Semantic Theory
Week 8 – Discourse Representation Theory

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Recap: DRS Syntax

A discourse representation structure (DRS) K is a pair $\langle U_K, C_K \rangle$, where:

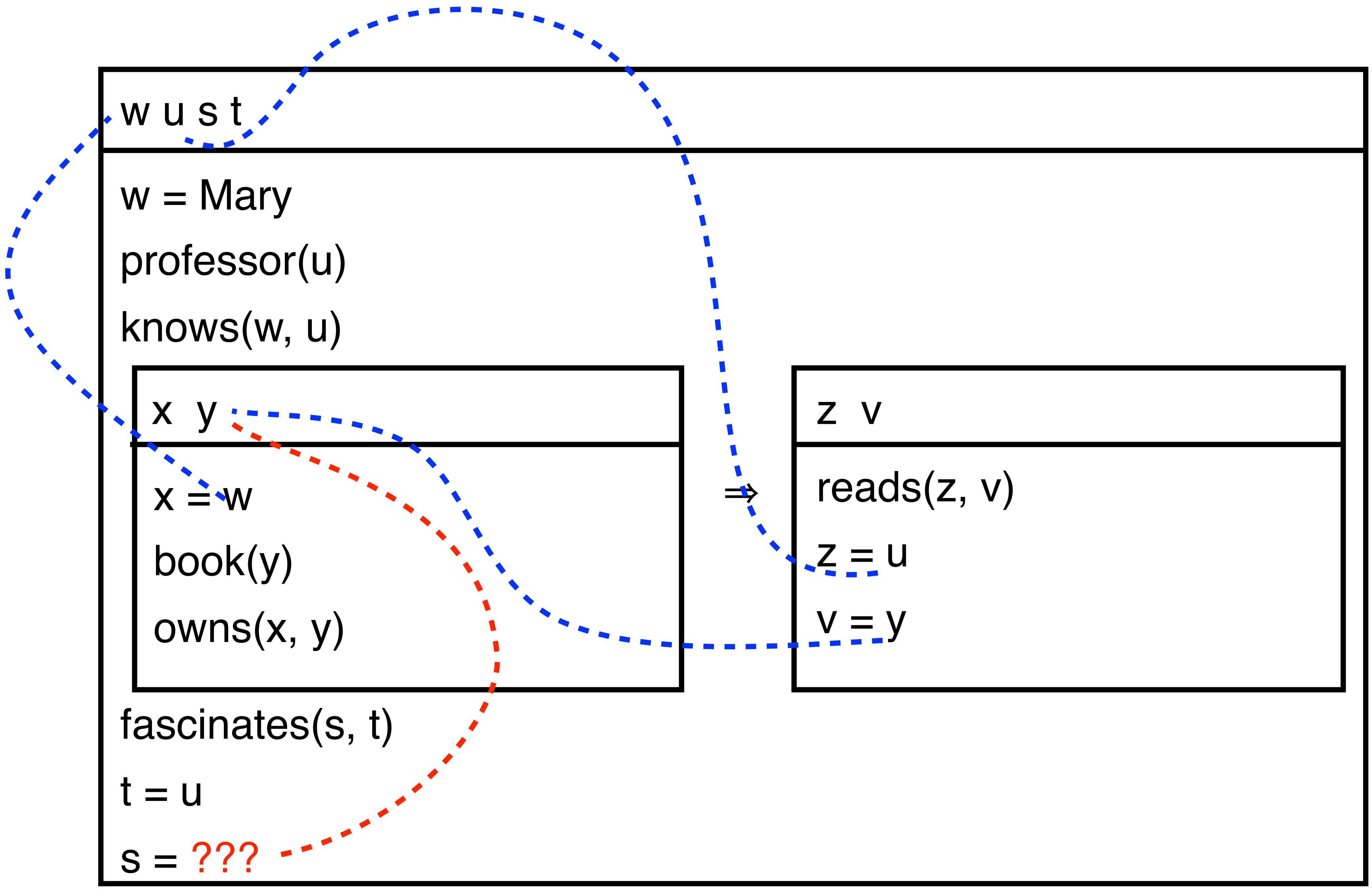
- $U_K \subseteq U_D$ and U_D is a set of discourse referents, and
- C_K is a set of well-formed DRS conditions

