differences at this level may be based upon even more fundamental differences concerning the concept of theory and the philosophy of science. By this time the levels of abstraction have become sufficiently complicated that some terminological specifications are necessary.

THEORY

In the sciences the term theory is often used in a relatively imprecise manner. In mathematics, however, the term theory is primarily restricted to axiomatic systems. In an axiomatic system there are undefined terms (or primitives) and axioms; either explicitly or implicitly these axioms include the basis of logical inference. From the undefined terms and axioms, definitions are formed; and from the complete system of undefined terms, axioms, and definitions, theorems may be constructed.

Scientific theories are not independent of mathematical theories. For the physical, biological, and social sciences, mathematics provides the theoretical abstractions by means of which facts and data can be related and interpreted. It is unnecessary that a mathematical theory be related to some aspect of the real world; however, there is no doubt that the real world has prompted many of the abstract axiom systems which have been developed in mathematics.

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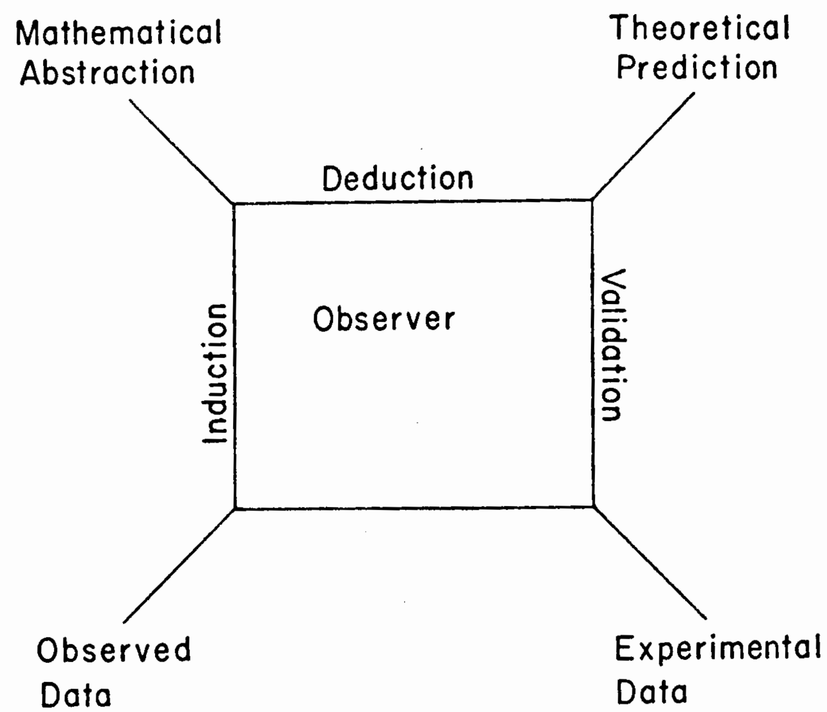


Fig. 1. The basic form and processes which constitute a scientific theory.

A scientific theory, then, involves a mathematical theory, but it also must include statements of the correspondences between the various properties of the real world and the mathematical theory. The mathematical theory may be well developed, but until the correspondences are clearly specified, we have a hypothesis rather than a scientific theory. The problem of constructing a scientific theory is thus basically that of finding the relevant mathematical system and of specifying appropriate relations or correspondences between the scientific observations or data and the mathematical system. Sometimes the relevant mathematics is trivial but the discovery of the appropriate correspondences is exceedingly difficult; sometimes the mathematics, in turn, is either exceedingly complex or has not yet been developed.

The properties of a scientific theory are suggested in Figure 1. An abstract mathematical interpretation of observed data is accomplished through induction, a process assigned to the observer shown at the centre of the figure. The mathematical model leads to deductions from which mathematical predictions can be made. The observer may verify or validate these predictions by obtaining new data through experimental procedures.

The nature of the theoretical predictions is an essential consideration. A theory rarely predicts the time and place of specific events; in fact, the prediction that a specific event will occur at all is exceptional. Rather, a theory generally expresses the laws which govern those events which do occur. Thus, usually it is the nature of an event, not the event itself which is predicted by scientific theory. If events occur which are different in nature from that required by a theory, then the theory must be revised or abandoned.

