Semantic Theory
Lecture 9: Generalized Quantifiers

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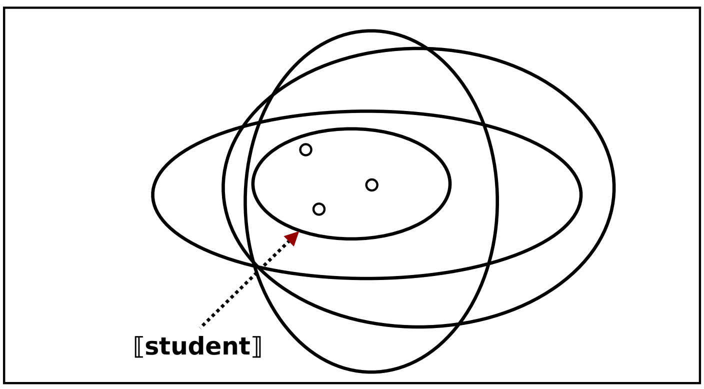
Generalized Quantifiers

- *Every student works*
  - $\forall x(\text{student}'(x) \rightarrow \text{work}'(x))$
  - *Every student* ↦ $\lambda Q \forall x(\text{student}'(x) \rightarrow Q(x))$
  - $[\![\textit{Every student}]\!]^M = \{ P \subseteq U_M \mid [\![\textit{student}]\!] \subseteq P \}$
- **A generalized quantifier** is a set of properties
  - property = set of individuals
- A sentence of the form $[_S$ NP VP$]$ is true iff $[\![VP]\!] \in [\![NP]\!]$
  - $[\![\textit{Every student works}]\!] = 1$ iff $[\![\textit{work}]\!] \in [\![\textit{every student}]\!]$

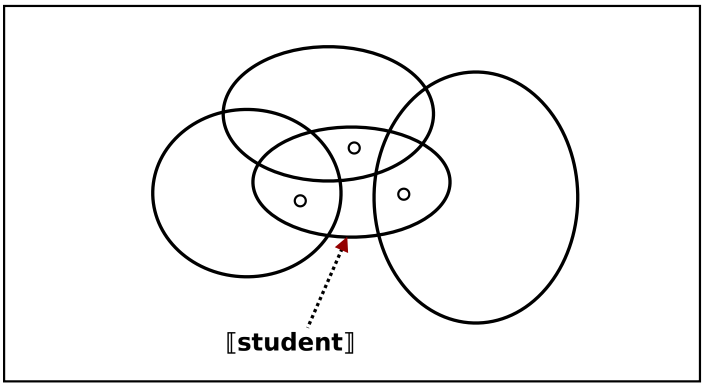
$[\![$*every student*$]\!]$

- $[\![$***every student***$]\!]$ denotes the set of properties that apply to every student (i.e., all supersets of $[\![$student$]\!]$)



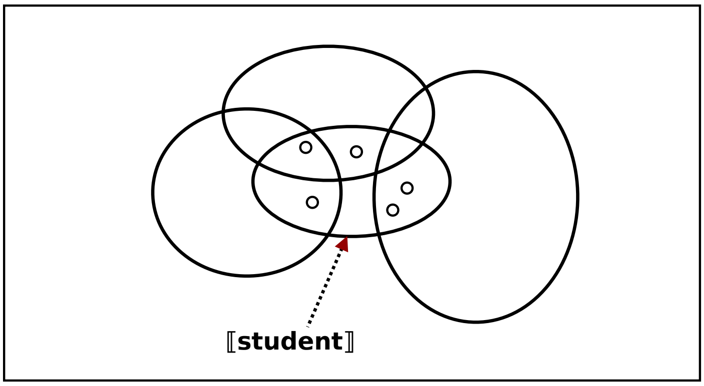
⟦*a student*⟧

- ⟦***a student***⟧ denotes the set of properties that apply to at least one student.



