Semantic Theory
week 8 – Plurals and Mass Nouns

Noortje Venhuizen

Universität des Saarlandes

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Plural NPs

(1) *Bill and Mary work* ⊨ *Bill works*

$$\text{work'}(b) \wedge \text{work'}(m) \models \text{work}(b)$$

(2) *Bill and Mary work* ⊨ *Mary works*

$$\text{work'}(b) \wedge \text{work'}(m) \models \text{work}(m)$$

(3) *All students work, John is a student* ⊨ *John works*

$$\forall x(\text{student}(x) \rightarrow \text{work'}(x)), \text{student'}(j) \models \text{work}(j)$$

But this pattern does not hold for all predicates…

(1) *Bill and Mary met ⊭ Bill met*

(2) *The students met, John is a student ⊭ John met*

(3) *The committee will dissolve. John is member of the committee ⊭ John will dissolve.*

“meet” is a collective predicate.

Distributive vs. Collective predicates

Distributive predicates

- Applicable to singular and plural NPs;
- Predication with a plural NP "distributes" over the individual objects covered by the NP;
- Examples: work, sleep, eat, tall, …

Collective predicates

- Only applicable to plural or group NPs;
- Semantics cannot be reduced to atomic statements about single standard individuals;
- Examples: meet, gather, unite, agree, be similar, compete, disperse, dissolve, disagree, be numerous, ...

Modeling plural terms

Desiderata for a model with plurality:

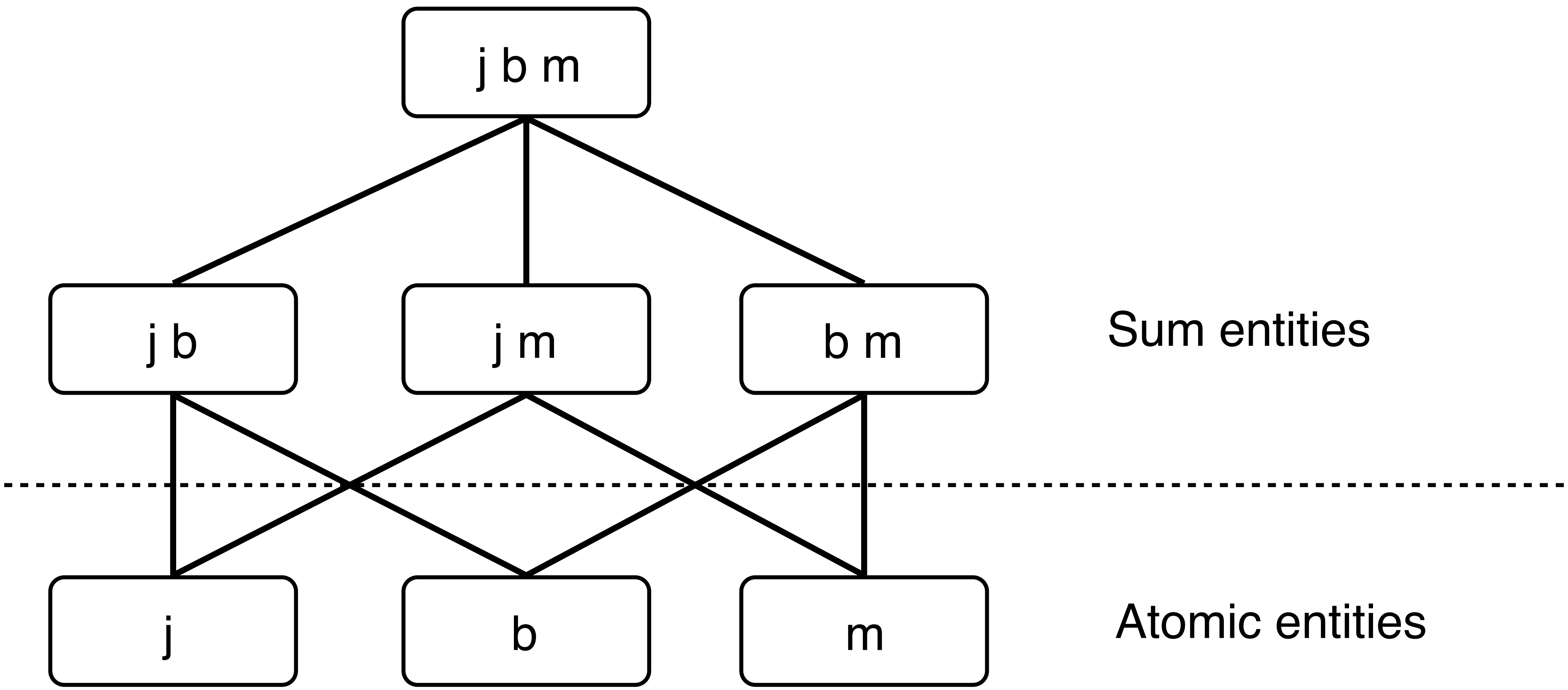
- A representation of plural terms that is not (only) defined in terms of atomic entities (to account for collective predicates)