In effect, the diagram of Figure 1 is an operational statement (indeed a hypothesis) of scientific theory. The primary thesis of the above development is that data, mathematical abstractions, and correspondences are the essence of scientific theory. There may be semantic disagreements over the details, but it is proposed here that any theory of scientific import must have these three major components.

SPEECH SYMBOLIZATION

The above considerations may seem far removed from the problems of the theory of phonemic analysis. If substantial linguistic theories are to be constructed, however, they must have the properties of a scientific theory. It should be noted that a scientific theory, rather than only a mathematical theory, is required; for language involves a real system rather than an abstraction, and as such comprises a portion of behavioral science. The notations often employed in describing specific dialects of languages, however, suggest the application of mathematical concepts and methods to language description.

The characteristics of spoken language may be considered in terms of the properties of a communication channel. The transmitting terminal of such a channel may be analyzed into three basic components: an information source, an encoder, and a transducer. While much is yet to be learned about the manner in which the information is organized by the human nervous system, this system is clearly the information source for speech. It is the central nervous system of the human which performs the encoding for speech production. The nature of the neural code and the encoding process is again a subject about which little is known. Possibly at relatively high functional levels within the brain there are simple organizations of neural impulses which form the basic code. It is certain, however, that the pattern of impulses in the motor nerves which control the vocal mechanism is highly complex and must be highly coordinated for normal speech production. According to this view, the motor mechanism of speech is a transducer which generates the discontinuous analog acoustical functions of speech. The entire mechanism acts as a servosystem with special, nonlinear properties. The prescribed or target values of the servo are not external, but are stored in the neural control system. The speech signal which results may be considered a unidimensional function of time, but upon analysis it is found to involve a highly complicated set of discontinuous parameters.

By means of a rather complicated set of procedures, it is possible to develop a discrete symbolization which will effectively represent the speech signal. It seems reasonable to say that the *symbolization is valid* if all utterances which are represented by corresponding symbol sequences have equivalent information content. Such utterances may be considered semantically equivalent, and may be defined as those utterances which are functionally equivalent in the control of behavior. It seems possible that more than one symbolization may be valid for a given language. The efficiency of valid symbolizations, however, may differ and may be evaluated in terms of a criterion of simplicity. Since the properties of the neural code itself are not known, there is at present no evidence that a one-to-one correspondence exists between the elements of some particular valid symbolization and the elements of the neural code by which the motor mechanism of speech is controlled.

THE NATURE OF PHONOLOGY

Phonemic theory is, of course, basic to the theory of phonemic analysis. Analysis procedures are essential to the development of a phonemic theory, and thus are also essential to the phonemicization of a specific dialect of a language. The phonological description of a specific dialect of a language includes its phonemicization and also a specification of the allowable phonemic and prosodemic sequences within the language. The concept of prosodeme is employed here in the manner specified in the paper on "Foundations of Phonemic Theory" by Peterson and Harary (3). In that development vowel and consonant duration, fundamental laryngeal frequency, and speech production power are defined as the physiological prosodic parameters. A prosody is a vowel or consonant duration or a prosodic parameter value or sequence of such values which contains an approximation to a steady-state or a steady-state with an associated controlled movement. Prosodemes are related to prosodies in a form which is parallel to the relation of phonemes to phones. In the present paper the theory of phonemic analysis will be considered in relation to general phonemic theory; the format for presenting the phonological description of specific languages will also be considered. Phonemic theory and the nature of phonological descriptions will be given priority over problems in the phonemic analysis of individual dialects of languages, since they are basic to such analyses.

It is convenient to distinguish those units of a linguistic system which are encompassed by phonology. Those elementary properties and events of speech and recurrent sequences of those properties and events which do not necessarily transmit information when produced as isolated entities constitute the subject matter of phonology. Thus, as defined here, phonemes and prosodemes, and recurrent phoneme and prosodeme sequences which do not normally constitute grammatical units are basic concepts within phonology.

