FSLT Homework due Nov. 22nd

Please read all of the instructions before starting. The following is to be done in preparation for the class on Friday, November 22nd.

1. Please watch these videos on matrices and matrix multiplication.
   - `https://www.youtube.com/watch?v=kYB8IZa5AuE`
   - `https://www.youtube.com/watch?v=XkY2DOUCWMU`
2. Read the following parts of the Warwick lecture notes:
   `https://homepages.warwick.ac.uk/~maskal/MA106NotesMaclaganTesta.pdf`
   - All of Section 2 (Matrix recap). As usual, feel free to skip the proofs.
   - Update: you are no longer required to read anything in Section 7.
   - Definition 8.1 and Lemma 8.2
   - Proposition 8.4
   - Definitions 8.5, 8.6, 8.7
   - The start of Section 8.3, up to and including Theorem 8.8
   - Proposition 8.11 and Theorem 8.12

   This may look like a lot, but much of it is along the lines of "this works indeed the way we would expect it to work".

   One more thing: the lecture notes formulate things very generally, for any kind of vector space and basis. Try to every now and then replace these abstract concepts with more concrete examples, such as $\mathbb{R}^2$ for the vector spaces, using the standard basis
   $$\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}.$$
   This may make things easier to follow. But also keep the general perspective in mind! Bouncing back and forth between concrete examples and a more abstract point of view is a useful technique in mathematics.
3. When watching the video and reading the text, answer the following questions. We will then discuss them in class.

   Update: Hint: Especially for questions (a) and (b), try using the perspective of using $\mathbb{R}^2$ for the vector spaces, using the standard basis
   $$\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}.$$

a) At 3:47 in the first video, 3Blue1Brown says "It turns out that you only need to record where the two basis vectors (...) land, and everything else will follow from that." This is essentially Proposition 8.4 in the lecture notes. Spell out how exactly the example in the video and Proposition 8.4 relate: What are K, U, V, S and f in the video?

b) Also have a look at the proof of Proposition 8.4. How does it relate to 3Blue1Brown's argument of grid lines remaining parallel and evenly spaced?

c) The start of Section 8.3 in the lecture notes contains a general definition of how to obtain a matrix for any linear transformation. Apply this definition to the example at 6:03 in the first video and check that it gives the same matrix as in the video. Hint: Use the standard basis

$$\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}.$$

d) Consider the following matrices (hint: B shows up in the videos):

$$A = \begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix} \qquad B = \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}.$$

Mentally (or on paper) visualize the corresponding linear transformations. Can you describe what they do with just a few words? Further, obtain the matrix AB both by calculation and by following where the basis vectors

$$\hat{\imath} = \begin{bmatrix}1\\0\end{bmatrix} \qquad \hat{\jmath} = \begin{bmatrix}0\\1\end{bmatrix}$$

land, as in the video. Do you get the same result both ways? Can you follow $\hat{\imath}$ and $\hat{\jmath}$ purely visually in your head?

Bonus question: Consider a set of matrices, e.g. $\mathbb{R}^{2x2}$ with addition and multiplication. Which of the field axioms (c.f. Definition 4.1) does this satisfy, which ones does it not satisfy, and for which ones do you not know yet?