We extend the universe of our model structures with "groups" (or: "sums")

- A relation between atomic and plural entities (to account for the entailment pattern of distributive predicates)

We add a membership relation (or: "individual part" relation) to the model structure

Structured Universe - Example



Lattices and Semi-lattices

A **partial order** is a structure ⟨A, ≤⟩ where ≤ is a reflexive, transitive, and antisymmetric relation over A.

- The **join** of a and b ∈ A (Notation: $a \sqcup b$) is the lowest upper bound for a and b.

- The **meet** of a and b ∈ A (Notation: $a \sqcap b$) is the highest lower bound for a and b.

A **lattice** is a partial order ⟨A, ≤⟩ that is closed under meet and join.

A **join semi-lattice** is a partial order ⟨A, ≤⟩ that is closed under join

Lattices and Semi-lattices (cont.)

A **bounded lattice** is a lattice with a maximal element (1) and a minimal element (0).

- An element $a \in A$ is an **atom**, if $a \neq 0$ and there is no $b \neq 0$ in A such that $b<a$.
- A lattice $\langle A, \leq \rangle$ is **atomic**, if for every $a \neq 0$ there is an atom b such that $b \leq a$.

Model structures for plural terms

A model structure is a pair $M = \langle\langle U, \leq\rangle, V\rangle$, where

- $\langle U, \leq\rangle$ is an atomic join semi-lattice with universe U and individual part relation $\leq$.

- V is an interpretation function.

In addition, we define:

