Computational Psycholinguistics

Lecture 10: Pattern Associators and Competitive Networks

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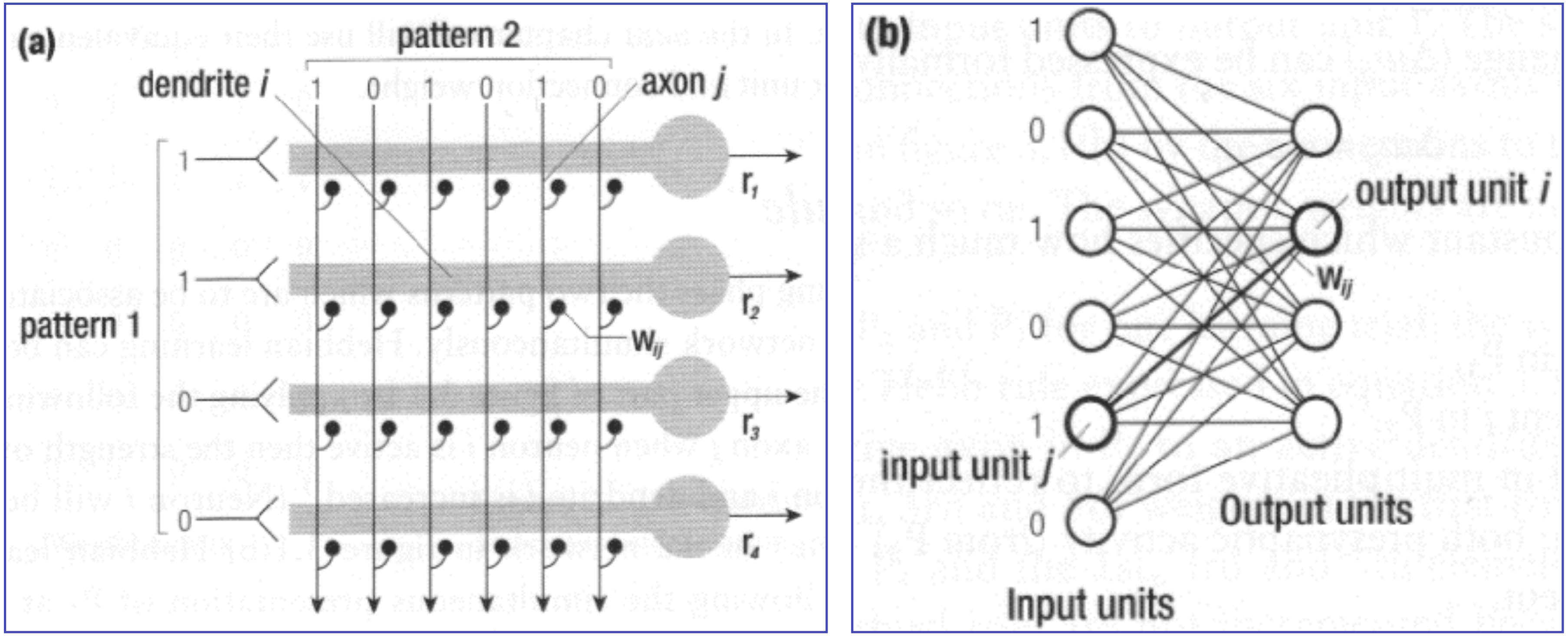
Overview