Well-formed DRS conditions:

• $R(u_1, \ldots, u_n)$	*where:*	R is an n-place relation, $u_i \in U_D$
• $u = v$		$u, v \in U_D$
• $u = a$		$u \in U_D$, a is a constant
• $\neg K_1$		K_1 is a DRS
• $K_1 \Rightarrow K_2$		K_1 and K_2 are DRSs
• $K_1 \vee K_2$		K_1 and K_2 are DRSs

Anaphora and accessibility

Mary knows a professor. If she owns a book, he reads it. ?It fascinates him.



Non-accessible discourse referents

Cases of non-accessibility:

(1) *If a professor owns a book, he reads it. It has 300 pages.*

(2) *It is not the case that a professor owns a book. He reads it.*

(3) *Every professor owns a book. He reads it.*

(4) *If every professor owns a book, he reads it.*

(5) *Peter owns a book, or Mary reads it.*

(6) *Peter reads a book, or Mary reads a newspaper article. It is interesting.*

Accessible discourse referents

The following discourse referents are accessible for a condition:

- DRs in the same local DRS
- DRs in a superordinate DRS
- DRs in the universe of an antecedent DRS, if the condition occurs in the consequent DRS.

We need a formal notion of DRS subordination

Subordination

A DRS K_1 is an immediate sub-DRS of a DRS $K = \langle U_K, C_K \rangle$ iff C_K contains a condition of the form

- $\neg K_1$, $K_1 \Rightarrow K_2$, $K_2 \Rightarrow K_1$, $K_1 \vee K_2$ or $K_2 \vee K_1$.

K_1 is a sub-DRS of K (notation: $K_1 \leq K$) iff

- $K_1 = K$, or
- K_1 is an immediate sub-DRS of K, or
- there is a DRS K_2 such that $K_1 \leq K_2$ and K_2 is an immediate sub-DRS of K (i.e. reflexive, transitive closure)

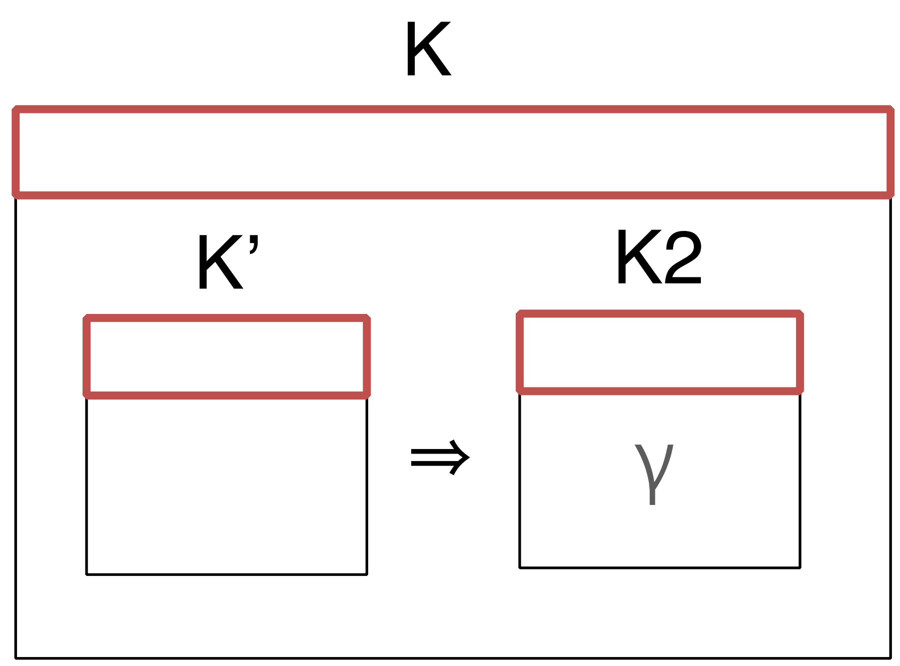
K_1 is a proper sub-DRS of K iff $K_1 \leq K$ and $K_1 \neq K$.

