# Semantic Theory: Discourse Representation Theory II

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## DRT: Denotational Interpretation

- Let
  - $U_D$ a set of discourse referents,
  - $K = \langle U_K, C_K \rangle$ a DRS with $U_K \subseteq U_D$,
  - $M = \langle U_M, V_M \rangle$ a FOL model structure appropriate for K.
- An *embedding* of K into M is a (partial) function f from $U_D$ to $U_M$ such that $U_K \subseteq$ Dom(f).



## Verifying embedding

- An embedding f of K in M verifies K in M:
  $f \models_M K$ iff f verifies every condition $\alpha \in C_K$.
- f verifies condition $\alpha$ in M ($f \models_M \alpha$):
  - (i) $f \models_M R(x_1,\ldots, x_n)$ iff $\langle f(x_1), \ldots , f(x_n)\rangle \in V_M(R)$
  - (ii) $f \models_M x = a$ iff $f(x) = V_M(a)$
  - (iii) $f \models_M x = y$ iff $f(x) = f(y)$



## Example Computation

Let K be the example DRS from above:

K = < {x, y, z, u},
{professor(x), book(y), own(x,y), read(z,u), z=x, u=y} >

$f \models_M K$ iff f verifies every condition $\alpha \in C_K$, i.e.:
$f \models_M$ professor(x) ∧ $f \models_M$ book(y) $f \models_M$ ∧ own(x,y) ∧
$f \models_M$ read(z,u) ∧ $f \models_M$ z=x ∧ $f \models_M$ u=y
which holds iff:
$f(x) \in V_M(\text{professor}) \wedge f(y) \in V_M(\text{book}) \wedge \langle f(x), f(y)\rangle \in V_M(\text{own}) \wedge$
$\langle f(z), f(u)\rangle \in V_M(\text{read}) \wedge f(z)=f(x) \wedge f(u)=f(y)$

## Simplification

$f \models_M K$ iff

$f(x) \in V_M(\text{professor}) \wedge f(y) \in V_M(\text{book}) \wedge \langle f(x), f(y)\rangle \in V_M(\text{own}) \wedge$
$\quad \langle f(z), f(u)\rangle \in V_M(\text{read}) \wedge f(z) = f(x) \wedge f(u) = f(y)$

iff

$f(x) \in V_M(\text{professor}) \wedge f(y) \in V_M(\text{book}) \wedge \langle f(x), f(y)\rangle \in V_M(\text{own}) \wedge$
$\quad \langle f(x), f(u)\rangle \in V_M(\text{read}) \wedge f(u) = f(y)$

iff

$f(x) \in V_M(\text{professor}) \wedge f(y) \in V_M(\text{book}) \wedge \langle f(x), f(y)\rangle \in V_M(\text{own}) \wedge$
$\quad \langle f(x), f(y)\rangle \in V_M(\text{read})$



## Truth

- Let K be a closed DRS and M be an appropriate model structure for K.

  K is true in M iff there is a verifying embedding f of K in M such that $Dom(f) = U_K$
- Let D be a discourse/text, K a DRS that can be constructed from D.

  D is true with respect to K in M iff K is true in M.
- Let D be a discourse/text, which is true with respect to all DRSes that can be consructed from D:

  D is true in M iff D is true with respect to all DRSes that can be constructed from D.



## DRS: Computation of truth conditions

- Compute conditions for verifying embedding.
- Simplify.
- Specify truth, based on (simplified) conditions for verifying embedding.



## Basic features of DRT

- DRT models linguistic meaning as anaphoric potential (through DRS construction) plus truth conditions (through model embedding).
- In particular, DRT explains the ambivalent character of indefinite NPs: Expressions that introduce new reference objects into context, and are truth conditionally equivalent to existential quantifiers.

## Translation of DRSes to FOL

- DRS K = $\langle\{x_1, ..., x_n\}, \{c_1, ..., c_k\}\rangle$

| $x_1 \ . \ . \ . \ x_n$ |
| --- |
| $c_1 \ . \ . \ . \ c_n$ |

is truth-conditionally equivalent to the following FOL formula:

$\exists x_1 ... \exists x_n [c_1 \wedge ... \wedge c_k]$



## DRT II: Extensions

- Conditionals, indefinites and anaphora
- Complex conditions
- Accessibility



## Indefinite NPs and conditionals

Indefinite NPs and conditional clauses:

- *If a student works, the professor is happy.*
  (1) $\exists x[\text{student}(x) \wedge \text{work}(x)] \rightarrow \text{happy\_prof}$
  (2) $\forall x[\text{student}(x) \wedge \text{work}(x) \rightarrow \text{happy\_prof}]$
- Formulas (1) and (2) are logically equivalent:
  $\exists xA \rightarrow B \Leftrightarrow \forall x[A \rightarrow B]$
  given that x doesn't occur free in B.



