Exercises are due on: Tuesday, May 3rd, 10 AM (before class)

Semantic Theory 2016: Exercise sheet 1

Exercise 1

Translate the following sentences into first-order predicate logic. You can freely introduce predicates, but try to retain as much of the structure as possible. Also provide the key to the translation.

- a. Jon Snow knows nothing.
- b. A dire wolf is not a pet.
- c. Every Lannister pays his debt.
- d. Fire-breathing dragons only obey Khaleesi.
- e. If one family rules the throne, all other families will fight for it.
- f. Although Jaime lost a hand, he wins every fight unless he loses his other hand.

Exercise 2

Consider the following model $M_1 = \langle U_1, V_1 \rangle$, with $U_1 = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. The interpretation function V_1 is defined as follows:

- $V_1(j) = e_1$
- $V_1(m) = e_4$
- $V_1(b) = e_6$
- $V_1(A) = \{e_2, e_3\}$
- $V_1(B) = \{e_2, e_4, e_5\}$
- $V_1(C) = \{e_2, e_4, e_5, e_6\}$
- $V_1(R) = \{\langle e_2, e_3 \rangle, \langle e_3, e_2 \rangle \langle e_4, e_1 \rangle, \langle e_4, e_2 \rangle, \langle e_5, e_6 \rangle \}$

M1: e_1 e_2 e_3 e_4 e_6 e_6

Let the assignment function g_1 be defined as follows: $g_1(x) = e_4$, $g_1(x') = e_2$, $g_1(x'') = e_3$ and for all other variables x'^* : $g_1(x'^*) = e_5$.

- **2.1** Evaluate the following formulas in model M_1 , with respect to assignment function g_1 , showing the crucial steps.
- a. $[R(x', x'') \land R(x''', b)]^{M_1, g_1} = ?$
- b. $[\exists x''(A(x'') \to R(x'', j))]^{M_1, g_1} = ?$
- c. $[\forall x' \exists x''' (R(x''', x') \lor R(x', x'''))]^{M_1, g_1} = ?$
- d. $[\forall x (B(x) \to (A(x) \lor \neg \exists x''(R(x'', x))))]^{M_1, g_1} = ?$
- **2.2** Provide the full definition of a model M_2 and assignment function g_2 that satisfy the following formulas (NB: c_1 and c_2 are constants):
 - \bullet R(x,x')
 - $\forall x (A(x) \lor \exists x' (R(x, x')))$
 - $\neg \exists x (R(x, c_1))$
 - $\exists x''(A(x'') \land \neg \exists x'(A(x') \land R(x', x'')))$
 - $\forall x'(B(x') \to (A(x') \lor R(x', c_2)))$
- **2.3 (Bonus)** Can you think of a sensible (or: funny) interpretation for the predicates A, B and R, and the constants c_1 and c_2 in your model of the previous exercise? Given this interpretation, what is the natural language translation of the formulas given in exercise 2.2?