Accessibility

Let K, K_1, K_2 be DRSs such that $K_1, K_2 \leq K$, $x \in U_{K_1}$, $\gamma \in C_{K_2}$

x is accessible from γ in K iff

- $K_2 \leq K_1$ or
- there are K_3, $K_4 \leq K$ such that $K_1 \Rightarrow K_3 \in C_{K4}$ and $K_2 \leq K_3$



Free and bound variables in DRT

A DRS variable x, introduced in DRS K_i, is bound in global DRS K iff there exists a DRS $K_j \leq K$, such that:

(i) $x \in U(K_j)$.

(ii) K_j is accessible for K_i in K

Properness: A DRS is *proper* iff it does not contain any free variables

Purity: A DRS is *pure* iff it does not contain any *otiose declarations* of variables

$x \in U(K_1)$ and $x \in U(K_2)$ and $K_1 \leq K_2$

From text to DRS

Text	$\Sigma = \langle \; S_1,$	$S_2,$	$\ldots,$	$S_n \; \rangle$
	$\downarrow$	$\downarrow$		$\downarrow$
Syntactic Analysis	$P(S_1)$	$P(S_2)$	$\ldots$	$P(S_n)$
	$\downarrow$	$\downarrow$		$\downarrow$
DRS Construction	$K_1 \longrightarrow$	$K_2 \longrightarrow$	$\ldots \longrightarrow$	K_n
				$\downarrow$

Interpretation by
model embedding:
Truth-conditions of Σ

DRS Construction Algorithm

Let the following be a well-formed, *reducible* DRS condition:

- Conditions of form α or α(x1, …, xn), where α is a context-free parse tree.

DRS construction algorithm:

- Given a text $\Sigma = \langle S_1, \ldots, S_n \rangle$, and a DRS K_0 ($= \langle \varnothing, \varnothing \rangle$, by default)
- Repeat for i = 1, …, n:
  - Add parse tree $P(S_i)$ to the conditions of K_{i-1}.
  - Apply DRS construction rules to reducible conditions of K_{i-1}, until no reduction steps are possible any more.
  - The resulting DRS K_i is the discourse representation of text $\langle S_1,\ldots,S_i \rangle$.

DRS Interpretation

Given a DRS $K = \langle U_K, C_K \rangle$, with $U_K \subseteq U_D$

Let $M = \langle U_M, V_M \rangle$ be a FOL model structure appropriate for K, i.e. a model structure that provides interpretations for all predicates and relations occurring in K

DRS K is *true* in model M **iff**

- there is an embedding function for K in M which verifies all conditions in K

… where: an embedding of K into M is a (partial) function **f** from U_D to U_M such that $U_K \subseteq \mathrm{Dom}(\mathbf{f})$.