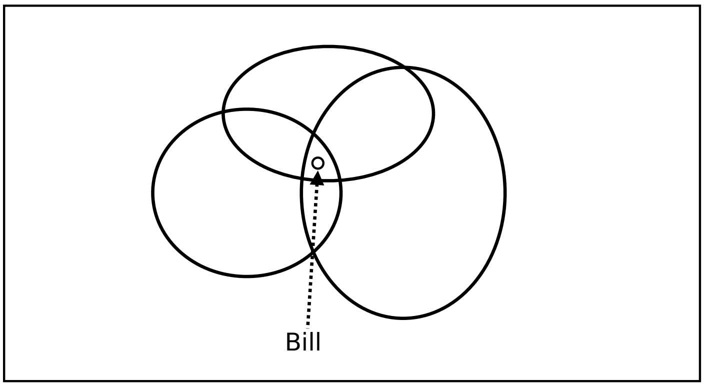
⟦*two students*⟧

- ⟦***two students***⟧ denotes the set of properties that apply to at least (exactly) two students.



⟦*Bill*⟧

- ⟦***Bill***⟧ denotes the set of properties that apply to Bill



Noun Phrase Interpretations

$[\![\textit{all } N]\!]^M = \{ P \subseteq U_M \mid [\![N]\!] \cap P = [\![N]\!] \}$

$[\![\textit{a } N]\!]^M = \{ P \subseteq U_M \mid [\![N]\!] \cap P \neq \emptyset \}$

$[\![\textit{not all } N]\!]^M =$

$[\![\textit{no } N]\!]^M =$

$[\![\textit{exactly } n \textit{ } N]\!]^M =$

$[\![\textit{at most } n \textit{ } N]\!]^M =$

$[\![\textit{at least } n \textit{ } N]\!]^M =$

Generalized Quantifier Theory

- What formal properties do quantifiers have?
- What natural subclasses can be distinguished?
- Which subclasses actually represent meanings of natural language noun phrases?

Negative Polarity Items

(1) a. *John **need**n't go there*
b. **John **need** go there*

(2) a. *Nobody saw **any**thing*
b. **Somebody saw **any**thing*

(3) a. *No student has **ever** been in Saarbrücken*
b. **Some student has **ever** been in Saarbrücken*

- **Negative polarity items** (any, ever, …)
⇒ items that can occur only in "negative contexts"
- **Question:** What licenses negative polarity items?

There-Sentences

(1) *There is someone in the garden*

(2) *There is no one in the garden*

(3) *There are two unicorns in the garden*

(4) **There is/are everyone in the garden*

(5) **There is John in the garden*

(6) **There are the two unicorns in the garden*

- **Question:** which noun phrases can appear in “there” sentences (and why)?
- Note: as exclamations (4) - (6) are correct

Coordination

(1) *No man and few women walked*

(2) *None of the girls and at most three boys walked*

(3) **A man and few women walked*

(4) **John and no woman saw Jane*

- **Question:** which noun phrases can be coordinated?

Inference Patterns

(1) *All men **walked rapidly*** $\models$ *All men **walked***

(2) *No man **walked*** $\models$ *No man **walked rapidly***

(3) *A girl **smoked a cigar*** $\models$ *A girl **smoked***

(4) *Few girls **smoked*** $\models$ *Few girls **smoked a cigar***

Upward Monotonicity

- *All men walked rapidly* ⊨ *All men walked*
  - Note: ⟦walked rapidly⟧ ⊆ ⟦walked⟧
- *A girl smoked a cigar* ⊨ *A girl smoked*
  - Note: ⟦smoked a cigar⟧ ⊆ ⟦smoked⟧
- **Observation:**
  A sentence [$_S$ NP VP] remains true if the denotation of the verb phrase is made “larger”

Upward Monotonicity

- A quantifier Q is **upward monotonic** in M = ⟨U, V⟩ iff Q is closed under supersets:
  - for all X, Y ⊆ U: if X ∈ Q and X ⊆ Y, then Y ∈ Q
- A noun phrase is upward monotonic if it denotes an upward monotonic quantifier.