## Indefinite NPs, Conditionals, and Anaphora

- *If a student works, he will be successful.*
  (1) $\exists x[\text{student}(x) \wedge \text{work}(x)] \rightarrow \text{successful}(x)$
  (2) $\exists x[\text{student}(x) \wedge \text{work}(x) \rightarrow \text{successful}(x)]$
  (3) $\forall x\ [\text{student}(x) \wedge \text{work}(x) \rightarrow \text{successful}(x)]$
  (1) is not closed
  (2) has wrong truth conditions (much too weak)
  (3) is correct, but how do you derive this compositionally?
- This is called the donkey sentence problem, with reference to the classical example by P.T. Geach (1967): *If a farmer owns a donkey, he beats it.*

## Indefinite NPs and Discourse Structure

- *A car is parked in front of Peter's garage. Peter needs to get to the office quickly. He doesn't know who owns the car. He calls the police, and it is towed away.*
- *Suppose a car is parked in front of Peter's garage. Peter needs to get to the office quickly. He doesn't know who owns the car. Then he will call the police, and it will be towed away.*
- *Let a and b be two positive integers. Let b further be even. Then the product of a and b is even too.*



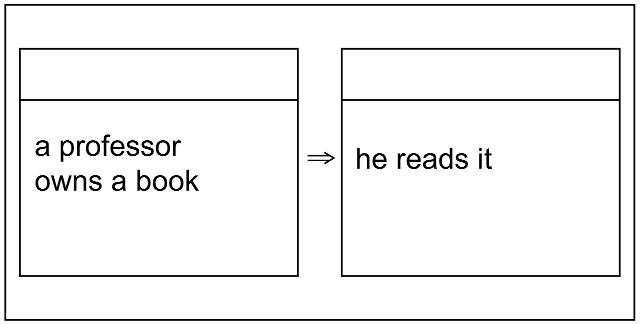
## Context-dependent interpretation of indefinites

- The „quantificational force“ of indefinites depends on context:
  - Existential in plain assertions and narrative contexts
  - Universal in conditional or hypothetical reasoning.
- DRT offers uniform treatment in DRS construction, different truth conditional interpretation induced is by the respective context.



## DRS for conditionals: An example

- *If a professor owns a book, he reads it.*



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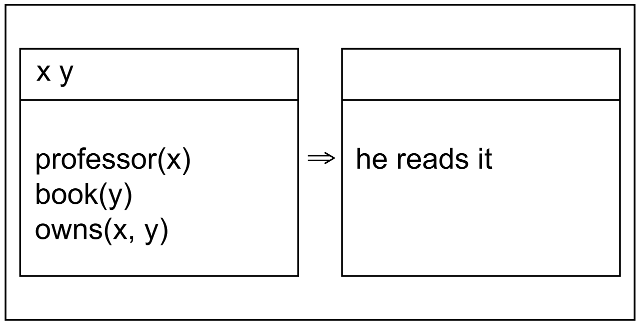


# DRS for conditionals: An example

- *If a professor owns a book, he reads it.*

| x y | | z v |
|---|---|---|
| professor(x)<br>book(y)<br>owns(x, y) | $\Rightarrow$ | reads(z, v)<br>z = x<br>v = y |



# DRS (1st Extension)

- A discourse representation structure (DRS) K is a pair $\langle U_K, C_K \rangle$, where
  - $U_K$ is a set of discourse referents
  - $C_K$ is a set of conditions
- (Irreducible) conditions:
  - $R(u_1, \ldots, u_n)$ — R n-place relation, $u_i \in U_K$
  - $u = v$ — $u, v \in U_K$
  - $u = a$ — $u \in U_K$, a is a proper name
  - $K_1 \Rightarrow K_2$ — $K_1$ and $K_2$ DRSes
- Reducible conditions: as before