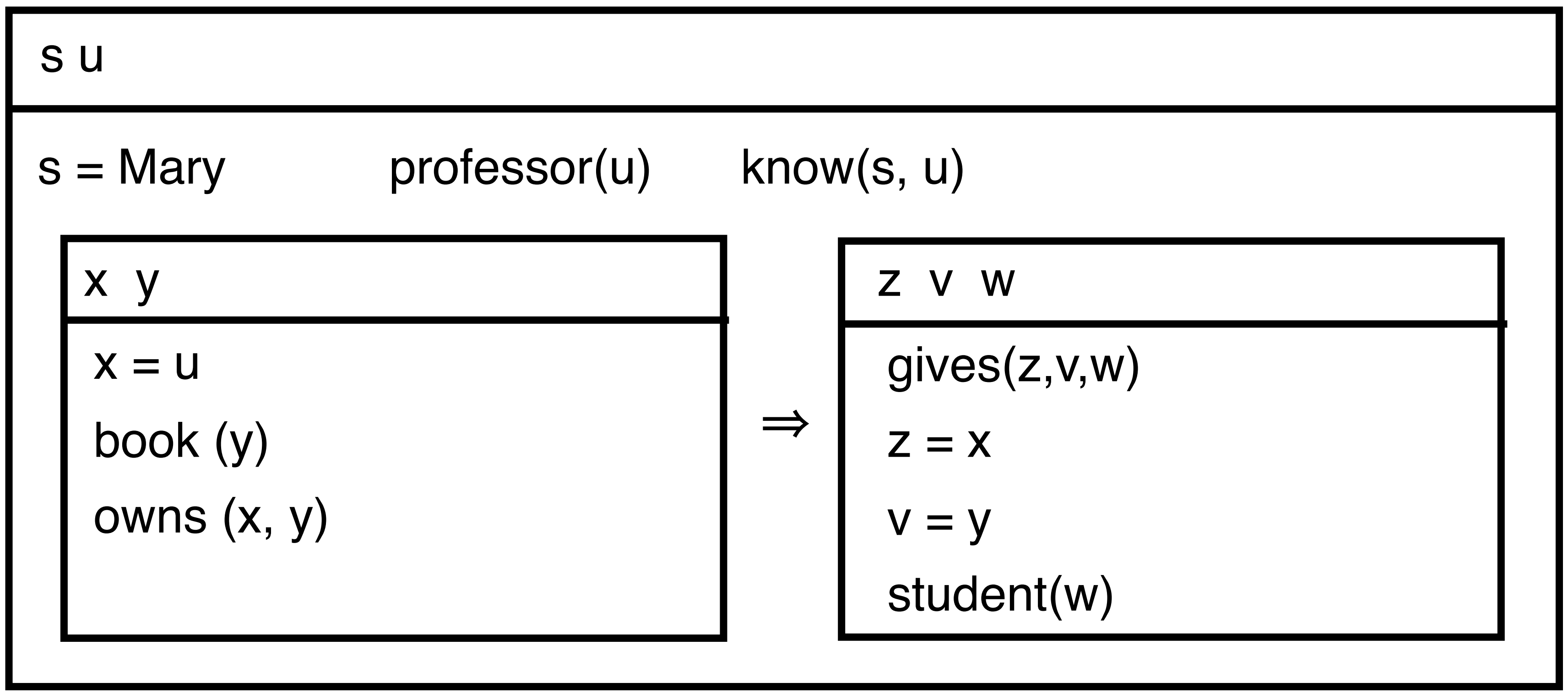
Verifying embedding

An embedding **f** of K in M verifies K in M (**f** $\vDash_M$ K) iff **f** verifies every condition $\alpha \in C_K$

- **f** $\vDash_M R(x_1, \ldots, x_n)$ iff $\langle \mathbf{f}(x_1), \ldots, \mathbf{f}(x_n)\rangle \in V_M(R)$
- **f** $\vDash_M x = y$ iff $\mathbf{f}(x) = \mathbf{f}(y)$
- **f** $\vDash_M x = a$ iff $\mathbf{f}(x) = V_M(a)$
- **f** $\vDash_M \neg K_1$ iff there is no **g** $\supseteq_{U_{K1}}$ **f** such that g $\vDash_M K_1$
- **f** $\vDash_M K_1 \Rightarrow K_2$ iff for all **g** $\supseteq_{U_{K1}}$ **f** such that **g** $\vDash_M K_1$
  there is a **h** $\supseteq_{U_{K2}}$ **g** such that **h** $\vDash_M K_2$
- **f** $\vDash_M K_1 \vee K_2$ iff there is a $\mathbf{g_1} \supseteq_{U_{K1}}$ **f** such that $\mathbf{g_1} \vDash_M K_1$
  or there is a $\mathbf{g_2} \supseteq_{U_{K2}}$ **f** such that $\mathbf{g_2} \vDash_M K_2$

Verifying embedding: example

Mary knows a professor. If he owns a book, he gives it to a student.