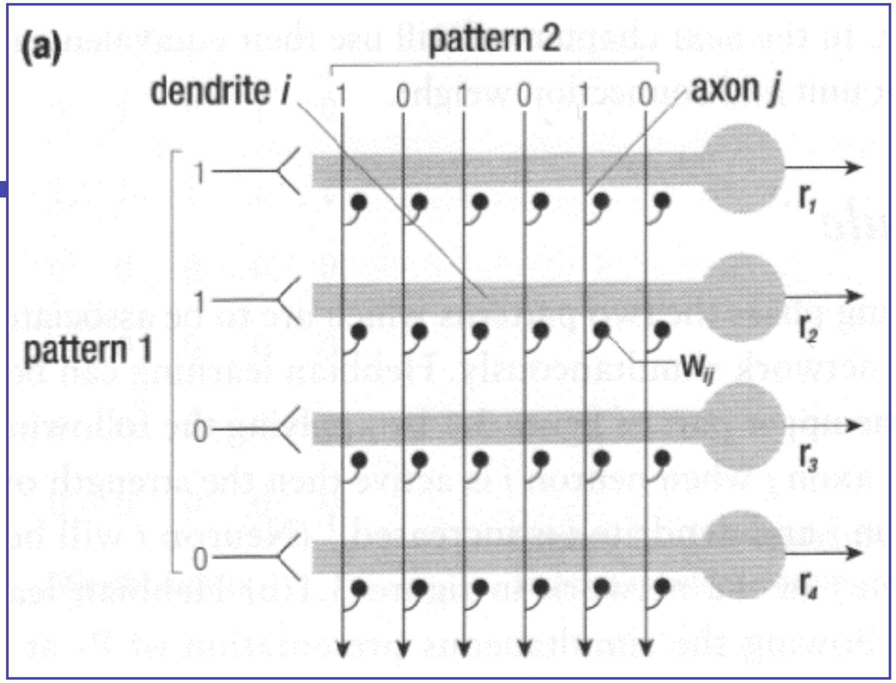
- Learning:
  - The delta rule
    - The perceptron convergence rule
    - Gradient descent learning
  - Back-propagation of error with hidden-layers
    - The Generalized Delta Rule
    - Not generally considered as biologically plausible
- Pattern Associators:
  - Single-layer networks
  - Networks as matrices
  - Associating distributed representations
  - Hebbian learning
  - Generalisation in learning
  - Biological plausibility
- Competitive networks and unsupervised learning

Pattern Associators

- Learn to associate one stimulus with another, e.g.:
  - Sight of chocolate associates with taste of chocolate
  - The string “yacht” associates with the pronunciation /y/ /o/ /t/
  - Etc.



Learning: Hebb's rule



- The idea behind Hebbian learning is simple: **reinforcement**
- The two patterns to be associated are presented simultaneously
- If there is activity on input axon *j*, when neuron *i* is active, then the connection weight w_{ij} (between axon *j* and dendrite *i*) is increased
- The Hebb rule: $$\Delta w_{ij} = \varepsilon a_i a_j$$
  - a_i is the activity of element *i* in P_1
  - a_j is the activity of element *j* in P_2
  - ε is the learning rate parameter

An example

- Assume binary neuron activations (0 or 1)
- Suppose the sight of chocolate is represented as: (1 0 1 0 1 0)
- The taste of chocolate is represented as (1 1 0 0)
- We can represent the weights as a 6x4 matrix of “synapses”

Weights *before* learning:

		1	0	1	0	1	0
		↓	↓	↓	↓	↓	↓
1	→	0	0	0	0	0	0
1	→	0	0	0	0	0	0
0	→	0	0	0	0	0	0
0	→	0	0	0	0	0	0

Weights *after* learning:

		1	0	1	0	1	0
		↓	↓	↓	↓	↓	↓
1	→	1	0	1	0	1	0
1	→	1	0	1	0	1	0
0	→	0	0	0	0	0	0
0	→	0	0	0	0	0	0

Assume that $\varepsilon=1$

Recall from a Trained Matrix

- $\text{netinput}_i = \sum_j a_j w_{ij}$
- This is just the *dot product* of 2 vectors, i.e.:

$$(1\ 0\ 1\ 0\ 1\ 0) \bullet (1\ 0\ 1\ 0\ 1\ 0)$$

$$= (1 \times 1 + 0 \times 0 + 1 \times 1 + 0 \times 0 + 1 \times 1 + 0 \times 0) = 3$$

- Thus for the recall cue (1 0 1 0 1 0), the output pattern is:

1	0	1	0	1	0
↓	↓	↓	↓	↓	↓
1	0	1	0	1	0
1	0	1	0	1	0
0	0	0	0	0	0
0	0	0	0	0	0

- If we assume a threshold of 2, where values <2 are 0 and others are 1:
  - Then the output pattern of activity is (1 1 0 0)