# DRS Construction Rule for Conditionals

- Triggering configuration:
  - $\alpha$ is a reducible condition in DRS K of the form $[_S \text{ if } [_S\ \beta] \text{ (then) } [_S\ \gamma]]$
- Action:
  - Remove $\alpha$ from $C_K$ .
  - Add $K_1 \Rightarrow K_2$ to $C_K$, where
    - $K_1 = \langle \varnothing, \{ \beta \} \rangle$ and
    - $K_2 = \langle \varnothing, \{ \gamma \} \rangle$
- Remark: $K_1 \Rightarrow K_2$ is called a duplex condition; $K_1$ the "antecedent DRS" and $K_2$ the "consequent DRS".



# Recap: DRT Embeddings

- Let
  - $U_D$ a set of discourse referents,
  - $K = \langle U_K, C_K \rangle$ a DRS with $U_K \subseteq U_D$,
  - $M = \langle U_M, V_M \rangle$ an FOL model structure appropriate for K.
- An *embedding* of K into M is a (partial) function f from $U_D$ to $U_M$ such that $U_K \subseteq \mathrm{Dom}(f)$.

## Verifying embeddings (1st extension, preliminary)

- An embedding f of K into M verifies K in M:

  $f \models_M K$ iff f verifies every condition $\alpha \in C_K$.
- f verifies condition $\alpha$ in M ($f \models_M \alpha$):

  (i) $f \models_M R(x_1,\ldots, x_n)$ iff $\langle f(x_1), \ldots , f(x_n)\rangle \in V_M(R)$

  (ii) $f \models_M x = a$ iff $f(x) = V_M(a)$

  (iii) $f \models_M x = y$ iff $f(x) = f(y)$

  (iv) $f \models_M K_1 \Rightarrow K_2$ iff for all $g \supseteq f$ s.t. $Dom(g) = Dom(f) \cup U_{K_1}$ and $g \models_M K_1$, we also have $g \models_M K_2$



## Notation: Extending embeddings

Let f, g be partial functions (embeddings) on $U_D$;

$U \subseteq U_D$ ; $x, y \in U_D$

We write

– $f \supseteq_U g$ for "$f \supseteq g$ and $Dom(f) = Dom(g) \cup U$"

– $f \supseteq_x g$ for "$f \supseteq_{\{x\}} g$".

So we can write (iv) as follows:

(iv) $f \models_M K_1 \Rightarrow K_2$ iff for all $g \supseteq_{U_{K_1}} f$ s.t. $g \models_M K_1$, we have $g \models_M K_2$



## The definition seems to work ...

- *If a professor owns a book, he reads it.*

K1:

K2:

x y

professor(x)
book(y)
owns(x, y)

⇒

K3:

z v

reads(z, v)
z = x
v = y



## ... but it doesn't really!

A slightly more complex example:

- *Mary knows a professor.*
  *If he owns a book, he gives it to a student.*

s u

s = Mary professor(u)
Know(s, u)

x y

x = u
book (y)
owns (x, y)

⇒

z v w

gives(z,v,w)
z = x
v = y
student(w)

## Verifying embeddings for conditionals (final)

- An embedding f of K into M verifies K in M:

  $f \models_M K$ iff f verifies every condition $\alpha \in C_K$.

- f verifies condition $\alpha$ in M ($f \models_M \alpha$):

  (i) $f \models_M R(x_1,\ldots, x_n)$ iff $\langle f(x_1), \ldots , f(x_n)\rangle \in V_M(R)$

  (ii) $f \models_M x = a$ iff $f(x) = V_M(a)$

  (iii) $f \models_M x = y$ iff $f(x) = f(y)$

  (iv) $f \models_M K_1 \Rightarrow K_2$ iff for all $g \supseteq_{U_{K_1}} f$ s.t. $g \models_M K_1$ there is a $h \supseteq_{U_{K_2}} g$ s.t. $h \models_M K_2$



## DRS construction rule for universal NPs

- Triggering configuration:
  - $\alpha$ is a reducible condition in DRS K; $\alpha$ contains a subtree $[_S [_{NP}\ \beta] [_{VP}\ \gamma]]$ or $[_{VP} [_V\ \gamma] [_{NP}\ \beta]]$
  - $\beta$ = every $\delta$
- Action:
  - Remove $\alpha$ from $C_K$.
  - Add $K_1 \Rightarrow K_2$ to $C_K$, where
    - $K_1 = \langle\{x\}, \{\delta(x)\}\rangle$ and
    - $K_2 = \langle\varnothing, \{\alpha'\}\rangle$
    - obtain $\alpha'$ from $\alpha$ by replacing $\beta$ by x