…is **true** in $M = \langle U_M, V_M \rangle$ *iff* there is an $\mathbf{f} :: U_D \rightarrow U_M$, (with $\{s,u\} \subseteq Dom(\mathbf{f})$) such that:

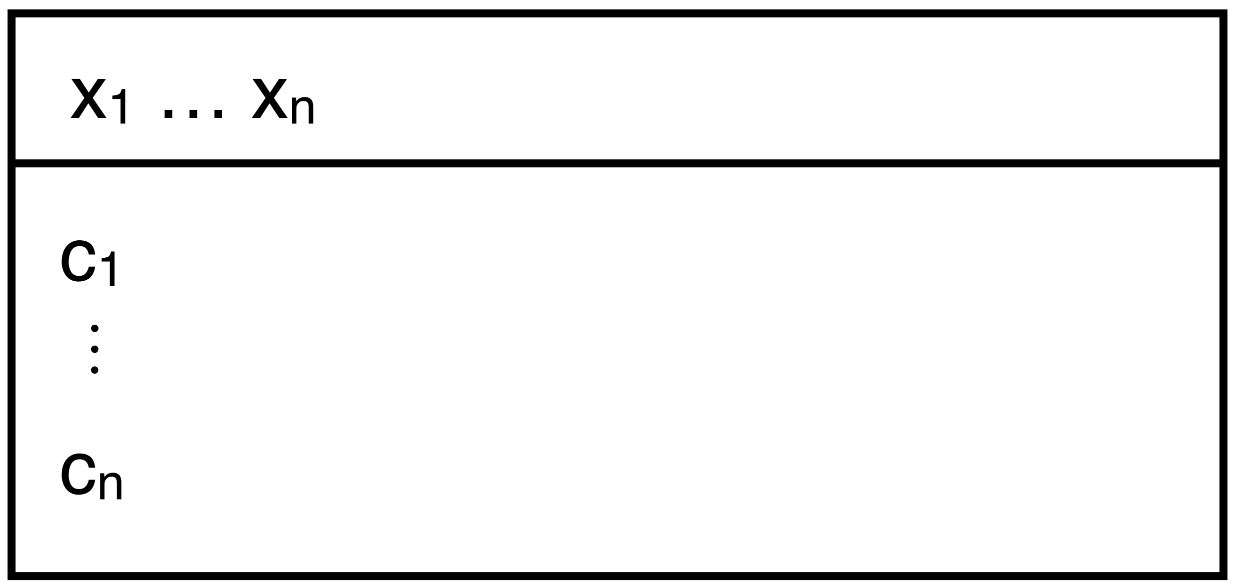
$\mathbf{f}(s) = V_M(Mary)$ & $\mathbf{f}(u) \in V_M(prof')$ & $\langle \mathbf{f}(s), \mathbf{f}(u) \rangle \in V_M(know)$,

and for all $\mathbf{g} \supseteq_{\{x,y\}} \mathbf{f}$ s.t. $\mathbf{g}(x) = \mathbf{g}(u)$ $(=\mathbf{f}(u))$ & $\mathbf{g}(y) \in V_M(book)$ & $\langle \mathbf{g}(x), \mathbf{g}(y) \rangle \in V_M(own)$,

there is a $\mathbf{h} \supseteq_{\{z,\ v,\ w\}} \mathbf{g}$ s.t. $\langle \mathbf{h}(z), \mathbf{h}(v), \mathbf{h}(w) \rangle \in V_M(give)$ & $\mathbf{h}(z) = \mathbf{h}(x)$ $(=\mathbf{g}(x))$ & … etc.

Translation of DRSs to FOL

Consider DRS $K = \langle\{x_1, \ldots, x_n\}, \{c_1, \ldots, c_k\}\rangle$



K is truth-conditionally equivalent to the following FOL formula:

$$\exists x_1 \ldots \exists x_n [c_1 \wedge \ldots \wedge c_k]$$

DRT and compositionality

- DRT is non-compositional on truth conditions: The difference in discourse-semantic status of the text pairs is not predictable through the (identical) truth conditions of its component sentences.

- Since structural information which cannot be reduced to truth conditions is required to compute the semantic value of texts, DRT is called a *representational* theory of meaning.