Learning Multiple Associations

- It is not very "computationally" surprising that an array of 24 can store the relationship between two vectors of size 6 and 4 respectively
- What happens if we try to store different associations with the same weight matrix?
  - Appearance of apricots: (1 1 0 0 0 1)
  - Taste of apricots: (0 1 0 1)

Change in weights for apricots:

		1	1	0	0	0	1
		↓	↓	↓	↓	↓	↓
0	→	0	0	0	0	0	0
1	→	1	1	0	0	0	1
0	→	0	0	0	0	0	0
1	→	1	1	0	0	0	1

The combined weight matrix:

1	0	1	0	1	0
2	1	1	0	1	1
0	0	0	0	0	0
1	1	0	0	0	1

Recall of multiple associations

- We can now see how well the pattern associator can perform recall for the two patterns
- Assume a threshold of 2
- Apricots:
  - Netinput: (1 4 0 3)
  - Output: (0 1 0 1)
- Chocolate:
  - Netinput: (3 4 0 1)
  - Output: (1 1 0 0)
- Both are correctly recalled

1	1	0	0	0	1		
↓	↓	↓	↓	↓	↓		
1	0	1	0	1	0	→	1
2	1	1	0	1	1	→	4
0	0	0	0	0	0	→	0
1	1	0	0	0	1	→	3

1	0	1	0	1	0		
↓	↓	↓	↓	↓	↓		
1	0	1	0	1	0	→	3
2	1	1	0	1	1	→	4
0	0	0	0	0	0	→	0
1	1	0	0	0	1	→	1

Recall, Similarity and Linear Algebra

- We have seen so far how network behaviour can be understood in terms of vectors, matrices, and operations thereon.
  - If an input pattern **a**, and the weights **w** leading from the inputs to some node are represented as vectors. Netinput to that node is the *dot product*.

$$\text{netinput}_i = \sum_j a_j w_{ij} = \mathbf{a} \bullet \mathbf{w}$$

  - If the current weights are represented by a matrix **m1**, and the change in weights by a matrix **m2**, then the new weight matrix is simply: **m1** + **m2**

- Observe: the dot product is highest when two vectors are *similar*:
  - Numbers in vector 1 are similar to those in the *corresponding positions* in vector 2
  - Thus netinput is highest for similar input/weights
  - Each dissimilarity reduces the netinput
  - Vectors with a dot product of 0 are said to be *orthogonal*

Properties of Pattern Associators

- Similarity in vectors
  - p: 1 0 0 0 0 1 1 1
  - w_1: 1 0 0 0 0 1 1 1   = 4
  - w_2: 1 0 0 0 1 0 1 1   = 3
  - w_3: 0 0 1 1 1 0 1 1   = 2
  - w_4: 0 1 1 1 1 0 0 0   = 0

- Operation of pattern associators using the Hebb rule:
  - Learning: if a neuron i is activated by P_1, an increment Δw_i that has the same pattern as P_2, is added to the weight vector of neuron i .
  - Recall: since patterns presented during learning are directly reflected in the weight vector for neuron i , the output at neuron i reflects the similarity of the recall cue to patterns presented during learning
- Properties
  - Generalisation
  - Fault tolerance
  - Prototype extraction
  - Speed

Generalisation

- If a presented cue is similar to one that is learned, a pattern associator will often produce a similar response for the new as for the old pattern
  - This means networks can associate "imperfect/noisy" stimuli

- I.e. Insensitive to relatively small differences in input stimuli:
  - E.g. (1 1 0 1 0 0) is slightly different from (1 1 0 0 0 1)
  - Or, (1 0 1 0 0 0) is slightly different from (1 0 1 0 1 0)

1	1	0	1	0	0		
↓	↓	↓	↓	↓	↓		
1	0	1	0	1	0	→	1
2	1	1	0	1	1	→	3
0	0	0	0	0	0	→	0
1	1	0	0	0	1	→	2