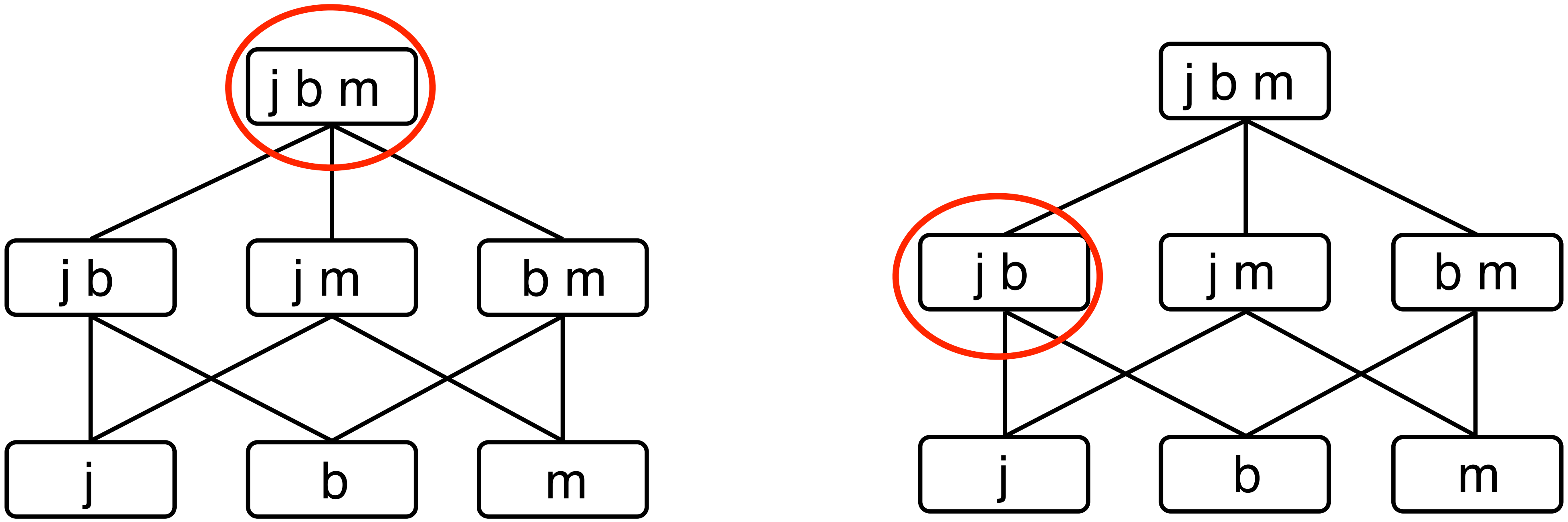
- $A \subseteq U$ is the set of atoms in $\langle U, \leq\rangle$.

- $U \setminus A$ is the set of non-atomic elements, i.e., the set of proper sums or groups in U.

Collective predicates

Let P_C be the set of collective predicates (*meet, collaborate, …*)

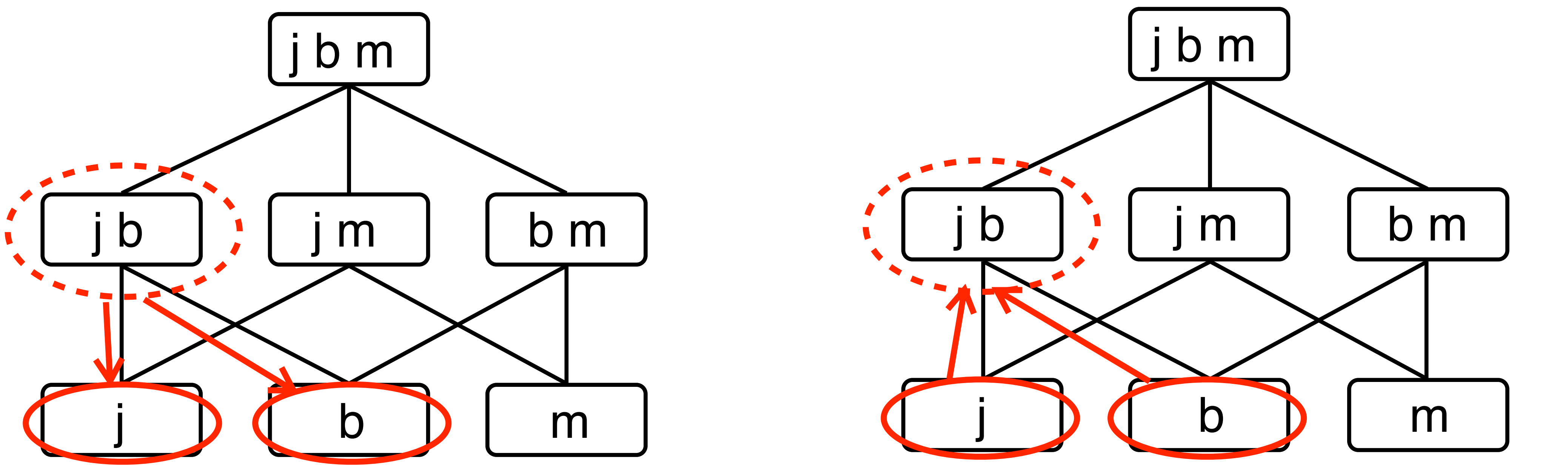
- The domain of P_C is restricted to non-atomic elements: $V_M(P_C) \subseteq U \setminus A$