However…

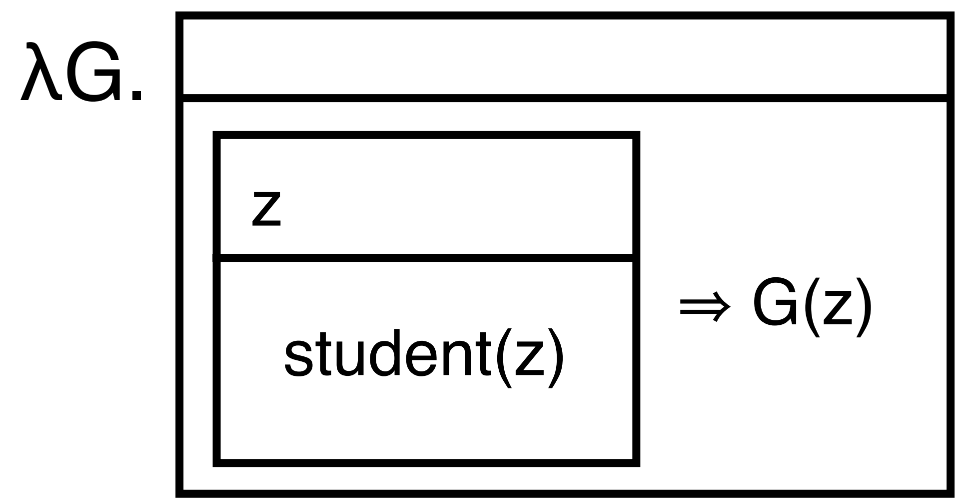
Wait a minute …

- Why can't we just combine type theoretic semantics and DRT?
- Use λ-abstraction and reduction as we did before, but:
- Assume that the target representations which we want to arrive at are not First-Order Logic formulas, but DRSs.
- The result is called λ-DRT.

λ-DRSs

An expression in λ-DRT consists of a lambda prefix and a partially instantiated DRS.

- *every student* :: $\langle\langle e, t\rangle, t\rangle \mapsto$



Alternative notation: $\lambda G\ [\ \varnothing \mid [\ z \mid \text{student}(z)\ ] \Rightarrow G(z)\ ]$

- *works* :: $\langle e, t\rangle \mapsto \lambda x\ [\ \varnothing \mid \text{work}(x)\ ]$

λ-DRT: β-reduction

Every student works

$\mapsto \lambda G[\ \varnothing \mid [\ z \mid \text{student}(z)\ ] \Rightarrow G(z)\ ]](\lambda x\ [\ \varnothing \mid \text{work}(x)\ ])$

$\Rightarrow^{\beta} [\ \varnothing \mid [\ z \mid \text{student}(z)\ ] \Rightarrow (\lambda x\ [\ \varnothing \mid \text{work}(x)\ ])(z)\ ]$

$\Rightarrow^{\beta} [\ \varnothing \mid [\ z \mid \text{student}(z)\ ] \Rightarrow [\ \varnothing \mid \text{work}(z)\ ]]$

How do we define conjunction on DRSs?

(Naïve) Merge

The "merge" operation on DRSs combines two DRSs (conditions and universes).

- Let $K_1 = [U_1 | C_1]$ and $K_2 = [U_2 | C_2]$.

Merge: $K_1 + K_2 = [U_1 \cup U_2 | C_1 \cup C_2]$

Merge: An example

- *a student* $\mapsto$ $\lambda G\ ([\ z \mid \text{student}(z)\ ] + G(z))$
- *works* $\mapsto$ $\lambda x\ [\ \varnothing \mid \text{work}(x)\ ]$

A student works $\mapsto$ $\lambda G\ ([\ z \mid \text{student}(z)\ ] + G(z))\ (\lambda x[\ \varnothing \mid \text{work}(x)])$

$\Rightarrow^{\beta}\ [\ z \mid \text{student}(z)\ ] + \lambda x[\ \varnothing \mid \text{work}(x)](z)$

$\Rightarrow^{\beta}\ [\ z \mid \text{student}(z)\ ] + [\ \varnothing \mid \text{work}(z)]$

$\Rightarrow^{\beta}\ [\ z \mid \text{student}(z), \text{work}(z)]$

Compositional analysis

- *Mary* $\mapsto$ $\lambda G\ ([\ z \mid z = \text{Mary}\ ] + G(z))$
- she $\mapsto$ $\lambda G.G(z)$

Mary works. She is successful.

