1 Type theory: Semantics

Let U is a non-empty set of entities. For every type τ, the domain of possible denotations D_τ is given by:

- $D_e = U$
- $D_t = \{0, 1\}$
- $D_{\langle\sigma,\tau\rangle}$ is the set of functions from D_σ to D_τ.

A model structure is a pair $M = \langle U_M, V_M\rangle$ such that

- U_M is a non-empty set of individuals
- V_M is a function assigning every non-logical constant of type τ a member of D_τ.

Interpretation:

- $[\![\alpha]\!]^{M,g} = V_M(\alpha)$ if α is a constant
- $[\![\alpha]\!]^{M,g} = g(\alpha)$ if α is a variable
- $[\![\alpha(\beta)]\!]^{M,g} = [\![\alpha]\!]^{M,g}([\![\beta]\!]^{M,g})$
- $[\![\lambda v\alpha]\!]^{M,g} =$ that function $f : D_\sigma \to D_\tau$ such that for all $a \in D_\sigma$, $f(a) = [\![\alpha]\!]^{M,g[v/a]}$ (for v a variable of type σ)
- $[\![\alpha = \beta]\!]^{M,g} = 1$ iff $[\![\alpha]\!]^{M,g} = [\![\beta]\!]^{M,g}$
- $[\![\neg\phi]\!]^{M,g} = 1$ iff $[\![\phi]\!]^{M,g} = 0$
- $[\![\phi \wedge \psi]\!]^{M,g} = 1$ iff $[\![\phi]\!]^{M,g} = 1$ and $[\![\psi]\!]^{M,g} = 1$
- $[\![\phi \vee \psi]\!]^{M,g} = 1$ iff $[\![\phi]\!]^{M,g} = 1$ or $[\![\psi]\!]^{M,g} = 1$
- $[\![\phi \to \psi]\!]^{M,g} = 1$ iff $[\![\phi]\!]^{M,g} = 0$ or $[\![\psi]\!]^{M,g} = 1$
- $[\![\exists v\phi]\!]^{M,g} = 1$ iff there is an $a \in D_\tau$ such that $[\![\phi]\!]^{M,g[v/a]} = 1$ (for v a variable of type τ)
- $[\![\forall v\phi]\!]^{M,g} = 1$ iff for all $a \in D_\tau$, $[\![\phi]\!]^{M,g[x/a]} = 1$ (for v a variable of type τ)

2 Cooper Storage

Transitive verbs are analysed as constants of type $\langle\langle\langle e,t\rangle,t\rangle,\langle e,t\rangle\rangle$.

Storage: $\langle Q, \Delta\rangle \Rightarrow_S \langle\lambda P.P(x_i), \Delta \cup \{Q_i\}\rangle$

if A is an noun phrase whose semantic value is $\langle Q, \Delta\rangle$, then $\langle\lambda P.P(x_i), \Delta \cup \{Q_i\}\rangle$ is also a semantic value for A, where $i \in N$ is a new index.

Retrieval: $\langle\alpha, \Delta \cup \{Q_i\}\rangle \Rightarrow_R \langle Q(\lambda x_i.\alpha), \Delta\rangle$

if A is any sentence with semantic value $\langle\lambda\alpha, \Delta \cup \{Q_i\}\rangle$, then $\langle Q(\lambda x_i.\alpha), \Delta\rangle$ is also a semantic value for A.

3 Nested Cooper Storage

Storage: $\langle Q, \Delta \rangle \Rightarrow_S \langle \lambda P.P(x_i), \{\langle Q, \Delta \rangle_i\} \rangle$

if A is an noun phrase whose semantic value is $\langle Q, \Delta \rangle$, then $\langle \lambda P.P(x_i), \{\langle Q, \Delta \rangle_i\} \rangle$ is also a semantic value for A, where $i \in N$ is a new index.

Retrieval: $\langle \alpha, \Delta \cup \{\langle Q, \Gamma \rangle_i\} \rangle \Rightarrow_R \langle Q(\lambda x_i.\alpha), \Delta \cup \Gamma \rangle$

if A is any sentence with semantic value $\langle \alpha, \Delta \cup \{\langle Q, \Gamma \rangle_i\} \rangle$, then $\langle Q(\lambda x_i.\alpha), \Delta \cup \Gamma \rangle$ is also a semantic value for A.

4 DRT: Syntax

A discourse representation structure (DRS) K is a pair $\langle U_K, C_K \rangle$ where

- U_K is a set of discourse referents
- C_K is a set of conditions.

Conditions:

$R(u_1, \ldots, u_n)$	R is an n-place relation, $u_i \in U_K$
$u = v$	$u, v \in U_K$
$u = a$	$u \in U_K$, a a proper name
$K_1 \Rightarrow K_2$	K_1 and K_2 DRSs
$K_1 \vee K_2$	K_1 and K_2 DRSs
$\neg K_1$	K_1 is a DRS

5 DRT: Embedding, verifying embedding

Let U_D be a set of discourse referents, $K = \langle U_K, C_K \rangle$ a DRS with $U_K \subseteq U_D$, $M = \langle U_M, V_M \rangle$ a model structure of first-order predicate logic that is suitable for K. An *embedding* of U_D into M is a (partial) function from U_D to U_M that assigns individuals from U_M to discourse referents.

An embedding f *verifies* the DRS K in M ($f \models_M K$) iff

1. $U_K \subseteq \mathrm{Dom}(f)$ and
2. f verifies each condition $\alpha \in C_K$.

f verifies a condition α in M ($f \models_M \alpha$) in the following cases:

$f \models_M R(u_1, \ldots, u_n)$	iff $\langle f(u_1), \ldots, f(u_n) \rangle \in V_M(R)$
$f \models_M u = v$	iff $f(u) = f(v)$
$f \models_M u = a$	iff $f(u) = V_M(a)$
$f \models_M K_1 \Rightarrow K_2$	iff for all $g \supseteq_{U_{K_1}} f$ such that $g \models_M K_1$, there is $h \supseteq_{U_{K_2}} g$ such that $h \models_M K_2$
$f \models_M \neg K_1$	iff there is no $g \supseteq_{U_{K_1}} f$ such that $g \models_M K_1$
$f \models_M K_1 \vee K_2$	iff there is a $g_1 \supseteq_{U_{K_1}} f$ such that $g_1 \models_M K_1$, or there is a $g_2 \supseteq_{U_{K_2}} f$ such that $g_2 \models_M K_2$.