# Semantic Theory
# Week 3 – Typed Lambda Calculus

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# Compositionality

The principle of compositionality: "The meaning of a complex expression is a function of the meanings of its parts and of the syntactic rules by which they are combined" (Partee et al.,1993)

Compositional semantics construction:

- compute meaning representations for sub-expressions
- combine them to obtain a meaning representation for a complex expression.

Problematic case: "[Not smoking$_{\langle e,t\rangle}$] [is healthy]

Logical connectives only apply to type t expressions—i.e., the connective ¬ can be interpreted as a function of type ⟨t,t⟩ that reverses truth values: [1→0, 0→1]

# Lambda abstraction

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λ-abstraction is the operation that transforms expressions of any type τ into a function ⟨σ,τ⟩, where σ is the type of the λ-variable.

Formal definition:

If $\alpha$ is in $WE_{\tau}$, and $x$ is in $VAR_{\sigma}$ then $\lambda x(\alpha)$ is in $WE_{\langle\sigma, \tau\rangle}$

- The scope of the λ-operator is the smallest WE to its right. Wider scope must be indicated by brackets.
- We often use the “dot notation” $\lambda x.\phi$ indicating that the λ-operator takes widest possible scope (over $\phi$).

# Interpretation of Lambda-expressions

If $\boldsymbol{\alpha} \in WE_\tau$ and $v \in VAR_\sigma$, then $[\![\lambda v \boldsymbol{\alpha}]\!]^{M,g}$ is that function $f : D_\sigma \rightarrow D_\tau$ such that for all $a \in D_\sigma$, $f(a) = [\![\boldsymbol{\alpha}]\!]^{M,g[v/a]}$

If the λ-expression is applied to some argument, we can simplify the interpretation:

- $[\![\lambda v \boldsymbol{\alpha}]\!]^{M,g} ([\![x]\!]^{M,g}) = [\![\boldsymbol{\alpha}]\!]^{M,g[v/[\![x]\!]^{M,g}]}$

Example: *"Bill doesn't smoke" ("Bill is a non-smoker")*

$[\![\lambda x(\neg S(x))(b')]\!]^{M,g} = 1$

iff $[\![\lambda x(\neg S(x))]\!]^{M,g}([\![b']\!]^{M,g}) = 1$

iff $[\![\neg S(x)]\!]^{M,g'} = 1$ where $g'=g[x/[\![b']\!]^{M,g}]$

iff $[\![S(x)]\!]^{M,g'} = 0$

iff $[\![S]\!]^{M,g'}([\![x]\!]^{M,g'}) = 0$

iff $V_M(S)(V_M(b')) = 0$

# β-Reduction (Function application)

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$$[\![\lambda v(\boldsymbol{\alpha})(\boldsymbol{\beta})]\!]^{M,g} = [\![\boldsymbol{\alpha}]\!]^{M,g[v/[\![\beta]\!]M,g]}$$

⇒ all (free) occurrences of the λ-variable in **α** get the interpretation of **β** as value.

This operation is called β-reduction

- **λv**(**α**)(**β**) ⇔ **α**[**β**/**v**]
- **α**[**β**/**v**] is the result of replacing all free occurrences of **v** in **α** with **β**

Achtung: The equivalence is not unconditionally valid …

# Variable capturing

Q: Are $\lambda v(\alpha)(\beta)$ and $\alpha[\beta/v]$ always equivalent?

- $\lambda x(\text{drive}'(x) \wedge \text{drink}'(x))(j') \Leftrightarrow \text{drive}'(j') \wedge \text{drink}'(j')$
- $\lambda x(\text{drive}'(x) \wedge \text{drink}'(x))(y) \Leftrightarrow \text{drive}'(y) \wedge \text{drink}'(y)$
- $\lambda x(\forall y\ \text{know}'(x)(y))(j') \Leftrightarrow \forall y\ \text{know}(j')(y)$
- $\lambda x(\forall y\ \text{know}'(x)(y))(y) \not\Leftrightarrow \forall y\ \text{know}(y)(y)$ Problem: y is not "free for x"

**Definition:** Let v, v' be variables of the same type, and let $\boldsymbol{\alpha}$ be any well-formed expression.

- v is free for v' in $\boldsymbol{\alpha}$ iff no free occurrence of v' in $\boldsymbol{\alpha}$ is in the scope of a quantifier or a λ-operator that binds v.