Upward Monotonicity Tests

- **If ⟦VP_1⟧ ⊆ ⟦VP_2⟧, then NP VP_1 ⊨ NP VP_2**
  - *All men walked rapidly* ⊨ *All men walked*
  - *No man walked rapidly* ⊭ *No man walked*
  - Note: ⟦walked rapidly⟧ ⊆ ⟦walked⟧
- **NP VP_1 and VP_2 ⊨ NP VP_1 and NP VP_2**
  - *All men smoked and drank* ⊨
    All men smoked and all men drank
  - *No man smoked and drank* ⊭
    No man smoked and no man drank
  - Note: ⟦VP_1 and VP_2⟧ = ⟦VP_1⟧ ∩ ⟦VP_2⟧

Upward Monotonicity

- The set of upward monotonic quantifiers is closed under conjunction and disjunction:
  - the intersection (union) of two upward monotonic quantifiers is an upward monotonic quantifier.
- *All boys and a girl walked rapidly* ⊨ *All boys and a girl walked*
- Note:
  - $[\![NP_1 \text{ and } NP_2]\!] = [\![NP_1]\!] \cap [\![NP_2]\!]$
  - $[\![NP_1 \text{ or } NP_2]\!] = [\![NP_1]\!] \cup [\![NP_2]\!]$

Downward Monotonicity

(1) *No man walked* ⊨
No man walked rapidly

(2) *Not every woman was asleep* ⊨
Not every woman was dreaming

(3) *Less than half of the girls smoked* ⊨
Less than half of the girls smoked cigars

(4) *Few boys were playing* ⊨
Few boys were playing out on the street

Downward Monotonicity

- A quantifier Q is **downward monotonic** in $M = \langle U, V\rangle$ iff Q is closed under inclusion:
  - for all $X, Y \subseteq U$: if $X \in Q$ and $X \supseteq Y$, then $Y \in Q$
- A noun phrase is downward monotonic if it denotes a downward monotonic quantifier.

Downward Monotonicity Tests

- **If $[\![VP_1]\!] \supseteq [\![VP_2]\!]$, then NP $VP_1 \vDash$ NP VP_2**
  - *All men walked* $\nvDash$ *All men walked rapidly*
  - *No man walked* $\vDash$ *No man walked rapidly*
  - Note: $[\![\text{walked}]\!] \supseteq [\![\text{walked rapidly}]\!]$
- **NP VP_1 or $VP_2 \vDash$ NP VP_1 and NP VP_2**
  - *Neither girl was drinking or smoking* $\vDash$
    Neither girl was drinking and neither girl was smoking.
  - *All boys sing or dance* $\nvDash$
    All boys sing and all boys dance.
  - Note: $[\![VP_1 \text{ or } VP_2]\!] = [\![VP_1]\!] \cup [\![VP_2]\!]$

Negative Polarity Items

(1) a. *John needn't go there*
b. **John need go there*

(2) a. *Nobody saw anything*
b. **Somebody saw anything*

(3) a. *No student has ever been in Saarbrücken*
b. **Some student has ever been in Saarbrücken*

- ⇒ negative polarity items are licensed only in downward monotonic contexts.

Coordination

(1) *No man and few women walked*

(2) *None of the girls and at most three boys walked*

(3) **A man and few women walked*

(4) **John and no woman saw Jane*

- Non-comparative noun phrases can be coordinated iff they have the same direction of monotonicity.

Coordination

(1) **A man and few women walked*

(2) *A man but few women walked*

(3) **John and no woman saw Jane*

(4) *John but no woman saw Jane*

- Coordination with the connective “but” requires noun phrases of different direction of monotonicity.

Language Universals

- **Monotonicity Constraint** (Barwise & Cooper 1981)
  The simple noun phrases of any natural language express monotone quantifiers or conjunctions of monotone quantifiers.
- **Simple noun phrase:** Proper names or noun phrases of the form $[_{NP}$ DET N$]$

Negation of Quantifiers

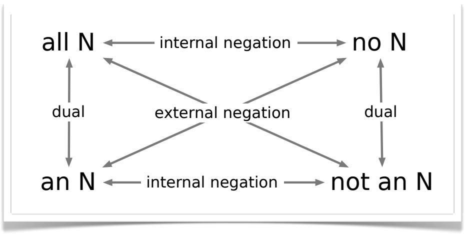
- **External negation:** $\neg Q = \{ P \subseteq U_M \mid P \notin Q \}$
  - $\neg[\![\text{all N}]\!] = \{ P \subseteq U_M \mid P \notin [\![\text{all N}]\!] \}$
    $= \{ P \subseteq U_M \mid [\![N]\!] \cap P \neq [\![N]\!] \}$
  - $\neg[\![\text{all N}]\!] = [\![\text{not all N}]\!]$
- **Internal negation:** $Q\neg = \{ P \subseteq U_M \mid (U_M - P) \in Q \}$
  - $[\![\text{all N}]\!]\neg = \{ P \subseteq U_M \mid (U_M - P) \in Q \}$
    $= \{ P \subseteq U_M \mid [\![N]\!] \cap (U_M - P) \neq \emptyset \}$
    $= \{ P \subseteq U_M \mid [\![N]\!] \cap P = \emptyset \}$
  - $[\![\text{all N}]\!]\neg = [\![\text{no N}]\!]$