The specification of grammatical units and the relationships among them are essential problems of a general grammatical theory. A format is required for the description of the grammatical system of specific languages. A complete linguistic description of a specific dialect of a language not only presents its grammatical and phonological structure, but also specifies the manner in which its grammatical units are related to its phonological units.

Domain. There are several different domains of speech which may be considered in the development of a phonemic theory and in the phonological description of a language. These will not be examined in detail here, but the major factors are suggested as follows:

1. Phonetic
   a. Syllables
   b. Speech between pauses
2. Grammatical
   a. Speech between junctures
   b. Morphemes
   c. Words
   d. Phrases
   e. Sentences
3. Stylistic
   a. Formal or maximally distinct
   b. Casual or conversational
   c. All forms of discourse
4. Dialectal
   a. A single speaker at a particular time
   b. A single speaker
   c. A single dialect
   d. All dialects of a particular language

The power of a phonemic theory is determined by the extent of the domain for which it provides successful predictions.

Aspect. The particular aspect of the speech process within which a phonemic theory may be formulated has received frequent consideration. At least five different stages of the process of speech communication can be distinguished:

1. Neurological innervation of the motor speech mechanism
2. Physiological production of speech
3. Acoustical speech waves
4. Physiological response of the auditory mechanism
5. Neural activities involved in speech perception

Various aspects have been considered as the basis for phonemic theory. Some authors have suggested that the choice of aspect is irrelevant, but most of those working with speech have chosen the physiological mechanism of production as fundamental. This results from the fact that the restrictions and constraints on the motor mechanism of speech production appear to be more severe than on other aspects of the speech communication process. Thus it appears that the speech signal is organized primarily in terms of the contraints and the possible formations of this mechanism.

Spectrographic studies have aided greatly in determining the nature of physiological speech formations. With the many advances in specifying the transformation from physiological to acoustical speech parameters, it may become possible to state phonemic theory in acoustical terms. There is a certain appeal in this possibility, since the acoustical speech signal is much more accessible for detailed measurement and study than are other aspects of the speech process.

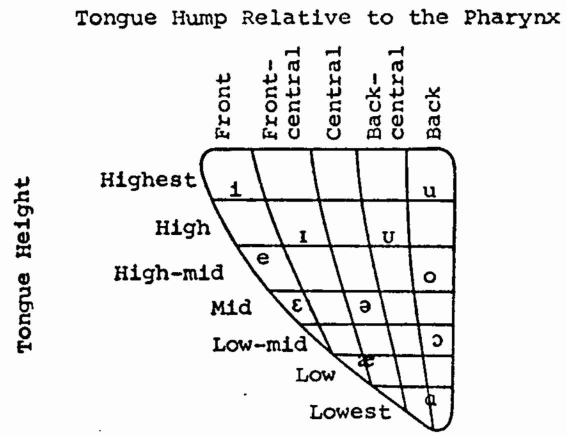
It should be noted that at present it would be impossible to develop a precise phonemic theory based upon the psychophysics of speech perception or upon the neural patterns associated with the interpretation of speech. Only the most elementary quantitative knowledge of these subjects is currently available. The work of Lane, Catania, and Stevens (2), for example, is a contribution to the psychophysics of speech perception, but very few studies of this general nature have thus far been conducted. There is no basic theory of the psychophysics of speech perception, for example, which is parallel in extent to the acoustic theory of speech production by Fant (1).

PHONETIC THEORY

In a phonemic theory the correspondences must be defined between properties and events of some specific aspect of speech (e.g. speech physiology) and the abstractions of the theory. The speech must first be described in terms of a set of variables or parameters. In effect, such a description is a phonetic theory, and in such a theory both the mathematical abstractions and the correspondences may be simple. If speech physiology is chosen as the basis for the theory, then a set of terms such as those given in Figure 2 may be employed in the description of speech. The constructs of this figure are obviously closely related to those conventional in descriptive phonetics. (The symbols shown in the figure identify selected canonical allophones of the phonemes of Midwestern American English according to the specifications of the phonemic theory of Peterson and Harary.)