# Conversion rules

- β-conversion: $\lambda v(\boldsymbol{\alpha})(\boldsymbol{\beta}) \Leftrightarrow \boldsymbol{\alpha}[\boldsymbol{\beta}/v]$

  (if all free variables in $\boldsymbol{\beta}$ are free for v in $\boldsymbol{\alpha}$)

- α-conversion: $\lambda v.\boldsymbol{\alpha} \Leftrightarrow \lambda w.\boldsymbol{\alpha}[w/v]$

  (if w is free for v in $\boldsymbol{\alpha}$)

- η-conversion: $\lambda v.\boldsymbol{\alpha}(v) \Leftrightarrow \boldsymbol{\alpha}$

# Determiners as lambda-expressions

- a student works ➔ $\exists x(\text{student}'(x) \wedge \text{work}'(x))$ :: t
  - a student ➔ $\lambda P \exists x(\text{student}'(x) \wedge P(x))$ :: $\langle\langle e,t\rangle,t\rangle$
  - a, some ➔ $\lambda Q \lambda P \exists x(Q(x) \wedge P(x))$ :: $\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$
- every student ➔ $\lambda P \forall x(\text{student}'(x) \rightarrow P(x))$ :: $\langle\langle e,t\rangle,t\rangle$
  - every ➔ $\lambda Q \lambda P \forall x(Q(x) \rightarrow P(x))$ :: $\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$
- no student ➔ $\lambda P \neg \exists x(\text{student}(x) \wedge P(x))$ :: $\langle\langle e,t\rangle,t\rangle$
  - no ➔ $\lambda Q \lambda P \neg \exists x(Q(x) \wedge P(x))$ :: $\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$
- someone ➔ $\lambda F \exists x F(x)$ :: $\langle\langle e,t\rangle,t\rangle$

# NL Quantifier Expressions: Interpretation

- someone' $\in$ $CON_{\langle\langle e,t\rangle,t\rangle}$, so $V_M$(someone') $\in$ $D_{\langle\langle e,t\rangle,t\rangle}$

- $D_{\langle\langle e,t\rangle,t\rangle}$ is the set of functions from $D_{\langle e,t\rangle}$ to $D_t$ , i.e.,

  the set of functions from $\mathcal{P}(U_M)$ (the powerset of $U_M$) to {0,1},

  which in turn is equivalent to $\mathcal{P}(\mathcal{P}(U_M))$

From $V_M$(someone') $\in \mathcal{P}(\mathcal{P}(U_M))$ it follows that $V_M$(someone') $\subseteq \mathcal{P}(U_M)$

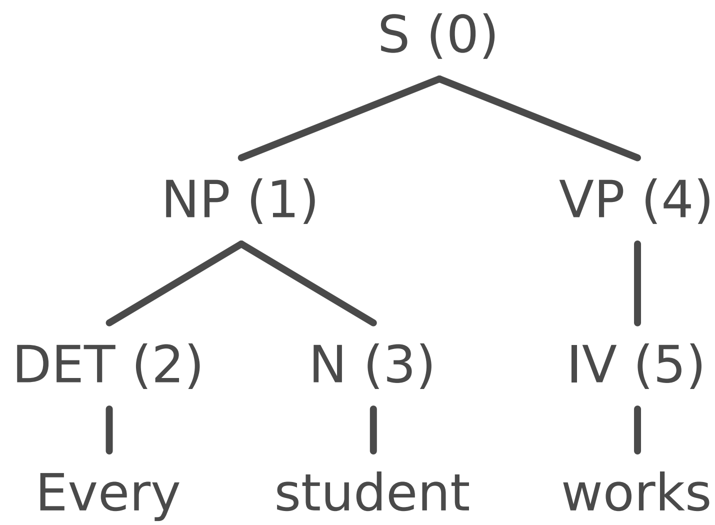
More specifically:

- $V_M$(someone') = $\{S \subseteq U_M \mid S \neq \varnothing\}$, if $U_M$ is a domain of persons

⇒ More on Natural Language Quantifiers in two weeks!