Negation of Quantifiers

- If Q is an upward monotonic quantifier, then both ¬Q and Q¬ are downward monotonic.
- If Q is an downward monotonic quantifier, then both ¬Q and Q¬ are upward monotonic.

Duals

- **The dual Q* of a quantifier Q in M**
  - $Q^* = \neg Q \neg = \{ P \subseteq U_M \mid (U_M - P) \in \neg Q \}$
    $= \{ P \subseteq U_M \mid (U_M - P) \notin Q \}$.
- If Q is upward monotonic, then Q* is upward monotonic.
- If Q is downward monotonic, then Q* is downward monotonic.



Determiners

- *Every man walked* ↦ $\forall x(\text{man}'(x) \rightarrow \text{walk}'(x))$
  - *Every* ⇒ $\lambda P \lambda Q \forall x(P(x) \rightarrow Q(x))$
  - $[\![\text{Every}]\!](A)(B) = 1$ iff $A \subseteq B$
- We can consider determiners as expressions that take a noun and a verb phrase to form a sentence.
- Semantically, the interpretation of a determiner can be seen as a relation between two sets.

Persistence

- **A determiner D is persistent** in M iff for all X, Y, Z:
  - if D(X, Z) and X ⊆ Y, then D(Y, Z)
- **Persistence test:**
  - If $[\![N_1]\!] \subseteq [\![N_2]\!]$, then DET N_1 VP ⊨ DET N_2 VP
  - *Some men walked* ⊨
    Some human beings walked
  - *At least four girls were smoking* ⊨
    At least four women were smoking.

Antipersistence

- **A determiner D is antipersistent** in M iff for all X,Y,Z:
  - if D(X, Z) and Y ⊆ X, then D(Y, Z)
- **Antipersistence test:**
  - If $[\![N_2]\!] \subseteq [\![N_1]\!]$, then DET N_1 VP ⊨ DET N_2 VP
  - *All children walked* ⊨
    All toddlers walked
  - *No woman was smoking* ⊨
    No girl was smoking
  - *At most three Englishmen agreed* ⊨
    At most three Londoners agreed.

Persistence and Monotonicity

- Persistence and monotonicity are closely related:
  - Persistence (antipersistence) is upward (downward) monotonicity of the first argument.
  - Upward (downward) monotonicity of noun phrases is upward (downward) monotonicity of the second argument of the determiner in the NP.
- Terminology:
  - left-monotonicity (↑mon and ↓mon)
  - right-monotonicity (mon↑ and mon↓)

Left and Right Monotonicity

↑mon↑	some, at least n, infinitely many
↓mon↑	all
↓mon↓	no, at most n, a finite number of
↑mon↓	not all

Conservativity

- **Conservativity:** for every $A, B \subseteq U$
  - $Q(A, B) \Leftrightarrow Q(A, A \cap B)$
- **Test:** D N VP ⇔ D N are N that VP
  - *All students work* ⇔
    All students are students that work
  - *Some girls are dancing* ⇔
    Some girls are girls that are dancing
  - *Most teachers are motivated* ⇔
    Most teachers are teachers that are motivated

Lives on

- **A quantifier Q lives on** X iff for all Y,
  - $Y \in Q$ iff $X \cap Y \in Q$
- **Universal** (Barwise & Cooper 1981, cited from Gamut)
  In every natural language, simple determiners together with an N yield an NP which lives on ⟦N⟧
- **Apparent exception:** only
  - *Only men smoke cigars* ⇎
    Only men are men that smoke cigars
  - ⇒ “only” not a determiner?

Literature

- L.T.F. Gamut. Logic, Language, and Meaning. Vol 2. Chapter 7.
- Partee, ter Meulen, Wall. Mathematical Methods for Linguists. Chapter 14.
- Jon Barwise & Robin Cooper. Generalized Quantifiers. Linguistics and Philosophy. 1981.