Other types of general phonetic descriptions may be employed; the format presented in Figure 2 is intended to provide a compact description of essentially all of the various speech sound formations which are employed in the transmission of information by means of speech. Each general division is treated as a parameter, and the subdivisions are considered to be parameter values. Thus any given single speech formation or phonetic unit is specified by a set of one or more parameter values. For example, [m] may be considered a bilabial-nasal-voiced-egressive speech formation. A set of several different types of phones is specified by the term bilabial, and a set which intersects with the bilabial set is specified by the term nasal; other sets of phone types are specified by the terms voiced and egressive. The intersection of these four sets is the phone type [m].

Primary Vowel Parameters



Primary Consonant Parameters

Place of Articulation

Manner of Articulation	Bilabial	Labio-dental	Dental	Alveolar	Alveolo-palatal	Palatal	Velar	Uvular	Pharyngeal	Glottal
Nasal	m			n			ŋ			
Sonorant	w			l	r	j				
Fricative	ʍ	fv	θð							h
Sibilant				sz	ʃʒ					
Trill										
Flap										
Plosive	pb			td			kg			
Click										

Modifying Vowel and Consonant Parameters

Air Direction
- Egressive - all
- Ingressive

Laryngeal Actions
- Whispered
- Breathy (V)
- Clear (V) - all
- Laryngealized (V)
- Voiceless (C) ʍ,f,θ,h,s,ʃ,p,t,k
- Voiced (C) m,n,ŋ,w,l,r,j,v,ð,z,ʒ,b,d,g

Secondary Articulations
- Spread (V)
- Rounded (V) u,ʊ,o,ɔ
- Labialized (C)
- Palatalized (C)
- Lateralized l
- Retroflexed r
- Velarized w,r,ʍ
- Nasalized
- Pharyngealized
- Glottalized (C)

PROSODIC PARAMETERS

Vowel and Consonant Duration
Short Mid Long Extra-long

Fundamental Laryngeal Frequency
Low Low-mid Mid Mid-high High Extra-high

Speech Production Power
Weakest Weak Mid Strong Extra-strong

Fig. 2. The information-bearing physiological parameters of speech, with the parameter values of the canonical allophones of Midwestern Amercan English.

Single terms may be introduced, of course, to specify sets of parameter values. An attempt may be made to determine these more general sets of parameter values in such a manner that they will apply equally well to phonological descriptions of various languages.

It may be noted that a particular phone or prosody type may be specified either by an individual symbol, when entered on such a phonetic chart, or by a set of parameter values. Obviously, specific phone or prosody types are most easily indicated by individual symbols, but sets of phone or prosody types are often most conveniently specified by one or more parameter values. Other than for convenience and simplicity, there is no particular virtue in the choice of one of these modes of expression over the other.

PHONEMIC THEORY

The individual units of phonology and the general relationships among these units are specified by a general phonemic theory which is equally applicable to various languages. These units are denoted by such terms as phone, prosody, allophone, alloprosody, etc. As in the case of any scientific theory, a phonemic theory will be concerned with correspondences between observed data and mathematical abstractions.

Such a general phonemic theory will impose constraints on specific phonological descriptions, and will provide a basis for decisions in the development of phonological descriptions for dialects of particular languages. It should be emphasized, however, that such a general phonemic theory is more than the arbitrary specification of a set of constraints and restrictions on the phonological descriptions of specific languages. Communication by means of speech is a behavioral process, and a phonemic theory is a scientific theory which establishes correspondences between sets of speech properties and events and mathematical abstractions. If the choice of mathematical abstractions is inadequate, or if the correspondences between these abstractions and speech behavior are improperly defined, then the resulting phonemic theory will lack predictive power and its validation with experimental linguistic data cannot be entirely successful.