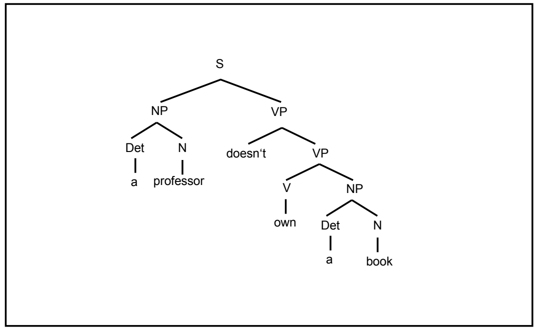
## DRS construction rule for negations

- Triggering configuration:
  - $\alpha$ is a reducible condition in DRS K of the form $[_S\ \beta\ [_{VP}$ doesn't $[_{VP}\ \gamma]]]$
- Action:
  - Remove $\alpha$ from $C_K$ .
  - Add $\neg K_1$ to $C_K$, where $K_1 = \langle\varnothing, \{[_S\ \beta\ [_{VP}\ \gamma]]\}\rangle$,



## Example

- *A professor doesn't own a book.*



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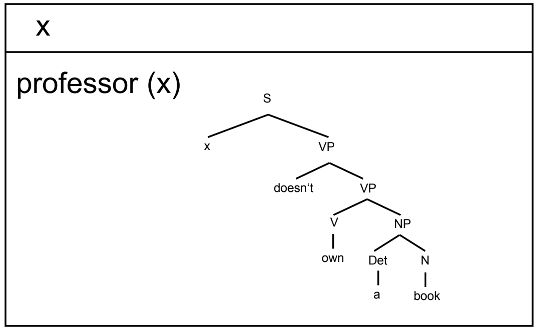




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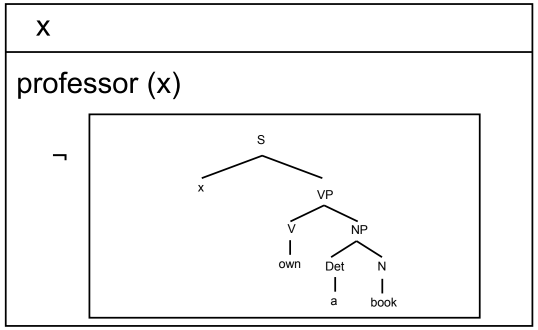





## Example

- *A professor doesn't own a book.*

x

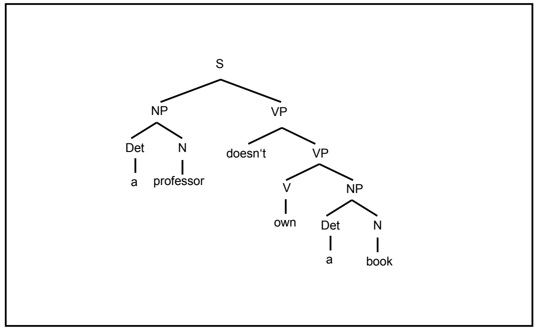
professor (x)

¬ [ y | book(y), owns(x, y) ]



## Example: A second reading

- *A professor doesn't own a book.*



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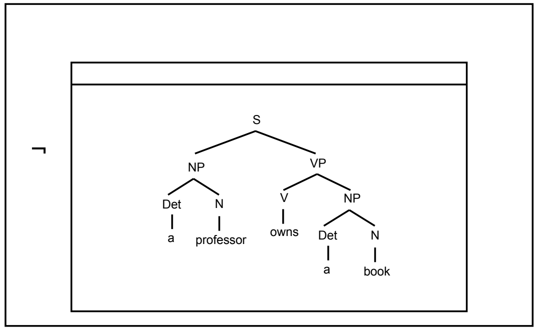
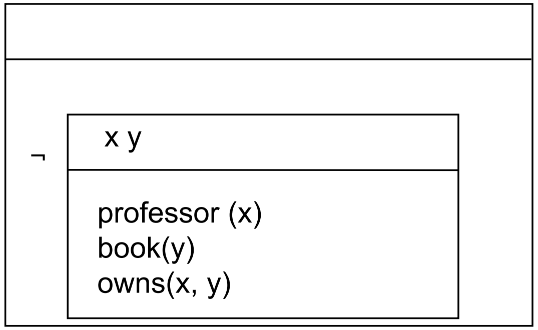