# β-Reduction Example

*Every student works.*




(2) $\lambda P \lambda Q \forall x(P(x) \rightarrow Q(x)) :: \langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle$

(3) $\lambda x.student'(x) \Leftrightarrow^{\eta} student' :: \langle e, t\rangle$

(1) $\lambda P \lambda Q \forall x(P(x) \rightarrow Q(x))(student')$

$\Leftrightarrow^{\beta} \lambda Q \forall x(student'(x) \rightarrow Q(x)) :: \langle\langle e, t\rangle, t\rangle$

(4)/(5) $\lambda x.work'(x) \Leftrightarrow^{\eta} work' :: \langle e, t\rangle$

(0) $\lambda Q \forall x(student'(x) \rightarrow Q(x))(work') \Leftrightarrow^{\beta} \forall x(student'(x) \rightarrow work'(x)) :: t$

# Transitive Verbs: Type Clash

- Someone reads a book

$$\dfrac{\text{someone} :: \langle\langle e, t\rangle,t\rangle \qquad \dfrac{\text{read} :: \langle e,\langle e, t\rangle\rangle \qquad \text{a book} :: \langle\langle e, t\rangle,t\rangle}{?? :: ??}}{?? :: t}$$

Solution: reverse functor-argument relation (again)

- Logical form: someone(read(a book))

- Adjust type of first argument of transitive verb: $\text{read}_{\langle\langle\langle e,\ t\rangle,t\rangle,\langle e,\ t\rangle\rangle}$ (*Type Raising*)

# Type Raising

It's not enough to just change the type of the transitive verb:

- read ➔ read' $\in$ CON$_{\langle\langle\langle e,t\rangle, t\rangle, \langle e, t\rangle\rangle}$

  *someone reads a book*:
  $\lambda F \exists x F(x)(read'(\lambda P \exists y(book'(y) \wedge P(y)))$
  $\Leftrightarrow^{\beta} \exists x(read'(\lambda P \exists y(book'(y) \wedge P(y)))(x)$ … No further reduction steps possible.

  Problem: this does not support the following entailment:
  *someone reads a book* $\vDash$ *there exists a book*

Hence, we need a more explicit λ-term:

- read ➔ $\lambda Q \lambda z.Q(\lambda x(read^*(x)(z))) \in$ WE$_{\langle\langle\langle e,t\rangle, t\rangle, \langle e, t\rangle\rangle}$
  where: read* $\in$ WE$_{\langle e, \langle e, t\rangle\rangle}$ is the "underlying" first-order relation

# Transitive Verbs: example

*someone reads a book: someone(read(a book))*

$\lambda F \exists x F(x)(\lambda Q \lambda z(Q(\lambda x(read^*(x)(z))))(\lambda R \lambda P(\exists y(R(y) \wedge P(y)))(book')))$

$\Leftrightarrow_\beta \lambda F \exists x F(x)(\lambda Q \lambda z(Q(\lambda x(read^*(x)(z))))(\lambda P(\exists y(book'(y) \wedge P(y)))))$

$\Leftrightarrow_\beta \lambda F \exists x F(x)(\lambda z(\lambda P(\exists y(book'(y) \wedge P(y)))(\lambda x(read^*(x)(z)))))$

$\Leftrightarrow_\beta \lambda F \exists x F(x)(\lambda z(\exists y(book'(y) \wedge \lambda x(read^*(x)(z))(y))))$

$\Leftrightarrow_\beta \lambda F \exists x F(x)(\lambda z(\exists y(book'(y) \wedge read^*(y)(z))))$

$\Leftrightarrow_\beta \exists x(\lambda z(\exists y(book'(y) \wedge read^*(y)(z)))(x))$

$\Leftrightarrow_\beta \exists x \exists y(book'(y) \wedge read^*(y)(x))$

# Reading material

- Winter: Elements of Formal Semantics (Chapter 3, Part III)
  http://www.phil.uu.nl/~yoad/efs/main.html