The statement of a general phonemic theory is primarily based upon the methods of speech analysis. The correspondence between speech and phone types is basically many-to-one, in which a certain aspect of many different speech formations is mapped onto a single phone type (normally indicated by a single symbol). The development of a phonemic theory, within which the basic concepts of phonology are specified, involves a set theoretic approach. Behavioral criteria are employed in organizing the general concepts of phonology into hierarchies of sets, and these behavioral criteria are thus involved in establishing the correspondences between the mathematical abstractions of set theory and the physiological (or physical) characteristics of speech.

In the development of such a theory, as in the development of all scientific theories, both simplicity and completeness are important considerations in the formulation. It is well known that there are no absolute tests for either simplicity or completeness in scientific theories. While both of these properties may be sought, the primary objective is to achieve consistency. In effect, this amounts to requiring that the theory have predictive value, which indeed it must if it is to be a theory at all. In particular, a phonemic theory predicts the general relation between the results of semantic or linguistic equivalence tests on utterances and the physiological or acoustical properties of those utterances.

PHONOLOGICAL DESCRIPTION

A phonological description of a specific dialect of a language may be considered a phonological hypothesis or theory of the structure of that dialect. Such a phonological description of the dialect may be formulated to provide as simple a description as possible within a specified criterion of completeness. Some linguists question whether such a description necessarily provides a corresponding simplicity for a complete linguistic description in which grammatical units are also included.

A phonological description of a dialect is not simply a collection of symbol types; it must also contain a specification of the conditions under which the symbols are to be used. A transcription of a specified corpus is one type of phonological description, but such a description obviously lacks such properties as simplicity, compactness, generality, etc. A description which contains a set of symbols and the rules governing the use of those symbols may be applicable to a specific corpus. Such a description does not generate that corpus to the exclusion of all others. In fact, a phonological description of a specific corpus is usually developed with the hope that it will apply to certain expansions of that corpus.

A corpus selected from a language can generally be expanded in such a manner that a description of the original corpus will not apply after the expansion. A general phonemic theory attempts to specify certain universal properties of language, and though such a theory may occasionally be modified and improved, it is assumed that the language universals do not alter. One universal, however, is that individual languages change in phonemic structure as time progresses. If the corpus on which the description of a dialect is based is expanded with time, then the description will not apply indefinitely; in fact, it may be assumed that new dialects are developed as time progresses. Thus the synchronic phonological description of a specific dialect lacks continuing predictive power, and such a description thereby lacks an essential property of a general theory.

Within the above indicated constraints, however, the phonological description of a specific dialect of a language has the form of a time-restricted theory, and as such has a limited predictive power. Such a phonological description cannot be developed unless inductions are made about observed data. The description will involve a specification of the correspondences between these data and certain mathematical abstractions. The system should provide theoretical predictions which can be

validated through experimental tests on the speech of the particular dialect under consideration.

Obviously, such a description will not predict the order in which a sequence of utterances will occur, nor even which utterances will occur. The description should, however, define the constraints (i.e. the laws) governing those utterances which may occur (within certain time and corpus restrictions). The laws may be made sufficiently specific that they apply only to types of utterances which have previously occurred within the dialect. It seems that their value would be greater however, if they have a sufficiently general form to encompass certain additions to the types of utterances which may occur within the dialect. The laws may intersect with those for other dialects. Obviously, however, they will be of limited value (in fact, be incomplete) if they are so general as to be congruent with the laws of other dialects. In effect then, the phonological description of a specific dialect of a language will predict (within the above indicated restrictions) the types of speech properties and events and the sequences of those various types of properties and events which may occur. A format for the phonological description of a specific dialect of a language will next be considered.

PHONOLOGICAL FORMAT

The phonological description of a specific dialect (at a particular period of time) may be determined according to the conditions of a general phonemic theory. If the theory is followed precisely, then the phonological description of the dialect provides a test (or validation) of the phonemic theory. The phonemic theory specifies the general nature of the units and their basic relationships; a phonological description specifies the individual units and their relationships within a particular dialect of a language.

In constructing a phonological description it is desirable to provide as simple and as concise a statement as possible of the structural relationships within the phonological system. There are three essential relationships which must be specified in the phonological description of a particular language. In the most elementary form, they may be considered as catalogs of relationships among the units of the system.