1	0	1	0	0	0		
↓	↓	↓	↓	↓	↓		
1	0	1	0	1	0	→	2
2	1	1	0	1	1	→	3
0	0	0	0	0	0	→	0
1	1	0	0	0	1	→	1

Fault tolerance

- Just as pattern associators can often deal with imperfect stimuli, they are often robust to damaged connections (synapses)

- This is because PAs compute a correlation of the pattern with the weights via a relatively large number of axons

1	1	0	0	0	1		
↓	↓	↓	↓	↓	↓		
1	0	1	0	1	0	→	1
2	1	1	0	X	1	→	4
0	0	0	X	0	0	→	0
1	X	0	0	0	1	→	2

- This can help explain continued (sometime partial) function in the event of normal cell loss, or certain kind of (distributed) brain damage.

More Properties

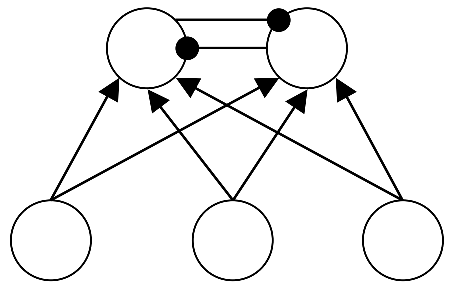
- Prototype Extraction & Noise Reduction
  - If the network is exposed to similar (but slightly different) P_2s for a given P_1 during training, the (scaled) weight vectors become the *average*
  - When tested, the best response is to the *average* pattern vector
  - Thus, even if trained on noisy instances, the network will have learned to respond to a prototype (which it has never explicitly seen).
- Interference
  - Not such a problem in distributed (non-local/symbolic) systems
  - Permits noise reduction, fault tolerance, generalisation, etc
  - Explains certain aspects of human memory and cognitive function
  - Robustness versus 100% accuracy
- Speed
  - Because computation is distributed across multiple neurons and synapses, the response to a stimulus can be determined in 1-2 steps
- Distributed Representations are important
  - Information about the stimulus is *distributed* over the population of elements, rather than encoded by a single element
  - Generalisation and graceful degradation rely on a continuous range of *dot products*

Competitive Networks: Overview

- Operation:
  - Given a particular input, output units compete with each other for activation
  - The winning output unit is the one with the greatest response activation
- During training:
  - Connections to the winning unit from the active input units are strengthened
  - Connections from inactive units are weakened or left unchanged
- Training is unsupervised
  - The is no external teacher (no target)
  - The network will categorise inputs, based on similarity
  - Learns to capture statistical properties of input space

Architecture of Competitive Networks

- A simple network:
  - Inputs are fully connected to outputs by feed-forword connections
  - Outputs may be connected to each other by *inhibitory* connections



- Outputs compete until only one remains active
  - Or, simply the unit with highest activation wins
- Excitation of outputs: $\text{netinput}_i = \sum_j a_j w_{ij}$
  - Dot product of input activations and the weight vector
- Competition:
  - Output activations are compared, unit with highest activation wins
  - Or, direct competition among outputs via inhibitory connections:
    - Active units force other units to become inactive

Adjusting Weights

- Weights are only adjusted on connections feeding into the winning output node:

$$\Delta w_{ij} = 0 \text{ if unit } i \text{ loses}$$
$$= \varepsilon\,(a_j - w_{ij}) \text{ if unit } i \text{ wins}$$

  - Where,

    ε is the learning rate parameter

    a_j is the activity of input unit j for pattern p

    w_{ij} is the weight of the connection from j to i before the trial

- Behaviour
  - The strengths of connections to the winning unit are adjusted until each weight is the same as the activity of its input
- Result
  - The winning unit's weight vector is changed to make it more similar to the input vector for which it is the winner

An example

Consider the following network:

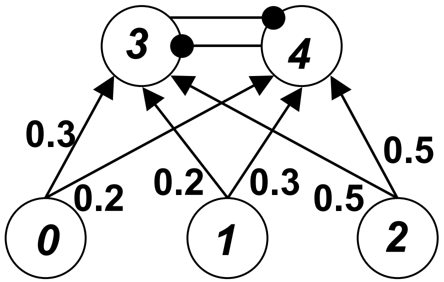
- Input pattern: (0 1 1)