Distributive predicates

Let P_d be the set of distributive predicates (*work, tall, student, …*)

- The domain of P_d is the universe of M: $V_M(P_d) \subseteq U_M$, such that $a \in V_M(F)$ and $b \in V_M(F)$ iff $a \sqcup b \in V_M(F)$



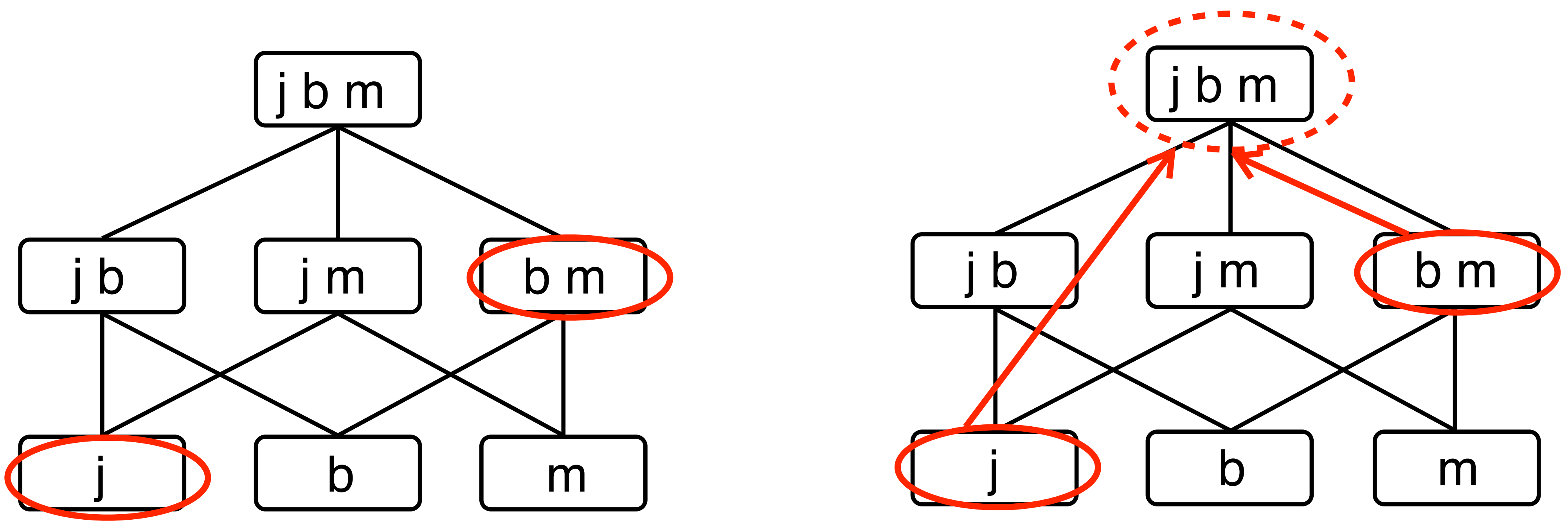
Distributivity

Closure under summation

Mixed predicates

Let P_m be the set of mixed predicates (*carry a piano, solve the exercise, …*)

- The domain of P_m is the universe of M: $V_M(P_m) \subseteq U$



Non-distributive   Closure under summation

Language for plural terms

We extend FOL with a summation operator ⊕, a one-place predicate At for “atom”, and a two-place relation ◅ for “(proper) individual part”

$j \oplus b$	“the group consisting of John and Bill”
$j \lhd j \oplus b$	“John is member of the group consisting of John and Bill”
$j \oplus b \lhd c$	“John and Bill are members of the committee”

