FSLT Homework due Nov. 29th

Please read all of the instructions before starting. The following is to be done in preparation for the class on Friday, November 29th.

1. Watch the following three videos in the linear algebra series on determinants, inverting matrices and non-square matrices:
   - `https://www.youtube.com/watch?v=Ip3X9LOh2dk`
   - `https://www.youtube.com/watch?v=uQhTuRlWMxw`
   - `https://www.youtube.com/watch?v=v8VSDg_WQlA`

2. Read and internalize the following:

   Definition: Let A be a square matrix. Then the **determinant** of this matrix, $\det(A)$, is the factor by which the linear transformation corresponding to A changes areas (if in 2 dimensions) or volumes (if in 3 dimensions or higher; if A operates on 4 dimensions then this is about 4-dimensional volumes and so on).

   Important determinant fact: For any two matrices A and B (that are square and of the same size), we have $\det(AB) = \det(A)\det(B)$.

   Definition: The **inverse** of a square matrix A is the unique matrix A^{-1} such that $A^{-1}A = I$ where I is the identity matrix (ones on the diagonal, zeros everywhere else). Note that this inverse A^-1 does not always exists.

   Definition: The **column space** of a matrix A is the space spanned by its column vectors. Sometimes this is also called the **image** of the corresponding linear transformation. In fact, this is the space that the linear transformation transforms the whole original space into.

   Definition: The **rank** of a matrix A is the dimension of its column space. A square matrix has **full rank** if the rank equals the number of columns of the matrix.

   Theorem: Let A be a square matrix. Then the following are equivalent:

   (i) $\det(A) \neq 0$

   (ii) A^{-1} exists

   (iii) A has full rank

   (iv) the columns of A are linearly independent

(v) any system of linear equations of the form $Ax = b$ has exactly one solution.

I will show you how to compute all of the above things in class.

3. When watching the video and reading the text, answer the following questions. We will then discuss them in class.

   a) All the above definitions (determinant, inverse, column space, rank, full rank) also occur in the videos. Note down in which video and at what time they are mentioned for the first time. Take this opportunity to compare the descriptions in the videos to the definitions given here.

   b) Answer the question from the first video: Can you explain why $\det(AB) = \det(A)\det(B)$ makes sense in just one sentence?

   c) Let A be a matrix that maps the whole 2-dimensional space onto a single line. An example of this would be the matrix

$$\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$$

   given around 9:13 in the second video. What dimension does its column space have? What is its rank? Does it have full rank? Can it be inverted, i.e. does its inverse A^{-1} exist? What is its determinant? You can answer all these questions using just the fact that the matrix maps the 2-dimensional space onto a single line, no computations required.

   d) From what dimension to what dimension does a matrix with 2 rows and 3 columns map? How about a matrix with 3 rows and 2 columns?