1. Correspondences between time functions of speech parameters and sequences of phone and prosody types.
2. Correspondences between sequences of phone and prosody types and phoneimc and prosodemic sequences.
3. Correspondences between well-formed or allowable phonemic and prosodemic sequences and a grammatical description.

It is often more convenient to express these relationships in terms of general statements than by cataloging specific items. In these general statements sets of the units under consideration, rather than individual units, are specified. The sets may be selected with the purpose of achieving as much simplicity as possible in the description, and also with the objective of minimizing the redundancy in the entire phonological description.

As indicated previously, at the phonetic level sets of phone and prosody types can be specified by sets of one or more physiological parameter values. The concept of parameter value, however, may be extended to apply to phonemes and prosodemes as well as phones and prosodies. For example, in the previously mentioned paper by Peterson and Harary an allophone is defined in such a manner that it will contain only one type of phone. The allophone type of an allophone, therefore, is the phone type of any phone which lies within that allophone. Particular allophones which generate phonemes are identified as canonical allophones. Many phonemes contain only one canonical allophone, in which case the phoneme type is specified as the allophone type of that canonical allophone. In instances where the phoneme is composed of more than one canonical allophone, the selection of a canonical allophone to specify the phoneme type is arbitrary.

By such procedures the classification properties of the parameter values specification may be applied to the description of sets of individual phonemes and prosodemes and to sets of phoneme and prosodeme sequences. Single terms indicating sets of parameter values may also be applied to individual phonemes and prosodemes and to phoneme and prosodeme sequences where convenient. Thus catalogs of correspondences between the output of a grammatical description and phoneme sequences, between phonemes and phones, and between phones and speech parameters may be, at least in part, reduced to general and more compact statements.

The relation between phones and phonemes is, of course, a many-to-one relation. There is no requirement on the nature of the mapping in specifying the correspondences. Phones may be mapped onto phonemes, or phonemes may be mapped into sets of phones. The former mapping would be employed in the approach of speech analysis, and the latter in a speech synthesis procedure. If bi-uniqueness is preserved in a phonemic theory then it is possible to employ either mapping in the phonological description of a specific dialect of a language. By bi-uniqueness it is meant that it must be possible by analysis to reconstruct any admissible discrete symbolic sequenec from speech which has been properly generated from the symbolic sequence; i.e. any utterance which is generated from a discrete symbolic sequence by synthesis must be convertible to the original symbolic sequence by analysis.

For some applications it is essential that the phonological description be expressed in an analysis format. Such a format is required in automatic speech recognition where it is necessary, for example, to identify the phoneme of the allophone which a particular phone represents.

When phonological descriptions are based on a bi-unique phonemic theory, then ambiguities are preserved in the phonemic transcriptions. Certain phonemic theories, however, do not maintain bi-uniqueness. Phonological descriptions based on these theories cannot be fully implemented in an analysis format, but can be in a synthesis

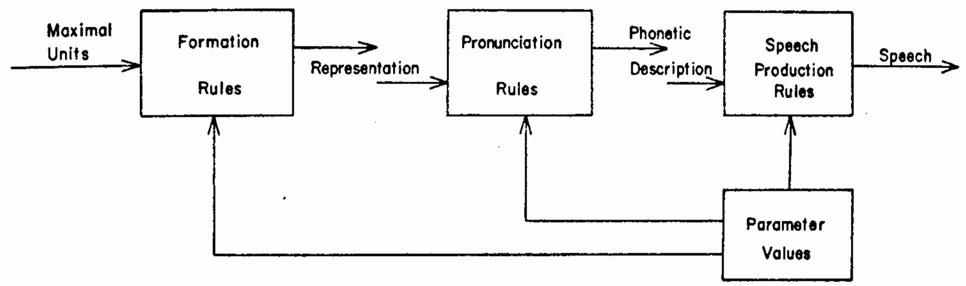


Fig. 3. A generative format for the description of the phonological system of a specific dialect of a language.

format. Thus a synthesis format for such phonological descriptions appears to be preferable.