  $netinput_3$ = (0x0.3+1x0.2+1x0.5)

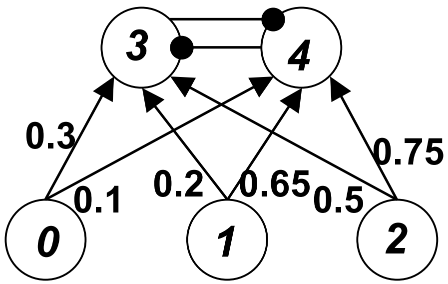
  = 0.7

  $netinput_4$ = (0x0.2+1x0.3+1x0.5)

  = 0.8



- Since $unit_4$ wins:
  - No changes in connections to $unit_3$
- For connections to $unit_4$:
  - $\Delta w_{ij} = \varepsilon\ (a_j - w_{ij})$
  - $\Delta w_{ij} = 0.5\ (0.0{-}0.2\ \ 1.0{-}0.3\ \ 1.0{-}0.5)$
  - $\Delta w_{ij} = 0.5\ ({-}0.2\ \ 0.7\ \ 0.5)$
  - $\Delta w_{ij} = ({-}0.1\ \ 0.35\ \ 0.25)$

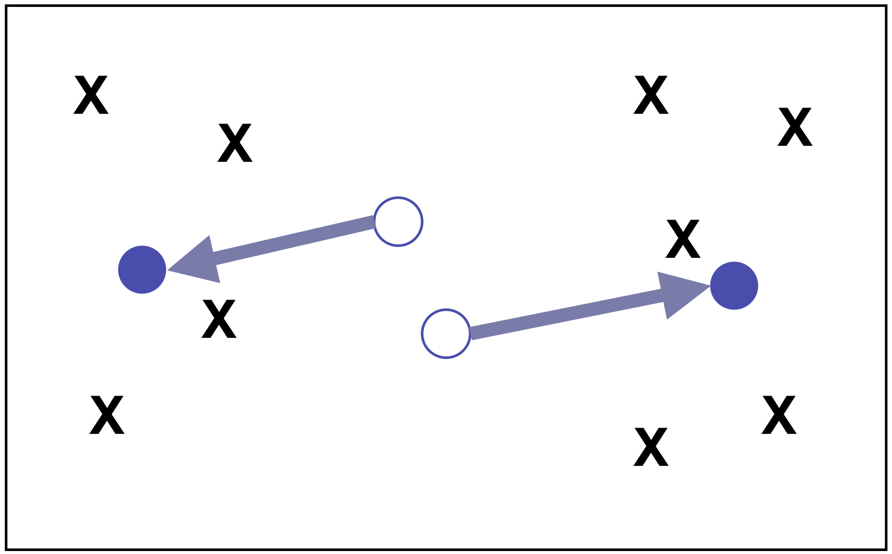


Overall behaviour

- Netinput to an output unit is greatest when its weight vector is most similar to the input vector
- Training makes the weight vector for a particular winning unit more similar to the input pattern
- It is therefore also likely to be the “winning unit” for similar patterns, and therefore learn to respond to those patterns as well
- The weight vector for a particular output unit learns to respond to similar input patterns
  - Because these patterns are all slightly different, the learned weights cannot exactly mimic the associated inputs
  - Rather, the learned weights will be an average of the patterns, based on the frequency of presentation during training
- The competitive network can therefore learn to categorise similar inputs without any “teacher”: unsupervised learning

Visualising competitive learning

- Represent input patterns and weight vectors in multi-dimensional space
  - Weight vectors for the output units have a random relation to the input patterns
  - Competitive learning changes the weight vector for a particular output so that it becomes the average for a subset of inputs
  - More outputs enable the network to more finely categorise the inputs