In addition, we introduce:

- Variables ranging over proper sums: X, Y, Z, …
- Number-specific constants: “student-sg”, “student-pl”

Interpretation of plural terms

$[\![a \oplus b]\!]^{M,g} = [\![a]\!]^{M,g} \sqcup [\![b]\!]^{M,g}$

$[\![a \lhd b]\!]^{M,g} = 1$ iff $[\![a]\!]^{M,g} < [\![b]\!]^{M,g}$

$[\![At(a)]\!]^{M,g} = 1$ iff $[\![a]\!]^{M,g} \in A$

Individual constants denote either atoms ($\in A$) or sums ($\in U \setminus A$)

Predicate expressions satisfy specific constraints:

- V_M(student-sg) $\subseteq A$
- V_M(student-pl) $\subseteq U \setminus A$

Interpretation of distributive predicates

If a distributive predicate applies to a set $X \subseteq A$, then the full denotation of the predicate is the join semi-lattice generated by X.

- The denotation of distributive predicates P_d is uniquely determined by their atomic members:

$$\forall x[P_d(x) \leftrightarrow \forall y[At(y) \wedge y \lhd x \rightarrow P_d(y)]]$$

Mass nouns

Mass nouns (*water, gold, wood, money, soup, …*) behave like plurals in different respects

(1) a. students + students = students
b. water + water = water

Closed under summation

(2) a. 5 students
b. 5 liters of water

Can combine with cardinalities

(3) a. #A students are hard workers
b. #A water is wet

Shared grammatical patterns

Mass Nouns vs. Plurals

Unlike plurals, mass nouns are divisive:

- An amount of water can always be subdivided into proper parts, which are water again.

The denotation of mass nouns cannot be reduced to model theoretic atomic individuals

- When talking about water, we are not talking about a collection of individual entities

Model structure for mass nouns

We add another sort of entities, the "portions of matter" M, to the model structure, and distinguish an part relation for individuals ($\leq_i$) and a part relation for materials ($\leq_m$):

$\mathcal{M} = \langle\langle U, \leq_i\rangle, \langle M, \leq_m\rangle, V\rangle$

- $U \cap M = \varnothing$
- $\langle U, \leq_i\rangle$ is an atomic join semi-lattice
- **$\langle M, \leq_m\rangle$ is a non-atomic and dense join semi-lattice**
- V is a value assignment function

Materialization

There is a close relation between the domain of material entities and the domain of (atomic and sum) individuals: *Each individual consists of a specific portion of matter*

Let $M = \langle\langle U, \leq_i\rangle, \langle M, \leq_m\rangle, h, V\rangle$ be a model structure in which h is a "materialization" function that models the object-matter relation:

- h is a homomorphism that maps (atomic and plural) individuals to the matter they consist of

- $a \leq_i b$ iff $h(a) \leq_m h(b)$

- $h(a \sqcup_i b) = h(a) \sqcup_m h(b)$

Representation of mass nouns

Additions to the logical representation language:

- Variables referring to matters: ***x***, ***y***, ***z***, ...
- A material fusion operation $\oplus_m$ and a material part relation $\lhd_m$ (to be distinguished from $\oplus_i$ and $\lhd_i$, respectively)
- A new logical operator m that expresses the materialization function:

  $[\![m(\alpha)]\!]^{M,g} = h([\![\alpha]\!]^{M,g})$, where $\alpha \in WE_e$ is a well-formed expression denoting an individual entity

Examples

(1) *The ring is made of gold*

$\mapsto \exists y[\text{ring}(y) \wedge \text{gold}(\text{m}(y))]$

(2) *The ring contains gold*

$\mapsto \exists y \exists \boldsymbol{x}[\text{ring}(y) \wedge \boldsymbol{x} \lhd_{\text{m}} \text{m}(y) \wedge \text{gold}(\boldsymbol{x})]$