The general characteristics of a synthesis format for the statement of the phonological structure of a specific dialect of a language is suggested in Figure 3. This type of format has also been called a generative format for phonological systems, which is perhaps more appropriate, since the emphasis is on description rather than on the synthesis of some specified set of utterances. An analysis format would be very similar, except that the direction of the operations (and thus the arrows) would be reversed, and the rules would be stated inversely to those indicated.

Obviously, such a generative format does not predict which utterances will occur within a dialect. A statistical property may be included in the generative description, based upon the analysis of a preceding corpus. In this instance, the format may prescribe those types of utterances which are most likely to occur, those which may occur, and those which will not occur unless the dialect is subject to major changes – in which case it may be considered to be a different dialect.

In the generative format of Figure 3, the largest or maximal units are shown at the left of the diagram. These are here considered to be the output of the grammatical description of the language and thus they convey information as isolated entities. They might, for example, be considered as the maximal units which lie between selected junctural positions. According to this view all grammatical units of a grammatical description are inherently separated by junctures. These junctures, however, are not necessarily all indicated in speech production. In a phonetic description a juncture is indicated by a position at which a pause may be introduced without altering the adjacent phonetic details (i.e. the associated phone and prosody types). Thus, at a junctural position the sequence of phone types on one side of the position is independent of the sequence of phone types on the other side. As is generally recognized, there are usually many more positions of juncture than are customarily marked by actual pauses in speech.

The units indicated at the left of the diagram form the bridge between grammar and phonology. Their full nature can be made clear only by the addition of a grammatical theory. To emphasize that each is a unit, it may be noted that each might be represented by a single separate symbol. The formation rules, to which the maximal units are applied, have as their output phonemic and prosodemic sequence representations of these maximal units. The rules may include general statements, expressed either in terms of sets of phonemes and prosodemes, or in terms of sets of one or more parameter values. If the generalized statements are made in terms of parameter value specifications, then at least two general levels of rules must be employed. In particular, statements for reducing the parameter values to individual phonemic and prosodemic units are necessary. Thus the order in which the formation rules are applied may be highly important.

The formation rules, then, operate on the maximal units to determine sequences of units of the next lower order. These rules represent a deductive system which specifies the well-formed sequences which may occur within the phonological system.

The pronunciation rules are applied to the representations to produce a specific phonetic description. The pronunciation rules, also, may employ general statements which are expressed either in terms of sets of allophones and alloprosodies or in terms of sets of one or more parameter values. In such cases, additional statements are required to reduce the general statements to specific phonetic descriptions. In general, the representations involve sequences of units, and at least part of the pronunciation rules must apply to sequences rather than to individual phonemes and prosodemes without regard to environment. Thus again, two general levels of statements may be distinguished within the pronunciation rules.

According to the system suggested in Figure 3, the output of the pronunciation rules is a phonetic description of the representations. Such a phonetic description may be in the form of a sequence of phone and prosody types. The speech production rules specify the conversion of such sequences to actual utterances. If the utterances are to be generated by the human vocal mechanism, then the vocabulary for the specification of the utterances may be taken from physiology (or perhaps acoustics). If the speech production rules are probabilistic, then the rules may account for all possible correspondences between phonetic descriptions and speech production. This possibility is related to an analysis format, in which any particular utterance may be represented by a sequence of discrete symbols.

SUMMARY

In this paper, the general nature of a theory of phonemic analysis is considered. A linguistic theory is viewed as a scientific theory, and as such must have the essential properties of any scientific theory. An attempt has been made to delimit phonology within linguistics and to identify those elements essential to a phonological system. Phonetic theory is considered basic to phonemic theory and to the phonological description of specific languages. A general phonemic theory specifies the individual

units and the basic relationships among these units in a manner that is equally applicable to various languages. The general properties of phonological descriptions of dialects of specific languages are considered, and formats are discussed in which the phonological description of specific dialects may be presented. Thus a general phonemic theory plus a format for expressing the phonemicization of specific dialects provide the essential components of the theory of phonemic analysis.

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