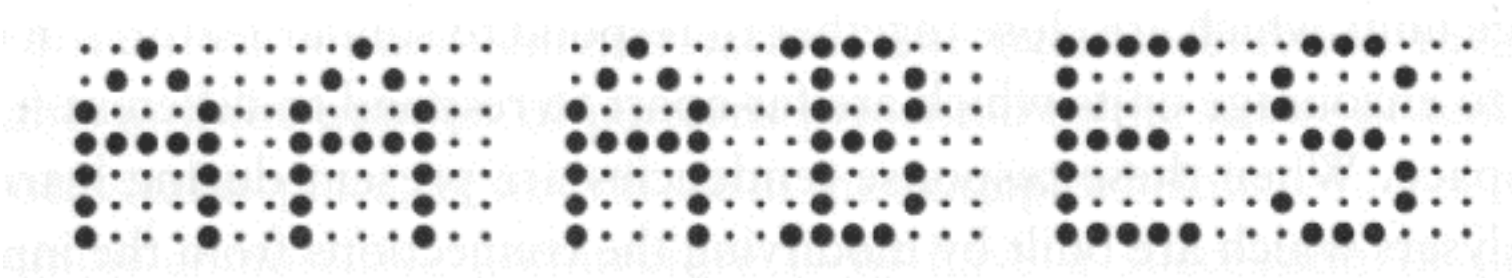


More on competitive networks

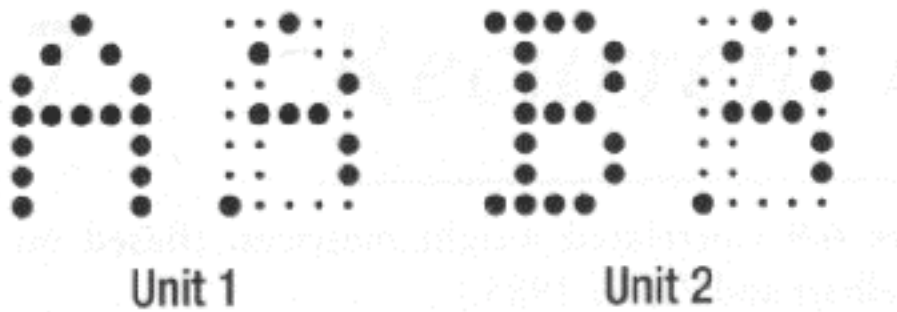
- Weight growth:
  - Depending on how training occurs, if many similar patterns are associated with one output, it may be impossible for other outputs to ever gain more activation, even for quite different input patterns
  - We could limit weight growth, by insisting that the sum of a weight vector equal some constant, and learning could only redistribute weight among connections to the winning unit
- As with Hebbian networks, learning is local:
  - Winner is found by competition of output: inhibitory connections leave only one neuron firing
  - Hebbian learning means that only connection weights to this node are changed
  - Information is available at the axon and dendrite of a connection
  - Also: no explicit teacher is required
- Remove redundancy: set of inputs associated with a single output
  - Sparsification: convert pattern stimuli to a localist representation
- Outputs are less correlated (possibly orthogonal) than inputs:
  - Useful as input to pattern associators (easier to learn less correlated patterns)

An example: Pattern classification

- We can use an unsupervised network to classify patterns of letters
- Input is a 7 x14 “retina”, connected to 2 outputs each with a 98 element weight vector, which is trained on pairs of letters:



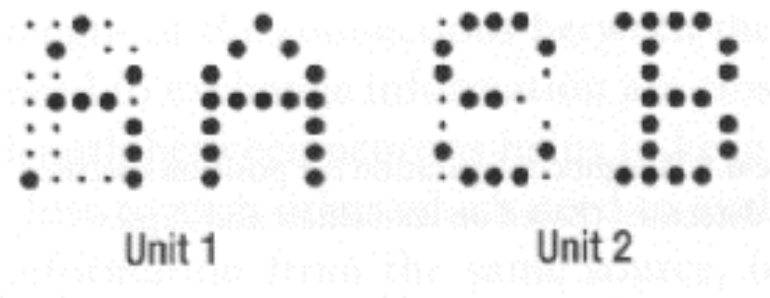
- First the network is trained on pairs of letters: AA, AB, BA, BB
- The resulting weights to the outputs are as follows:
  - Unit 1: AA, AB
  - Unit 2: BA, BB
- Why? What else could it learn?
- What would happen if the network had 4 output units?