$\mapsto \lambda K\ \lambda K'(K + K')([\ z \mid z = \text{Mary, work}(z)])([\ \mid \text{successful}(z)])$

$\Rightarrow^{\beta} \lambda K'([\ z \mid z = \text{Mary, work}(z)] + K')([\ \mid \text{successful}(z)])$

$\Rightarrow^{\beta} [\ z \mid z = \text{Mary, work}(z)] + ([\ \mid \text{successful}(z)])$

$\Rightarrow^{\beta} [z \mid z = \text{Mary, work}(z), \text{successful}(z)]$

Merge again

The "merge" operation on DRSs combines two DRSs (conditions and universes).

- Let $K_1 = [\ U_1 \mid C_1\ ]$ and $K_2 = [\ U_2 \mid C_2\ ]$.

Merge: $K_1 + K_2 \Rightarrow [\ U_1 \cup U_2 \mid C_1 \cup C_2\ ]$

under the assumption that no discourse referent $u \in U_2$ occurs free in a condition $\gamma \in C_1$.

Variable capturing

In λ-DRT, discourse referents are captured via the interaction of β-reduction and DRS-binding:

- λK'([z | student(z), work(z)] + K')([| successful(z)])

  $\Rightarrow^{\beta}$ [z | student(z), work(z)] + [| successful(z)]

  $\Rightarrow^{\beta}$ [z | student(z), work(z), successful(z)]

But the β-reduced DRS must still be *equivalent* to the original DRS!

So, the potential for capturing discourse referents must be captured into the interpretation of a λ-DRS. Possible, but tricky.

Playing in the sandbox

PDRT-SANDBOX is a Haskell library that implements Discourse Representation Theory (and the extension Projective DRT)

http://hbrouwer.github.io/pdrt-sandbox/
also available via: login.coli.uni-saarland.de:/proj/courses/semantics19

- Define your own DRSs, using the internal syntax or the set-theoretic notation
- Show the DRSs in different output formats (boxes, linear boxes, set-theoretic, internal syntax)
- Composition of DRSs (using lambda's)
- Translate DRSs to FOL formulas

PDRT SANDBOX

DRS Syntax in PDRT-SANDBOX

DRS: `DRS […] […]` referents conditions

Referents: `DRSRef “x”, DRSRef “Mary”`

Conditions:

- Relation: `Rel (DRSRel "man") [DRSRef “x"]`
- Identity: `Rel (DRSRel "=") [DRSRef “x”,DRSRef “y"]`
- Negation: `Neg (DRS […] […])`
- Implication: `Imp (DRS […] […]) (DRS […] […])`
- Disjunction: `Or (DRS […] […]) (DRS […] […])`

Properties: `isPure(DRS […] […]), isProper(DRS […] […])`

Using PDRT-SANDBOX on **coli**

```
~$ cp -r /proj/courses/semantics-19/pdrt-sandbox/ .
~$ cp /proj/courses/semantics-19/ghci .ghci
~$ cd pdrt-sandbox/
~/pdrt-sandbox$ make
[…]
~/pdrt-sandbox$ cd tutorials/
~/pdrt-sandbox/tutorials$ ghci DRSTutorial.hs
GHCi, version 7.10.3: http://www.haskell.org/ghc/  :? for help
[1 of 1] Compiling Main             ( DRSTutorial.hs, interpreted )
Ok, modules loaded: Main.
*Main>
```

Literature

Reading:

- Hans Kamp and Uwe Reyle: From Discourse to Logic, Kluwer: Dordrecht 1993.

Link:

- https://plato.stanford.edu/entries/discourse-representation-theory/