## Example: A second reading

- *A professor doesn't own a book.*





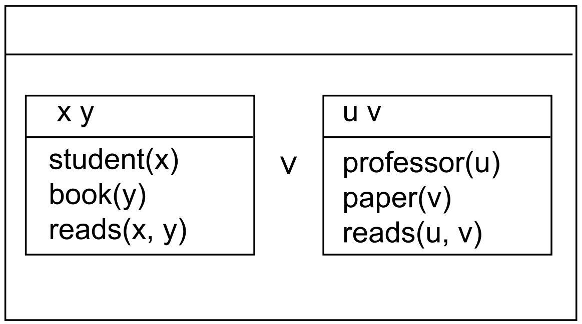

## DRS construction rule for clausal disjunction

- Triggering configuration:
  - $\alpha$ is a reducible condition in DRS K of the form $[_S [_S \beta]$ or $[_S \gamma]]$
- Action:
  - Remove $\alpha$ from $C_K$ .
  - Add $K_1 \vee K_2$ to $C_K$, where
    - $K_1 = \langle \emptyset, \{\beta\} \rangle$ and
    - $K_2 = \langle \emptyset, \{\gamma\} \rangle$



## An example

- *A student reads a book, or a professor reads a paper.*



# DRS (2nd Extension)

- A discourse representation structure (DRS) K is a pair $\langle U_K, C_K \rangle$, where
  - $U_K$ is a set of discourse referents
  - $C_K$ is a set of conditions
- (Irreducible) conditions:
  - $R(u_1, \ldots, u_n)$ — R n-place relation, $u_i \in U_K$
  - $u = v$ — $u, v \in U_K$
  - $u = a$ — $u \in U_K$, a is a proper name
  - $K_1 \Rightarrow K_2$ — $K_1$ and $K_2$ DRSs
  - $K_1 \vee K_2$ — $K_1$ und $K_2$ DRSs
  - $\neg K_1$ — $K_1$ DRS



# Verifying embeddings

- f verifies condition $\alpha$ in M ($f \models_M \alpha$):
  - (i) $f \models_M R(x_1,\ldots, x_n)$ iff $\langle f(x_1), \ldots, f(x_n)\rangle \in V_M(R)$
  - (ii) $f \models_M x = a$ iff $f(x) = V_M(a)$
  - (iii) $f \models_M x = y$ iff $f(x) = f(y)$
  - (iv) $f \models_M K_1 \Rightarrow K_2$ iff for all $g \supseteq_{U_{K_1}} f$ s.t. $g \models_M K_1$ there is a $h \supseteq_{U_{K_2}} g$ s.t. $h \models_M K_2$
  - (v) $f \models_M \neg K_1$ iff there is no $g \supseteq_{U_{K_1}} f$ s.t. $g \models_M K_1$
  - (vi) $f \models_M K_1 \vee K_2$ iff there is a $g_1 \supseteq_{U_{K_1}} f$ s.t. $g_1 \models_M K_1$ or there is a $g_2 \supseteq_{U_{K_2}} f$ s.t. $g_2 \models_M K_2$



# Translation from DRT to FOL

- DRSs

  $T(\langle\{u_1, \ldots, u_n\}, \{c_1, \ldots, c_n\}\rangle) = \exists u_1 \ldots \exists u_n[T(c_1) \wedge \ldots \wedge T(c_n)]$
- Conditions:

  $T(c) = c$ for atomic conditions c

  $T(\neg K_1) = \neg T(K_1)$

  $T(K_1 \vee K_2) = T(K_1) \vee T(K_2)$

  $T(K_1 \Rightarrow K_2) = \forall u_1 \ldots \forall u_n[(T(c_1) \wedge \ldots \wedge T(c_n)) \rightarrow T(K_2)]$,

  for $K_1 = \langle\{u_1, \ldots, u_n\}, \{c_1, \ldots, c_n\}\rangle$
- For every closed DRS K and every appropriate model M, K is true in M iff T(K) is true in M.