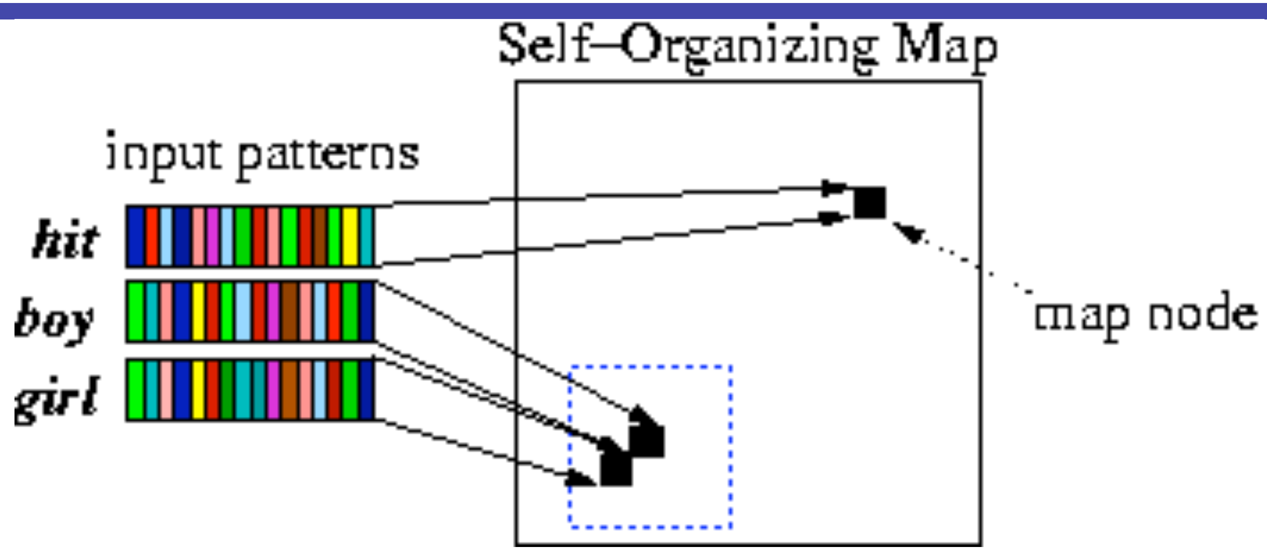
Pattern classification continued

- Consider the case where we train the network on individual letters, instead of pairs: A, B, E, S
- Cluster using 2 output units
- The result will be to cluster A & E and B & S, since they are the most similar: thus the classifier acts as a feature detector within letters

- What if the network is trained to classify AA, BA, SB, EB?
  - As we would expect, the network learns a letter-specific classification
  - But, we have forced A & B and S & E to be grouped together
  - In this way, we force the network to find whatever features do *correlate* for the letters in the 1 position
  - The 2nd letter acts as a teacher, since it forces the network into a specific solution



Self-Organizing Maps



- Competitive network that develops two-dimensional clustering of input space
- Preserves topology of input space by mapping similar inputs to nearby nodes
- Node response: $n_{ij} = 1.0 - || x - m_{ij} || / d_{max}$
- Winner: maximally responding unit: $i, j = \text{argmax } n_{ij}$
- Neighborhood: nearby nodes with similar weight vectors: N_c
- Weights updated: $\Delta\mu_{ij} = \alpha(t) [x - \mu_{ij}]$ for $(i,j) \in N_c(t)$

Summary

- Associate multiple stimulus-response patterns in a single network
  - Networks can be represented as a weight matrix
- Weights are sensitive to similarity
  - The more similar, the higher the netinput; the *dot product* of P and W
- Important properties
  - Generalisation: robust to noisy input
  - Fault tolerance: robust to loss/damage
  - Prototype extraction & noise reduction
- Biologically Plausible:
  - Learning is strictly local
  - Reinforcement based