## Exercises are due on: Thursday April 25, 10AM (before class)

## Semantic Theory 2019: Exercise sheet 1

## Exercise 1

Translate the following sentences into first-order predicate logic. You can freely introduce predicates, but try to retain as much of the structure as possible. Also provide the key to the translation.
a. Link is scared of nothing.
b. Every princess loves her saviour.
c. A sage has magical powers.
d. Zelda does not wear green clothes.
e. The Master Sword is the only sword that can defeat Agahnim.
f. If the leader of the Dark World is defeated, the Light World is free.
g. Although Zelda was saved from Ganon, Link has to save her again in another adventure.

## Exercise 2

Consider the following model $M_{1}=\left\langle U_{1}, V_{1}\right\rangle$, with $U_{1}=\left\{e_{1}, e_{2}, e_{3}, e_{4}, e_{5}, e_{6}\right\}$. The interpretation function $V_{1}$ is defined as follows:

- $V_{1}(j)=e_{1}$
- $V_{1}(m)=e_{4}$
- $V_{1}(b)=e_{6}$
- $V_{1}(A)=\left\{e_{2}, e_{3}\right\}$
- $V_{1}(B)=\left\{e_{2}, e_{4}, e_{5}\right\}$
- $V_{1}(C)=\left\{e_{2}, e_{4}, e_{5}, e_{6}\right\}$

- $V_{1}(R)=\left\{\left\langle e_{2}, e_{3}\right\rangle,\left\langle e_{3}, e_{2}\right\rangle\left\langle e_{4}, e_{1}\right\rangle,\left\langle e_{4}, e_{2}\right\rangle,\left\langle e_{5}, e_{6}\right\rangle\right\}$

Let the assignment function $g_{1}$ be defined as follows:
$g_{1}(x)=e_{4}, g_{1}\left(x^{\prime}\right)=e_{2}, g_{1}\left(x^{\prime \prime}\right)=e_{3}$ and for all other variables $x^{\prime *}: g_{1}\left(x^{\prime *}\right)=$ $e_{5}$.
2.1 Evaluate the following formulas in model $M_{1}$, with respect to assignment function $g_{1}$, showing the crucial steps.
a. $\llbracket R\left(x^{\prime}, x^{\prime \prime}\right) \wedge R\left(x^{\prime \prime \prime}, b\right) \rrbracket^{M_{1}, g_{1}}=$ ?
b. $\llbracket \exists x^{\prime \prime}\left(A\left(x^{\prime \prime}\right) \rightarrow R\left(x^{\prime \prime}, j\right)\right) \rrbracket^{M_{1}, g_{1}}=$ ?
c. $\llbracket \forall x^{\prime} \exists x^{\prime \prime \prime}\left(R\left(x^{\prime \prime \prime}, x^{\prime}\right) \vee R\left(x^{\prime}, x^{\prime \prime \prime}\right)\right) \rrbracket^{M_{1}, g_{1}}=$ ?
d. $\llbracket \forall x\left(B(x) \rightarrow\left(A(x) \vee \neg \exists x^{\prime \prime}\left(R\left(x^{\prime \prime}, x\right)\right)\right)\right) \rrbracket^{M_{1}, g_{1}}=$ ?
2.2 Provide the full definition of a model $M_{2}$ and assignment function $g_{2}$ that satisfy the following formulas (NB: $c_{1}$ and $c_{2}$ are constants):

- $R\left(x, x^{\prime}\right)$
- $\forall x\left(A(x) \vee \exists x^{\prime}\left(R\left(x, x^{\prime}\right)\right)\right)$
- $\neg \exists x\left(R\left(x, c_{1}\right)\right)$
- $\exists x^{\prime \prime}\left(A\left(x^{\prime \prime}\right) \wedge \neg \exists x^{\prime}\left(A\left(x^{\prime}\right) \wedge R\left(x^{\prime}, x^{\prime \prime}\right)\right)\right)$
- $\forall x^{\prime}\left(B\left(x^{\prime}\right) \rightarrow\left(A\left(x^{\prime}\right) \vee R\left(x^{\prime}, c_{2}\right)\right)\right)$
2.3 (Bonus) Can you think of a sensible (or: funny) interpretation for the predicates $A, B$ and $R$, and the constants $c_{1}$ and $c_{2}$ in your model of the previous exercise? Given this interpretation, what is the natural language translation of the formulas given in exercise 2.2?

