Formal Models of Language

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Introduction

Hi!

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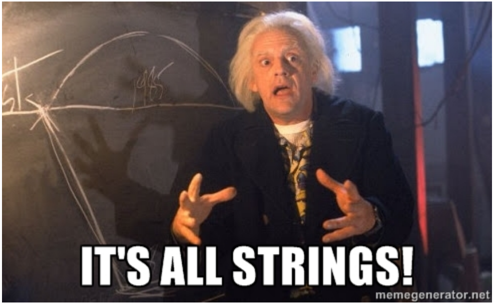

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- (Similar to musical/poetic form analysis)

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- If two different grammars can generate/accept the same structures as well, then they have the same **strong generative capacity**

Formal Language Hierarchy

	Formal Language
	Non-Turing-acceptable
0:	**Recursively enumerable**
	Recursive/ Decidable
1:	Context-sensitive
	Indexed
	Mildly context-sensitive
2:	**Context-free**
	Deterministic context-free
3:	**Regular**
	Finite

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- For example: long-distance dependencies, complex reordering in machine translation, reduplication, etc.
- You can also get an idea of how fast or slow it will take for a computer (or human) to process sequential stuff (like natural language!)

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- For natural language, this would correspond to having a finite number of possible sentences

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- (There's more discussion on the interwebs if you're interested)

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- Then we have something called big-$\mathcal{O}$ notation from computer science. To make a long story short:

$\mathcal{O}(1)$	"constant time"	# units unrelated to input
$\mathcal{O}(n)$	"linear time"	# units lin. proportional to input string
$\mathcal{O}(n^2)$	"quadratic time"	# unites quadrat. prop. to input string
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- Processing regular languages can be done in linear time ($\mathcal{O}(n)$), with a constant size of memory ($\mathcal{O}(1)$)

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- Processing CF languages can be done in about cubic time ($\mathcal{O}(n^3)$), with linear memory usage ($\mathcal{O}(n)$)

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- Processing MCS languages can be done in about $\mathcal{O}(n^6)$ time, with quadratic memory usage ($\mathcal{O}(n^2)$)
- Mildly context-sensitive is very different from context-sensitive, which is much more powerful
- Some grammar formalisms that can handle MCS langs:
  - Tree Adjoining Grammar (TAG)
  - Combinatory Categorial Grammar (CCG)
  - Linear Indexed Grammars (LIG) (easy to understand)
  - Head Grammars (HG)

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- For example: context-free languages harder to machine-learn than regular languages.

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  - Computer vision – maybe we really want explicit descriptions of objects in human language.

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- Dictionary problem: what is the meaning of a feature? Define words in terms of other words?

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- The main question of formal semantics: what do we need to reason about language?